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Overcoming Computational Difficulties in the Multinomial (Polynomial) Distribution with an Increased Number of Events (for Example, Municipal Elections). Using MATLAB

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Abstract: *When calculating the probabilities of a polynomial distribution with a large number of events* n_k, *the researcher encounters either very large or very small numbers that even powerful programs cannot master. For a fast enough and with a small error of calculation, the author compiled a program in MATLAB attached to municipal elections. The program is applicable to satellite launches, in military affairs, pharmacological tests, at work, in trade, in the services sector, in emergency situations and in many other things. All actions are presented in such detail that a non-specialist will quickly master them.*

Keywords: *multinomial (polynomial) distribution, MATLAB program, multinomial calculator, factorial calculator, election campaign.*

1. Introduction

The multinomial (polynomial) distribution in elections of various levels in science and life is reflected in perhaps the only publication. Dr. Lothar Sachs in the book Statistical Evaluation [1] considered the application of the multinomial distribution in the election of candidates. But Dr. Sachs did not consider:

1. Computational difficulties caused by large factorials manually and even using standard computer programs;
2. Difficulties of a computational nature, caused by rather small values of probabilities manually and even using standard computer programs;
3. Difficulties due to the participation in the elections of a significant number of candidates;
4. The substitution of unequal probabilities in the multinomial formula.

In the proposed article, the author considers the multinomial distribution on the example of the municipal elections of the head of the city of Vladivostok on September 8, 2013.

The author set a goal to investigate the use of the multinomial distribution formula for complications that block the outcome.

Lothar Zaks explains [1]: “In the binomial distribution, if the probability of a random choice of a smoker is p, and that of a non-smoker is – (1-p), then the probability of getting exactly x smokers from the total number n is equal to

$$P(x/n,\ p) = P_{n,p}(x) = \binom{n}{x} p^x q^{n-x} = C_x^n p^x q^{n-x} = \frac{n!}{x!(n-x)!} p^x q^{n-x}. \tag{1}$$

If instead of two outcomes there are several – $E_1, E_2, \ldots, E_k$ with probabilities $p_1, p_2, \ldots, p_k$, then the probability in n experiments to get exactly $n_1, n_2, \ldots, n_k$ of events $E_1, E_2, \ldots, E_k$ determined by expression 2; otherwise: if more than two outcomes are possible and the population consists of outcomes $A_1, A_2, \ldots, A_k$ with probabilities $p_1, p_2, \ldots, p_k$ at $\sum_{i=1}^{k} p_i = 1$, then the probability that in a sample of n independent observations n_1 times the outcome A_2 etc., distributed according to the multinomial law (polynomial distribution). The parameters of this distribution:″

$$average\ value: \mu_i = np_i;$$

$$dispersion: \sigma_i^2 = np_i(1-p_i) = np_i q_i.$$

$$p(n_1; n_2; \ldots; n_k / p_1; p_2; \ldots; p_k / n) = \frac{n!}{n_1! \times n_2! \times \ldots \times n_k!} \times p_1^{n_1} \times p_2^{n_2} \times \ldots \times p_k^{n_k}; \quad (2)$$

$$provided \sum_{i=1}^{k} n_i = n.$$

Dr. Sachs gives an example [1]: ″10 should choose one of three candidates (A, B, C). What is the probability of choice 8A, 1B и 1C?″

$$provided \sum_{i=1}^{k=3} n_i = 10.$$

Figure 1 shows a quick solution to this example.

