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# Hydromagnetic Boundary Layer Flow of Nanofluids over a Permeable Moving Surface with Partial Slip in the Presence of Newtonian Heating

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**Abstract:** *In this paper, the magnetohydrodynamics (MHD) boundary layer flow of nanofluids past a permeable moving flat plate with partial slip in the presence of convective heating has been studied. The nanofluids considered contain water as the base fluid with copper (Cu), Alumina (Al2O3) and Silver (Ag) as the nanoparticles. The model equations are obtained and solved numerically by applying the Keller box method. The influence of governing parameters on the velocity and temperature of the nanofluid are investigated. The obtained results are presented graphically and the physical aspects of the problem are discussed quantitatively.*

**Keywords:** *Hydromagnetic, nanofluids, permeable, partial slip, convective heating.*

## 1. INTRODUCTION

The study of the flow of an electrically conducting fluid past a permeable walls has many applications in manufacturing and natural process which include cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, cooling of an infinite metallic plate in a cooling bath, textile and paper industries, solar central receivers exposed to wind currents, geothermal energy extraction, MHD generators, the boundary layer control in the field of aerodynamics, plasma studies and blood flow problems.

Magnetohydrodynamics (MHD) is the study of the flow of electrically conducting fluids in a magnetic field. Many experimental and theoretical studies on conventional electrically conducting fluids indicate that magnetic field markedly changes their transport and heat transfer characteristics. The study of magnetohydrodynamic (MHD) has many important applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field.

Suction of a fluid on the boundary surface, can significantly change the flow field and, as a consequence, affect the heat transfer rate at the surface ([1]-[3]). The flow and heat transfer over a permeable surface has practical applications especially in geophysical fluid dynamics such as beach sand, wood, sandstone, limestone, the human lung and in small blood vessels.

In the study of boundary layer flow, using slip condition along the stretched surface tends to decrease wall shear stress along the sheet which has interesting applications in the nanofluidic devices. In recent years, some interest has been given to the study of convective transport of the slip flow regime due to its application in micro/nanofluids. Swati and Rama [4] have studied effects of partial slip on boundary layer flow past a permeable exponential stretching sheet in presence of thermal radiation. Several researches have been done on the boundary layer flow with partial slip ([5] - [9]).On the other hand, it has been establish that suction makes a significant effect on the fluid velocity adjacent to the wall in the presence of slip. Hydrodynamic nano boundary layer flow over permeable stretching surface using Homotopy analysis and boundary value problem solver (BVP) has been studied by Van Gorder et al [10]. Yazdi et al [11] have investigated slip MHD flow over permeable stretching surface with chemical reaction.

Fluid heating and cooling are important in many industries such as power, manufacturing, transportation, and electronics. Effective cooling techniques are greatly needed for cooling any sort of high-energy device. Common heat transfer fluids such as water, ethylene glycol, and engine oil have limited/poor heat transfer capabilities due to their low heat transfer properties. In contrast, metals have thermal conductivities up to three times higher than these fluids, so it is natural that it would be desired to combine the two substances to produce a heat transfer medium that behaves like a fluid, but has the thermal conductivity of a metal. A lot of experimental and theoretical researches has been made to improve the thermal conductivity of these fluids. In 1993, during an investigation of new coolants and cooling technologies at Argonne national laboratory in U.S, Choi invented a new type of fluid called Nanofluid [12]. Nanofluids are fluids that contain small volumetric quantities of nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid [13]. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil. Nanofluids commonly contain up to a 5% volume fraction of nanoparticles to see effective heat transfer enhancements. Nanofluids are studied because of their heat transfer properties: they enhance the thermal conductivity and convective properties over the properties of the base fluid. Moreover, the presence of the nanoparticles enhance the electrical conductivity property of the nanofluids, hence are more susceptible to the influence of magnetic field than the conventional base fluids. Typical thermal conductivity enhancements are in the range of 15-40% over the base fluid and heat transfer coefficient enhancements have been found up to 40% [14]. Increasing in thermal conductivity of this magnitude cannot be solely attributed to the higher thermal conductivity of the added nanoparticles, and there must be other mechanisms attributed to the increase in performance.

After the pioneer investigation of Choi, thriving experimental and theoretical researches were undertaken to discover and understand the mechanisms of heat transfer in nanofluids. The knowledge of the physical mechanisms of heat transfer in nanofluids is of vital importance as it will enable the exploitation of their full heat transfer potential. Masuda et al. [15] observed the characteristic feature of nanofluid is thermal conductivity enhancement. This observation suggests the possibility of using nanofluids in advanced nuclear systems [16]. A comprehensive survey of convective transport in nanofluids was made by Buongiorno [17], who says that a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity is yet to be found. He focused on further heat transfer enhancement observed in convective situations. Recently, Kuznetsov and Nield [18] have examined the influence of nanoparticles on natural convection boundary-layer flow past a vertical plate using a model in which Brownian motion and thermophoresis are accounted for. The authors have assumed the simplest possible boundary conditions, namely those in which both the temperature and the nanoparticle fraction are constant along the wall. Nield and Kuznetsov [19, 20] have studied the Cheng and Minkowycz [21] problem of natural convection past a vertical plate in a porous medium saturated by a nanofluid. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis for the porous medium. Nadeem and Lee [22] used the homotopy analysis method to study the boundary layer flow of nanofluid over an exponentially stretching sheet. The magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate has been discussed by Hamad et al. [23]. Haddad et al. [24] experimentally investigated natural convection in nanofluds by considering the role of thermophoresis and Brownian motion in heat transfer enhancement. They indicated that neglecting the role of Brownian motion and thermophoresis deteriorate the heat transfer and this deterioration elevated by increasing the volume fraction of nanoparticles. Anbuchezhian[25]studied the thermoporesis and Brownian motion effects on boundary layer flow of nanofluid in presence of thermal stratification due to solar energy. In addition to these studies, several researchers have investigated the flow and heat transfer of nanofluids ([27] - [38]).

