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The Probabilistic Heuristic Justification for the Goldbach's Strong Conjecture

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Abstract: *This paper doesn't provide any proper solution to the Goldbach's conjecture. Here we have demonstrated a probabilistic heuristic justification for the Goldbach's strong conjecture . In this paper we have used the prime number theorem to make a guess that every even integer has at least one representation as the sum of two primes. We have also made a guess that every large even integer has not just one representation as the sum of two primes but in fact has many such representations*

Keywords: *Number theory, proof of the Goldbach's conjecture, heuristic justification, unsolved problems.*

1. Introduction

The most well-known conjecture in additive number theory is the strong Goldbach's conjecture, which we formulate as follows:

Conjecture 1: *Every even integer n greater than 2 can be expressed as the sum of two primes.*

Goldbach's conjecture is one of the oldest unsolved problems in number theory. On 7 June 1742, the Prussian mathematician Christian Goldbach wrote a letter to Leonhard Euler in which he suggested the following conjecture, which would later be called Goldbach's strong conjecture. The conjecture has been shown to hold for all even integers less than 4×10^{18}, but remains unproven despite considerable effort. In this paper we didn't try to prove this conjecture. We have just shown to possibilities that the Goldbach's strong conjecture maybe true.

2. Main Results

Firstly we have shown that at least how many odd numbers are needed to express all the even numbers from 6 to n. Then we have shown that the number of odd numbers is less than the number of odd prime numbers less than n. As we need a few odd numbers to express all the even numbers from 6 to n. So there may be a possibility to express all the even numbers from 6 to n by using the prime numbers where the number of primes is greater than the odd numbers we need.

Secondly, by using the prime number theorem we have show that the total number of ways to write a large even integer n as the sum of two primes to be roughly

$$\frac{\frac{n}{2} - ln\left(\frac{n}{2}\right)}{ln(n) ln\left(\frac{n}{2}\right)}$$

The lowest number of odd numbers to express all the even numbers from 6 to n

There is no such a formula which can help us to calculate the accurate number of odd numbers (lowest) to express all the even numbers from 6 to n. Now look at the table below where we will take some value of n and calculate the lowest number of odd numbers and the number of odd primes less than n.

Value of n	The odd numbers we need to express all the even numbers from 6 to n (at least)	Number of odd numbers	Number of odd primes less than n
10	3,5	2	3

20	3,5,9,11	4	7
30	3,5,9,11,15,25	6	9
40	3,5,9,11,15,25,27	7	11
50	3,5,9,13,15,29,31,45	8	14
60	3,5,9,13,15,29,31,45,47	9	16
70	3,5,9,13,15,29,33,35,49,57	10	18
80	3,5,9,13,15,29,33,37,51,63	10	21
90	3,5,9,13,15,29,33,37,51,53, 63	11	23
100	3,5,9,13,15,29,33,37,51,63,77,89	12	24

Here we can see as we increase the value of n, the difference between the number of lowest odd numbers to express all the even numbers from 6 to n and the number of odd primes less than n is getting bigger. Now we have to put these data on a graph.

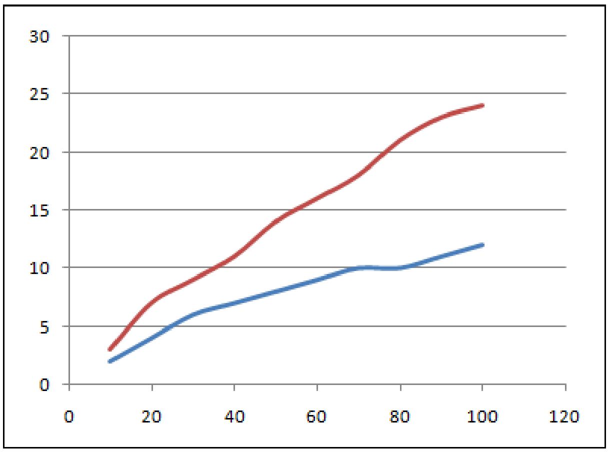

Figure1. *Here the blue line graph stands for the lowest number of odd numbers to express all the even numbers from 6 to n and the red line graph stands for the number of odd primes less than n.*

Here we can see if we increase the value of n the distance between those two graphs also increases. Again, if the lowest number of odd numbers we need to express all the even numbers from 6 to n becomes greater than the number of primes less than n, then it won't be possible to express all the even numbers as the sum of two primes up to n. As we know, the Goldbach's conjecture has been shown to hold for all even integers less than 4×10^{18}. That's why the number of lowest odd numbers to express all the even numbers from 6 to any even integer n less than 4×10^{18} is less than the number of primes less than n.

When we are calculating the lowest number of odd numbers to express all the even numbers from 6 to n, we are allowed to take any odd number. For this we have to take those odd numbers which will help us to express most of the even numbers from 6 to n. That's why we need a few odd numbers to express all the even numbers from 6 to n. So, there is a huge chance that the number of lowest odd numbers will be always less than the number of primes less than n.

We know, all prime numbers except 2 are odd. As we need a few odd numbers to express all the even numbers from 6 to n. So there may be a possibility to express all the even numbers from 6 to n by using the prime numbers where the number of primes is greater than the odd numbers we need. Here we didn't prove anything. We just made a guess.

