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An introduction of a novel group theorem to Abstract Algebra

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Abstract: *This paper proposes a group theorem in Abstract Algebra which is aimed at interestingly showcasing an undiscovered Mathematical concept, using the concept of Euler's Phi Function. The theorem states that: "for all n≥3 'n' divides the sum of all elements of U (n), where is the set of all positive integers less than 'n' and it is relatively prime to 'n'.". We also presented a logical proof to the theorem with a few examples in the fora clear understanding of the concept.*

Keywords: *Groups, Abstract Algebra, Euler's Phi Function*

1. Introduction

The branch of Mathematics 'Abstract Algebra' consist of a part called 'Group Theory' whose small part describes the set $U(n)$ "The set of all integers less than n and is relatively prime to n such that for all, $k \in U(n) g.c.d.(k,n) = 1$ where n belongs to the set of positive integers, that is, $n \in \boldsymbol{Z}^+$. $U(n)$ is a group under multiplication modulo 'n'.The group of units U(n) is a common groupstudied in an introductory Abstract Algebra class. As far as the above concepts it is easy to verify that one may choose any $n \in N$ or any $n \in \boldsymbol{Z}^+$ and $U(n)$ will be a group. However, in this concept, we shall deal with only those values of n which are greater than or equal to 3. But the values $n = 1$ *and* 2 are neglected because this leads to the contradiction of the theorem. Hence, the theorem is valid for only $n \geq 3$. However, the work of [1] contains the example which gives more information about the set $U(n)$. The information about Euler Phi function was also considered in a corollary. More information regarding group theory can be found [2, 3, 4].

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1.1. Definition

If $U(n)$ is the set of all prime numbers less than n.
In general, the sum of all elements of $U(n)$ (or sum of all prime factors of n) is given by

$$\sum_{n=1}^{p-1} U(n) = \frac{n}{2}(n-1), \qquad where\ n\ are\ prime\ and\ n > 2.$$

---------------- (1)

1.2. Remark

Note that these elements of $\sum_{n=1}^{p-1} U(n)$are $3, 5, 7, 9, \ldots, n-1$

1.3. Does the sum of prime factors of n divide n

In $U(n)$ for instance,

for $n = 3, we\ have$
$U(3) = 3\ which\ is\ divisible\ by\ n = 3$. That is $3\ |3 = 1 + 2$ . . . . (2)

For $n = 4, we\ have$
$U(4) = 4\ which\ is\ divisible\ by\ n = 4. That\ is\ 4|4 = 1 + 3$ . . . . (3)

For $n = 5, we\ have$
$U(5) = 10\ which\ is\ divisible\ by\ n = 5$. That is $5\ |10 = 1 + 2 + 3 + 4$ . . . . (4)

For $n = 6, we\ have$

$U(6) = 6 \, which \, is \, divisible \, by \, n = 6$. That is $6|6 = 1 + 5$ . . . . (5)

For $n = 7, we \, have$
$U(7) = 21 \, which \, is \, divisible \, by \, n = 7$. That is $7 \mid 21 = 1 + 2 + 3 + 4 + 5 + 6$ .. . . (6)

$$U(k_j) = \sum_{i=1}^{j-1} k_i$$

Which is divisible by k_j. That is

$$k_j \; divides \; U(k_j) = \sum_{i=1}^{j-1} k_i$$

.. . . (7)

j (which is the number of elements in the set) has to be even for the division to be possible:To see how true this is, notice that for the equations $(2), (3), (4), (5)$ and (6), we have that j are respectively $2, 2, 4, 2$ and 6 which are all even.

Hence, for all $n \geq 3$ 'n' divides the sum of all elements of $U(n)$.

2. Theorem

"If n is an integers, the sum of the prime factors of n divides n

Proof:

The proof is already as given in the illustration of section 1.3

(Proved)

3. Conclusion

The paper is based on the principle that the sum of all the elements of the set $U(n)$ is divisible by n if and only if $n \geq 3$ and n belongs to the set of all positive integers

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