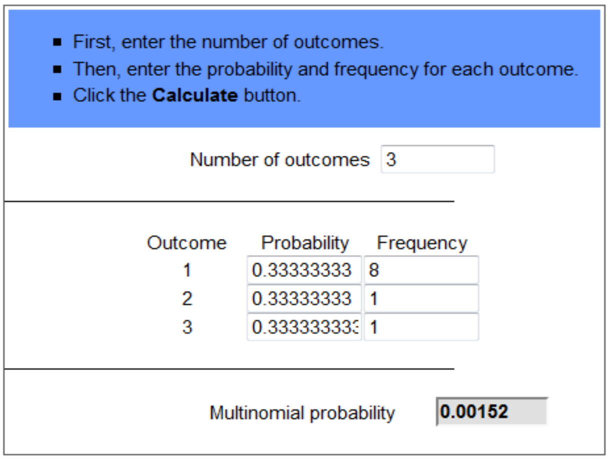

Fig1. *The multinomial calculator window*

So, the probability $P_{8A, 1B, 1C} = 0.00152$.

Using the formula (2) of the multinomial distribution proposed by L. Sachs, which gives a binomial distribution for k = 2 (special case), we determine the chances that for each of the 9 candidates (Andreichenko, Vasilyev, Velgodsky, Monastyrev, Protchenko, Pushkaryov, Ulyanov, the late V. I. Cherepkov, Yurtaev) a certain number of voters will vote for the position of mayor of Vladivostok (table 1).

Table 1. *Excerpts from the minutes of September 10, 2013. Vladivostok Municipal Election Commission [3] on the results of the election of the mayor of 08.09.2013*

<table>
<tr><td></td><td colspan="2">The number of voters listed at the end of voting</td><td>4</td><td>5</td><td>2</td><td>0</td><td>7</td><td>6</td></tr>
<tr><td colspan="2" rowspan="2">Surnames, first names, patronymic names of registered candidates included in the ballot paper</td><td colspan="7">The number of votes cast for each registered candidate</td></tr>
<tr><td colspan="3">absolute value</td><td colspan="4">Percentage of the number of voters who participated in</td></tr>
</table>

								the voting
19	Andreichenko Andrei Valerievich	0	0	1	5	1	1	1,80%
20	Vasilyev Viktor Evgenievich	0	0	0	8	8	4	1,05%
21	Velgodsky Oleg Nikolaevich	0	0	8	7	0	4	10,34%
22	Monastyryov Alexander Vyacheslavovich	0	0	2	2	1	3	2,63%
23	Protchenko Danil Aleksandrovich	0	0	0	3	7	9	0,45%
24	Pushkaryov Igor Sergeevich	0	5	0	0	2	5	59,45%
25	Ulyanov Ilya Mikhailovich	0	0	0	7	9	6	0,95%
26	Cherepkov Viktor Ivanovich	0	1	5	9	0	5	18,90%
27	Yurtaev Alexander Grigorievich	0	0	2	4	8	3	2,95%
								98,52%

Source: *"Election Commission of Primorsky Krai" [Electronic resource]. – Access mode: www.izbirkom.primorsky.ru/elections/show.php?id=558, (Access date 04.07.2014).*

The number of voters who took part in the elections: absolute - 84208, in percentage - 18.63%.

In accordance with Part 8 of Article 78 of the Electoral Code of the Primorsky Territory [4] Igor Sergeyevich Pushkarev, who received the largest number of votes of the voters who participated in the voting, was recognized as the elected head of the city of Vladivostok

Let us point out the mistakes of the Vladivostok city municipal election commission. The absolute number of voters who took part in the elections is equal to 82.900 people $\sum_{i=1}^{k=9} n_i = 82900$. Further, the relative number, as is evident from table 1, is equal to 98.52% in total, and not 100%. If we enter this data into the multinomial calculator[1], it will refuse to process it, demanding to bring it to 100% (Figure 2).

