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Using the Least Squares Method with Five Points to Solve Algebraic Equations Nonlinear

Nasr Al Din IDE[1*], Sundus Naji Al Aziz[2]

[1]*Aleppo University-Faculty of Science-Department of Mathematics, Syria*

[2]*Princess Nourah bint Abdulrahman University -Faculty of Science-Department of Mathematics, Riyadh, KSA*

Corresponding Author:** ***Nasr Al Din IDE, *Aleppo University-Faculty of Science-Department of Mathematics, Syria*

Abstract: *In this paper, we introduce an extension of previous research to estimate the parameters of nonlinear algebraic models using the least square method to find the roots of nonlinear algebraic equations that is an important problem in science and engineering later many methods have developed for solving nonlinear equations. The Least square method for five points is used to find the present methods (PM). We applied a number some examples and numerical results obtained show that the present method is faster or slowest than the other methods with the higher degrees of least square polynomial.*

Keywords: *Nonlinear equation; least square method; Newton's method; five points.*

1. Introduction

Finding root of non-linear equation $f(x)=0$, is a classical problem in numerical analysis which arise in many scientific and engineering fields [1]. Newton's method is the most well-known method for solving nonlinear equations. Various numerical methods have been developed using different techniques including finite differences [1-4], quadrature rules ,QiaolingXue, JianZhu [5], Nenad Ujevic 2006[6], Taylor's series, decomposition methods, homotopy techniques, Newton theorem (Nasr Al Din IDE, 2013), (Shijun Liao., 1997[7]) etc., in order to carry out the solution of non-linear equations with different convergence rates. Most commonly used numerical methods for root location of non-linear equations includes, Bisection/interval halving method, Regula-falsi/false position method.

Nonlinear Regression Method and several another methods see for example [2-30].Here we describe a new method by using least square method as a polynomial form of degree five.

The goal: is identify the coefficients ai's such that f(x) fits the data well:

$$f(x)=0 \tag{1}$$

where, f denotes a continuously differentiable function on $[a, b] \subset \mathscr{R}$, and has at least one root α, in [a, b] Such as Newton's Method, Bisection method, Regula Falsi method, Nonlinear Regression Method and several another methods see for example [10-30]. Here we describe a new method by using Least square method as a polynomial form a second degree and more than two (third, fourth and fifth degrees(PM)), then we find that, this procedure lead us to the root α of equation (1). Some test examples given to show the efficiency of the proposed methods and compared the results of these examples of present methods. The comparison with the famous methods of classical Newton's method (NM) [12] , Nasr Al Din IDE [2], Hou [19], New Eighth higher and Sixteenth-order iterative methods given by Rafiullah (R1)[9], the numerical results obtained show that the present method is faster than the other methods.

2. Proposed Method

In the first, we fitted a polynomial function by estimating the parameters using the least squares method to eliminate complex nonlinear functions to arrive at a solution for nonlinear equations. The general problem of fitting the best least squares line to a collection of data (xi,yi) minimizing the total error.

3. Algorithm

The present method has 6 steps:

- Take $[a, b]$ is an initial interval, which has at least a root in this interval.
- Compute ((x1), f (x1)), ((X2), f (X2)) , ((X3), f(X3),), ((X4), f(X4),), ((X5), f(X5)), and solve the equation of the fifth degree

$$a_0 + a_1x_i + a_2x^2{}_i + a_3x^3{}_i + a_4x^4{}_i + a_5x^5{}_i = 0 \qquad (2)$$

for determine the roots of (1) ,x= x1 , x= X2, x= x3 , x= x4 , x= x5.

- Determine the constants a0, a1, a2, a3, a4 and a5 by solving the system of five linear algebraic equations using least square method .
- Find iteration (X_{n+1}) from

$$X_{n+1} = v_n - \frac{f_{(v_n)}}{\acute{f}_{(v_n)}} \qquad (3)$$

- Return to step (2) until the absolute error $(x) < \varepsilon$.

4. Numerical Testing

In the first we fitted a polynomial function by estimating the parameters using the least squares method to eliminate complex nonlinear functions to arrive at a solution for nonlinear equations, By using maple , and test the effectiveness of the proposed method and compare it with other methods:

We start with quadratic equation, then we find that, this procedure lead us to the root α of equation (1) , let e_i is the error or the different value between the true value y_i and the estimated value $\hat{y}$,therefore.