Probabilistic heuristic justification for Goldbach's conjecture

In number theory, the prime number theorem (PNT) describes the asymptotic distribution of the prime numbers among the positive integers. The prime number theorem gives an asymptotic form for the prime counting function $\pi(n)$, which counts the number of primes less than any integer n. According to the prime number theorem the number of primes less than any integer n is approximately $\frac{n}{ln(n)}$.

$$\pi(n) \sim \frac{n}{ln(n)}$$

We know all primes except 2 are odd. By using the above prime counting function we can write that the number of odd primes less than any integer n is approximately

$$\frac{n}{ln(n)} - 1$$

The prime number theorem asserts that any integer less than n has roughly a $\frac{1}{ln(n)}$ chance of being prime.

2 is only one even prime. So, if we add 2 with any other prime we will get an odd number. For this the only way we can use 2 to represent an even number as the sum of two primes is 2+2=4. So we can rewrite the goldbach's conjecture that every even integer greater that 4 can be expressed as the sum of two primes.

Suppose, n is an even integer greater than 4. If we can express n as the sum of two odd primes, then one prime must be less than or equal to n/2 and another prime must be greater than or equal to n/2. Now, what is the number of odd primes less than or equal to n/2? We know the number is approximately, $\frac{n/2}{ln(n/2)} - 1$.

Suppose, the primes numbers are,

$$p_1, p_2, p_3, \ldots\ldots\ldots\ldots, p_{\frac{n/2}{ln(n/2)}-1}$$

Again, suppose,

$$p_1 + q_1 = n$$

$$p_2 + q_2 = n$$

$$p_3 + q_3 = n$$

……………………

$$p_{\frac{n/2}{ln(n/2)}-1} + q_{\frac{n/2}{ln(n/2)}-1} = n$$

If there is at least one prime between q_1 to $q_{\frac{n/2}{ln(n/2)}-1}$,then n can be obviously expressed as the sum of two primes. But it is not possible to say is there any prime between q_1 to $q_{\frac{n/2}{ln(n/2)}-1}$ or not. Now consider the two numbers used in every sum as a pair. So, the pairs will be

$$(p_1, q_1), (p_2, q_2), (p_3, q_3), \ldots\ldots, \left(p_{\frac{n/2}{ln(n/2)}-1}, q_{\frac{n/2}{ln(n/2)}-1}\right)$$

We know, the first number of every pair is a prime number. So, the probability of the first number of every pair is 1. Again, the second number of every pair is greater than or equal to n/2 and less than n.

$$\frac{n}{2} \le q_1, q_2, q_3, \ldots\ldots\ldots\ldots, q_{\frac{n/2}{ln(n/2)}-1} < n$$

As we know the prime number theorem asserts that any integer less than n has roughly a $\frac{1}{ln(n)}$ chance of being prime. So, the probability of the second number of every pair being prime to be roughly $\frac{1}{ln(n)}$.

Now, the probability of the both numbers of every pair simultaneously being prime to be $1 \times \frac{1}{ln(n)}$.

So, the total number of ways to write a large even integer n as the sum of two odd primes to be roughly

$$= \left(\frac{n/2}{ln(n/2)} - 1\right) \times \frac{1}{ln(n)}$$

$$= \frac{\frac{n}{2} - ln\left(\frac{n}{2}\right)}{ln(n)ln\left(\frac{n}{2}\right)}$$

Now, suppose, $f(n) = \frac{\frac{n}{2} - ln\left(\frac{n}{2}\right)}{ln(n)ln\left(\frac{n}{2}\right)}$

If $f(n) \geq 1$, then there may be a possibility of being true of the goldbach's conjecture.

Now, look at the graph below,

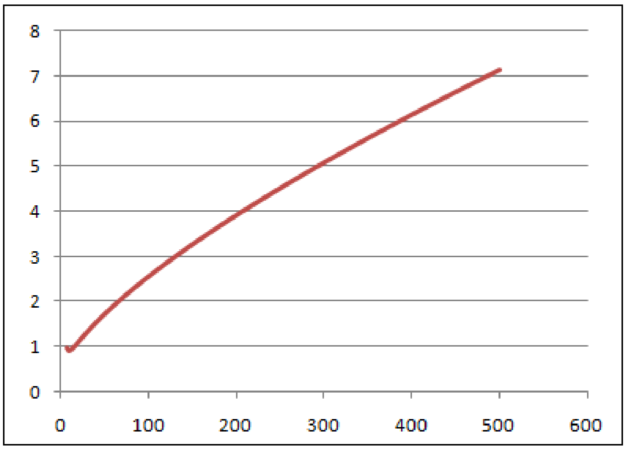

Figure2. *The line graph stands for the f(n) for the value of n from 6 to 500.*

This graph shows that for a large value of n, $f(n) > 1$. This means every large even integer has not just one representation as the sum of two primes but in fact has many such representations.

3. Conclusion

In this paper we have just shown a probabilistic heuristic justification of the goldbach's strong conjecture. This paper only tells us the possibility of being true of the goldbach's conjecture.

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Author's Biography

Salman Mahmud, studying at BIAM Model School and College, Bogura, Bangladesh. My core research interest is in modern physics and pure mathematics. I like to spend time with my father (MD. Shah Alam) and my mother (MST. Sabina Yesmin). I also like to quarrel with my younger brother (Suyaib Sadik)!

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