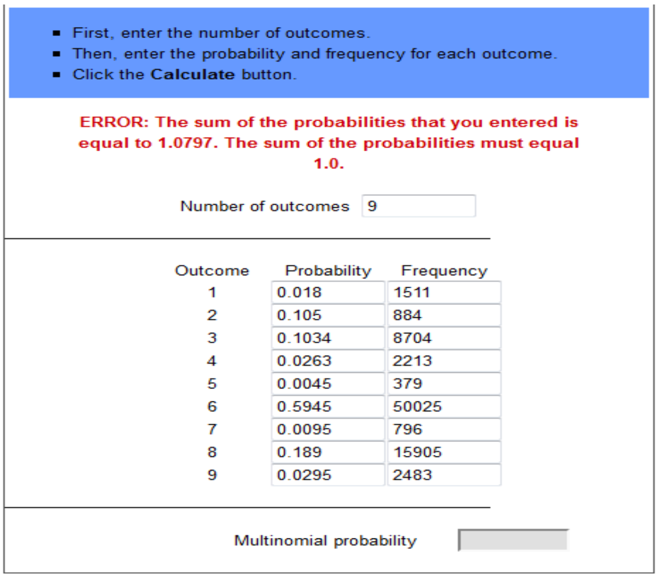

Fig2. *The multinomial calculator window*

But having corrected the mistakes of the commission, we cannot use the calculator. A manual solution will require a huge investment of time and effort.

It's a pity, but we come across limitations first, Microsoft Office Excel 2010, when using the "FACTORY" function, but large numbers, for example, the factorial of the number 171 program no longer masters, numerous factorials calculators can help (Figure 4). Note that this is a high power calculator; but it also has a limit, factorial 40,000 is its ceiling. Secondly, raising a fractional number to a power with a very high index in theory does not give 0, but in practice the program shows 0, that is, it is not able to show a very small fraction. And we should divide each number into 10, 100, 1000, etc. But since factorial is defined only for non-negative integers, we have to pay for this with a significant rounding error

```
> 0.603438^5002.5
[1] 0
> 0.603438^500.25
[1] 1.825937e-110
>
>
> (0.018227^15.11)*(0.010663^8.84)*(0.104994^87.04)*(0.026695^22.13)*(0.004572$
[1] 0
>
```

Fig3. *The window of a more powerful program*

As proof of the above in Figure 3 with a touchstone, we raised the probability of Pushkarev's victory to the level of 0.603438 (50025/10). The second line of Figure 3 showed 0 – this path is closed. By raising to the same degree (50025/100) we received a small number 1.825937e-110, and this is a good shot! But the complete solution of our example again gave 0. It is necessary to divide by 1000. In this division, the votes for Andreichenko, Monastyryova and Yurtaev equaled (2 each), for Vasilyev and Ulyanov 1 each, and for Protchenko – 0 votes.

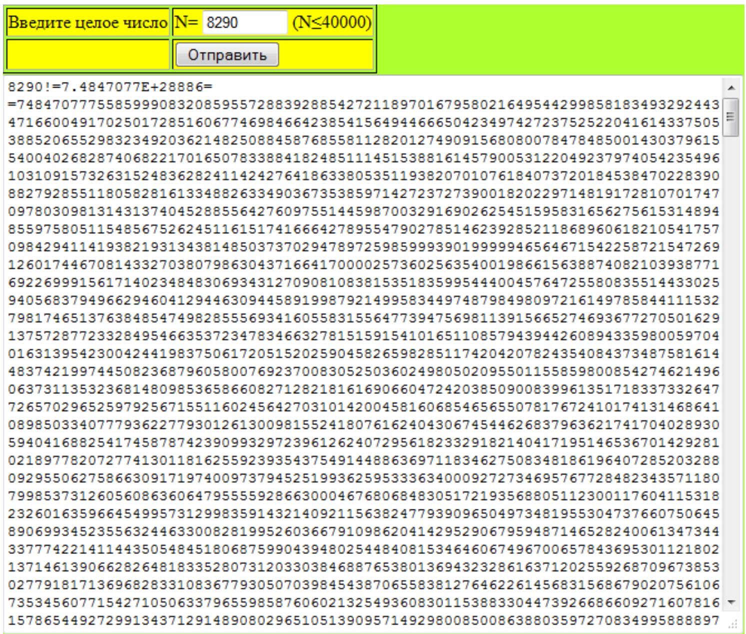

Fig4. *Factors calculator window* ($\frac{general\ \ population}{10}$)

That is why the author has compiled a solution program in MATLAB, which shows a solution that, although approximate, but with a significantly higher degree of hitting, does not harm the rights of candidates who have received modest support.