$$e_i = y_i - \hat{y}_i \qquad (4)$$

and the sum of square error for second ,Third , fourth and fifth degrees:

$$\sum_{i=1}^{5} e_i^2 = \sum_{i=1}^{5}(y_i - \hat{y}_i)^2 \qquad (5)$$

$$\sum_{i=1}^{2} e_i^2 = \sum_{i=1}^{2}(y_i - (a_0 + a_1x_i + a_2x^2{}_i))^2 \qquad (6)$$

$$\sum_{i=1}^{3} e_i^2 = \sum_{i=1}^{3}(y_i - (a_0 + a_1x_i + a_2x^2{}_i + a_3x^3{}_i))^2 \qquad (7)$$

$$\sum_{i=1}^{4} e_i^2 = \sum_{i=1}^{4}(y_i - (a_0 + a_1x_i + a_2x^2{}_i + a_3x^3{}_i + a_4x^4{}_i))^2 \qquad (8)$$

or $\sum_{i=1}^{5} e_i^2 = \sum_{i=1}^{5}(y_i - (a_0 + a_1x_i + a_2x^2{}_i + a_3x^3{}_i + a_4x^4{}_i + a_5x^5{}_i))^2$ (9)

To find a0 , a1 , a2, a3, a4 and a5 , we will minimize this function , taking the derivative of (5,6,7,8,9) equal to zero, we find the three normal equations for second degree:

$$\left.\begin{aligned}
&n\,a_0 + a_1\sum x_i + a_2\sum x^2{}_i = \sum y_i\\
&a_0\sum x_i + a_1\sum x^2{}_i + a_2\sum x^3{}_i = \sum x_i y_i\\
&a_0\sum x^2{}_i + a_1\sum x^3{}_i + a_2\sum x^4{}_i = \sum x^2{}_i y_i
\end{aligned}\right\} \qquad (10)$$

Find normal equations for third degree:

$$\left.\begin{aligned}
&n\,a_0 + a_1\sum x_i + a_2\sum x^2{}_i + a_3\sum x^3{}_i = \sum y_i\\
&a_0\sum x_i + a_1\sum x^2{}_i + a_2\sum x^3{}_i + a_3\sum x^4{}_i = \sum x_i y_i\\
&a_0\sum x^2{}_i + a_1\sum x^3{}_i + a_2\sum x^4{}_i + a_3\sum x^5{}_i = \sum x^2{}_i y_i\\
&a_0\sum x^3{}_i + a_1\sum x^4{}_i + a_2\sum x^5{}_i + a_3\sum x^6{}_i = \sum x^3{}_i y_i
\end{aligned}\right\} \qquad (11)$$

Calculate the normal equation for the fourth degree:

$$\left.\begin{aligned}
&n\,a_0 + a_1\sum x_i + a_2\sum x^2{}_i + a_3\sum x^3{}_i + a_4\sum x^4{}_i = \sum y_i\\
&a_0\sum x_i + a_1\sum x^2{}_i + a_2\sum x^3{}_i + a_3\sum x^4{}_i + a_4\sum x^5{}_i = \sum x_i y_i\\
&a_0\sum x^2{}_i + a_1\sum x^3{}_i + a_2\sum x^4{}_i + a_3\sum x^5{}_i + a_4\sum x^6{}_i = \sum x^2{}_i y_i\\
&a_0\sum x^3{}_i + a_1\sum x^4{}_i + a_2\sum x^5{}_i + a_3\sum x^6{}_i + a_4\sum x^7{}_i = \sum x^3{}_i y_i\\
&a_0\sum x^4{}_i + a_1\sum x^5{}_i + a_2\sum x^6{}_i + a_3\sum x^7{}_i + a_4\sum x^8{}_i = \sum x^4{}_i y_i
\end{aligned}\right\} \qquad (12)$$

And find the normal equation for the fifth degree:

$$\left.\begin{array}{l}
n\,a_0 + a_1\sum x_i + a_2\sum x^2{}_i + a_3\sum x^3{}_i + a_4\sum x^4{}_i + a_5\sum x^5{}_i = \sum y_i \\
a_0\sum x_i + a_1\sum x^2{}_i + a_2\sum x^3{}_i + a_3\sum x^4{}_i + a_4\sum x^5{}_i + a_5\sum x^6{}_i = \sum x_i y_i \\
a_0\sum x^2{}_i + a_1\sum x^3{}_i + a_2\sum x^4{}_i + a_3\sum x^5{}_i + a_4\sum x^6{}_i + a_5\sum x^7{}_i = \sum x^2{}_i y_i \\
a_0\sum x^3{}_i + a_1\sum x^4{}_i + a_2\sum x^5{}_i + a_3\sum x^6{}_i + a_4\sum x^7{}_i + a_5\sum x^8{}_i = \sum x^3{}_i y_i \\
a_0\sum x^4{}_i + a_1\sum x^5{}_i + a_2\sum x^6{}_i + a_3\sum x^7{}_i + a_4\sum x^8{}_i + a_5\sum x^9{}_i = \sum x^4{}_i y_i \\
a_0\sum x^5{}_i + a_1\sum x^6{}_i + a_2\sum x^7{}_i + a_3\sum x^8{}_i + a_4\sum x^9{}_i + a_5\sum x^{10}{}_i = \sum x^5{}_i y_i
\end{array}\right\} \quad (13)$$

Then, find fitted the parameters of cubic polynomial, fourth polynomial and fifth polynomial (PM).