Matfile in MATLAB:

a=[sqrt(2*pi) 82900 exp(1);

sqrt(2*pi) 1511 exp(1);

sqrt(2*pi) 884 exp(1);

```
sqrt(2*pi) 8704 exp(1);
sqrt(2*pi) 2213 exp(1);
sqrt(2*pi) 379 exp(1);
sqrt(2*pi) 50025 exp(1);
sqrt(2*pi) 796 exp(1);
sqrt(2*pi) 15905 exp(1);
sqrt(2*pi) 2483 exp(1)];
log10(a)
b=[1 0.5+82900 82900;
1 0.5+1511 1511;
1 0.5+884 884;
1 0.5+8704 8704;
1 0.5+2213 2213;
1 0.5+379 379;
1 0.5+50025 50025;
1 0.5+796 796;
1 0.5+15905 15905;
1 0.5+2483 2483];
b.*log10(a)
u= b.*log10(a)
c=[1511; 884; 8704; 2213; 379; 50025; 796; 15905; 2483]; d=[0.018226779; 0.01066345; 0.104993969; 0.026694813; 0.004571773; 0.603437877; 0.00960193; 0.19185766; 0.029951749]; log10(d)
c.*log10(d)
w=c.*log10(d)
q=w(1,1)+w(2,1)+w(3,1)+ w(4,1)+w(5,1)+w(6,1)+ w(7,1)+w(8,1)+w(9,1)
p=u(1,1)+u(1,2)-u(1,3)-((u(2,1)+u(2,2)-u(2,3))+(u(3,1)+u(3,2)-u(3,3))+(u(4,1)+u(4,2)-u(4,3))+
(u(5,1)+u(5,2)-u(5,3))+(u(6,1)+u(6,2)-u(6,3))+(u(7,1)+u(7,2)-u(7,3))+(u(8,1)+u(8,2)-
u(8,3))+(u(9,1)+u(9,2)-u(9,3))+(u(10,1)+u(10,2)-u(10,3)))+q
10^p
ans =
0.3991    4.9186    0.4343
0.3991    3.1793    0.4343
0.3991    2.9465    0.4343
0.3991    3.9397    0.4343
0.3991    3.3450    0.4343
0.3991    2.5786    0.4343
0.3991    4.6992    0.4343
0.3991    2.9009    0.4343
```

```
0.3991    4.2015    0.4343
0.3991    3.3950    0.4343
ans = 1.0e+05 *
0.0000    4.0775    0.3600
0.0000    0.0481    0.0066
0.0000    0.0261    0.0038
0.0000    0.3429    0.0378
0.0000    0.0740    0.0096
0.0000    0.0098    0.0016
0.0000    2.3508    0.2173
0.0000    0.0231    0.0035
0.0000    0.6683    0.0691
0.0000    0.0843    0.0108
u = 1.0e+05 *
0.0000    4.0775    0.3600
0.0000    0.0481    0.0066
0.0000    0.0261    0.0038
0.0000    0.3429    0.0378
0.0000    0.0740    0.0096
0.0000    0.0098    0.0016
0.0000    2.3508    0.2173
0.0000    0.0231    0.0035
0.0000    0.6683    0.0691
0.0000    0.0843    0.0108
ans = -1.7393; -1.9721; -0.9788; -1.5736; -2.3399; -0.2194; -2.0176; -0.7170;
-1.5236
ans = 1.0e+04 *
-0.2628; -0.1743; -0.8520; -0.3482; -0.0887; -1.0974; -0.1606; -1.1404;
-0.3783
w = 1.0e+04 *
-0.2628; -0.1743; -0.8520; -0.3482; -0.0887; -1.0974; -0.1606; -1.1404;
-0.3783
q = -4.5027e+04
p = -16.3263
ans = 4.7176e-17
```

So, the probability

$$P_{\substack{1511\,ANDREICHENKO,\,884\,VASILYEV,\,8704\,VELGODSKY,\,2213\,MONASTYRYOV,\,379\,PROTCHENCO,\,50025\,PUSHCARUOV,\,796\,ULYANOV,\\ 15905\,CHEREPKOV,\,2483\,YURTAEV}} =$$

$$= 4.7176 \times 10^{17}.$$

To solve the example of Dr. Sachs, the matfile looks like this:

```
a=[sqrt(2*pi) 10 exp(1);
sqrt(2*pi)  8   exp(1);
sqrt(2*pi)  1   exp(1);
sqrt(2*pi)  1   exp(1)];
log10(a)
b=[1 0.5+10 10;
1 0.5+8 8;
1 0.5+1 1;
1 0.5+1 1];
b.*log10(a)
u= b.*log10(a)
c=[8; 1; 1];  d=[1/3; 1/3; 1/3];  log10(d)
c.*log10(d)
w=c.*log10(d)
q=w(1,1)+w(2,1)+w(3,1)
p=u(1,1)+u(1,2)-u(1,3)-((u(2,1)+u(2,2)-u(2,3))+(u(3,1)+u(3,2)-u(3,3))+(u(4,1)+u(4,2)-u(4,3)))+q
10^p
ans =
0.3991    1.0000    0.4343
0.3991    0.9031    0.4343
0.3991         0    0.4343
0.3991         0    0.4343
ans =
0.3991   10.5000    4.3429
0.3991    7.6763    3.4744
0.3991         0    0.4343
0.3991         0    0.4343
u =
0.3991   10.5000    4.3429
0.3991    7.6763    3.4744
0.3991         0    0.4343
0.3991         0    0.4343
ans = -0.4771 -0.4771  -0.4771
ans = -3.8170 -0.4771 -0.4771
w = -3.8170 -0.4771 -0.4771
q = -4.7712
p = -2.7457
ans = 0.0018
```

So, the probability $P_{8A,\,1B,\,1C} = 0.0018$.

2. CONCLUSION

In conclusion, it should be noted that we did the calculation, looking ahead. Prior to the elections, the probabilities are calculated on the basis of sociological surveys or by a vote of a commission of experts. In this case, calculations are not carried out with the general population, and your task should be easier.

Of course, the multinomial distribution is also suitable for presidential elections. The election of candidates – is not the only field of application of the multinomial distribution. Instead of applicants, defense issues, satellite launches, manufacturing, trade data, services, emergency situations and much more can be explored.

Thus, having studied the application of the multinomial distribution in municipal elections, the author came to the following conclusions:

- Dr. Sachs, ahead of his time, suggested using a multinomial distribution when electing candidates, but using his formula (2) is expensive;
- Dr. Sachs used the equally probable multinomial model, as if the sympathies of the voters are distributed as when throwing a flawless three -, four -, five -, six - and so on. - an outside dice;
- Dividing each number of voters by 10, 100, 1000, etc., to simplify the calculations, requires us to pay an accuracy of the total data in relation to candidates “non-heavyweight”. In this case, two pairs of candidates artificially received an equal number of votes, and one remained without votes. For the election headquarters of these persons, this is unacceptable;

Using the matfile for MATLAB, proposed by the author of this article, allows you to perform calculations on many objects. But this also gives an error, the size of which you, the reader, can compare by comparing the solution of Dr. Sachs example in Figure 1, equal to 0.00152 with the solution of the same example in MATLAB, where the answer was 0.0018.

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