5. Numerical Examples

Consider the following examples to check the effectiveness of the least square estimation of the polynomial higher degrees .First we compare the present method with the method of (PM1) Nasr Al Din IDE [2], Rafiullah.M [9],the classical Newton's method [12]and Hou[13] and Hou[15].

The initial interval [1,2]

Example1:

Consider the equation:

Table1. *Show numerical results obtained the (PM)*

functions	Methods	No.of iteration	x0	xn	f(xn)	
$f_1(x) = \sin(x)^2 + x$	least square for polyonomial with 2nd degreee	2	1	5.4871E-08	0	
	least square for poly with 3rd degree	2	1	2.3743E-05	0	
	4th degree	2	1	-0.0017174	0	
	fifth degree	2	1	-0.0074584	0	
$f_3(x) = 4x^5 - 2x^4 + 2x^2 - 3$	least square for polyonomial with 2nd degreee	5	2	1.09427405	0.3833	the stopping criterion is not met
	least square for poly with 3rd degree	5	2	1.00713458	0	
	4th degree	5	2	1.04377126	-1E-07	
	fifth degree	5	2	1.00143	0.0005	
$f_4(x) = xe^{-x} - x$	least square for polyonomial with 2nd degreee	2	1	0.36751045	0	
	least square for poly with 3rd degree	4	1	0.27049737	2E-08	
	4th degree	2	1	0.18219121	-2E-11	
	fifth degree	3	1	0.28094432	3E-09	
$f_5(x) = x^3 - x^2 + \log(x)$	least square for polyonomial with 2nd degreee	5	1.5	0.9747202	4E-06	
	least square for poly with 3rd degree	4	1.5	0.99998227	3E-06	
	4th degree	4	1.5	0.99999969	3E-06	
	fifth degree	4	1.5	1.00000042	3E-06	
$f_6(x) = e^x\sin(x) + \log(1+x^2)$	least square for polyonomial with 2nd degreee	3	1.5	0.25922091	0	
	least square for poly with 3rd degree	2	1.5	0.43400233	-1E-10	
	4th degree	2	1.5	0.45934153	-3E-09	
	fifth degree	2	1.5	0.42640183	-1E-10	
$f_8(x) = 3tanx - x$	least square for polyonomial with 2nd degreee	1	1	0.00002104	0	
	least square for poly with 3rd degree	2	1	0.00000997	0	
	4th degree	2	1	0.00042496	0	
	fifth degree	2	1	0.16973787	-1E-10	

Comparing the results of this research with the results of previous methods of previous studies method PM1 [2] IDE Nasr-Al-Din, and the method of R1 exist in the research , where we observe a similarity in the result for the first function of the second class where the number of iterations 2 and better than Newton's classical method.

Table2. *The numerical results to compare the present method (PM) with another methods*

second degree						
function	PM	PM1	NM	R1	HOU	Compare
$f_1(x) = sin(x)^2 + x$	2	2	7	2	2	same
$f_3(x) = 4x^5 - 2x^4 + 2x^2 - 3$	5	2	9	3	2	bad
$f_4(x) = xe^{-x} - x$	2	9	9	13	3	best
$f_5(x) = x^3 - x^2 + \log(x)$	5	2	7	2	2	bad
$f_6(x) = e^x \sin(x) + \log(1 + x^2)$	3	2	8	2	2	bad
$f_8(x) = 3tanx - x$	1	2	6	3	slow	best

The comparison table(2) for the numerical examples of the functions between the method proposed(PM) with the methods used [2] Nasr Al Din IDE ,We compare (PM) with the method Nasr Al Din IDE [2], M. Rafiullah (R1) [9] with the classical Newton's method (NM) [12] ,Hou [19] which are eighth, second, twelfth and ninth order methods respectively. With the same function, numbering shows that the number of iterative to reach the roots was better for the proposed method for the fourth and eighth functions. The similarity in the result was in the first function, but the third, fifth and sixth results were bad at the second-degree case.

6. Conclusion

In this work, we have compared the result with Nasr Al Din IDE [2] (2018) results, note that there are no differences from the second degree of the same functions used in the research. Therefore, we observe convergence in the results of the second degree of the first, second, third , fourth, fifth and sixth, and eight functions from table(1), and we notice the number of iterative to reach the roots was better for the proposed method for the fourth and eighth functions from table(2)

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