Recently, several authors ([39]-[42]) numerically investigated the natural convection of nanofluids under the influence of a magnetic field. Their theoretical studies on magnetic nanofluids assumed that both the nanoparticles and the conventional base fluids have equal electrical conductivity properties. In reality, this may not hold because ignoring the difference in electrical conductivity property of both the nanoparticles and the conventional base fluids may affect the outcome of their investigation. To the best of the authors knowledge, no attempt has been made in the past to study the effects of the complex interaction between the electrical conductivity of the conventional base fluids and that of the nanoparticles on the hydromagnetic flow with partial slip in the presence of Newtonian heating. The main objective of this paper is to numerically investigate the change in the electrical conductivity of the conventional base fluids as a result 2 of the nanoparticles and the subsequent interaction with the magnetic field at the boundary layer flow over a flat permeable surface with partial slip in the presence of convective heating.

A similarity transformation is used to reduce the governing partial differential equations to a set of ordinary differential equations. The set of coupled non-linear ordinary differential equations for momentum and energy balance are solved numerically by applying the Keller box method. The numerical results showing velocity and temperature profiles are presented graphically and discussed quantitatively.

## 2. MATHEMATICAL FORMULATION

Consider a steady unidirectional boundary layer flow of an electrically conducting nanofluid (Ag -water, Cu-water and Al2O3-water) past a semi-infinite permeable moving flat plate with partial slip in the presence of a uniform transverse magnetic field of strength B0 applied parallel to the y-axis. It is assumed that the induced magnetic field and the external electric field are negligible. At the boundary, the permeable plate is moving at a velocity U0 with a hot convectional fluid of temperature Tf flowing below it and a cold nanofluid of temperature T < Tf flowing above the plate. Far away from the plate, u = 0, T = T∞. We choose the coordinate system such that x-axis is along the moving flat plate and y-axis is normal to the plate. The physical flow model and coordinate system is shown in Fig.1. The surface temperature is assumed to be maintained by convective heat transfer at a constant temperature Tf Under the boundary-layer approximations, the nanofluid equations for momentum and energy balance governing the problem

Under consideration are written as shown below.

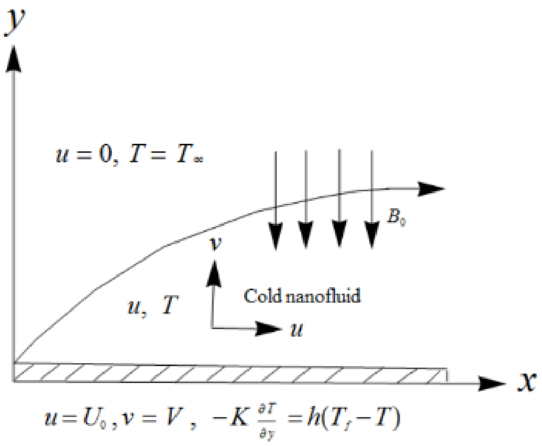

**Fig1.** *Physical flow model and coordinate system.*

**Table1.** *Thermophysical properties of water, Silver, Copper and Alumina ([43], [44]).*

| Physical properties | Fluid phase(water) | Ag | Cu | $Al_2O_3$ |
|---|---|---|---|---|
| $C_p$ (J.Kg$^{-1}$) | 4179 | 235 | 385 | 765 |
| $\rho$ (kg.m$^{-3}$) | 997.1 | 10500 | 8933 | 3970 |
| K(W.m$^{-1}$K$^{-1}$) | 0.613 | 429 | 400 | 40 |
| σ (S/m) | $5.5\chi10^{-6}$ | $63\chi10^{6}$ | $58\chi10^{6}$ | $35\chi10^{6}$ |

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

$$v\,\frac{\partial u}{\partial y} = -\nu_{nf}\,\frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{nf} B_o^2(x)}{\rho_{nf}}\,u. \tag{2}$$

$$v\,\frac{\partial T}{\partial y} = -\alpha_{nf}\,\frac{\partial^2 T}{\partial y^2} - \frac{\mu_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{\sigma_{nf} B_o^2(x)}{\rho c_{p_{nf}}}\,u^2. \tag{3}$$

The associated boundary conditions are

$$\begin{aligned} u &= U_0 + u_s, \;\; K_f\frac{\partial T}{\partial y} = h(T_f - T), \quad \text{at} \quad y = 0 \\ u &= 0, \;\; T = T_\infty, \quad \text{as } y \to \infty. \end{aligned} \tag{4}$$

Where u and v are the velocity components in the x and y directions respectively, T is the temperature of the nanofluid, and νn f , αn f , ρn f , μn f , kn f and (ρcp)n f , σn f are the kinematic viscosity, thermal diffusivity, density, viscosity, thermal conductivity, heat capacitance and electrical conductivity of the nanofluid, respectively, which are defined as([25], [26]):

$$v_f = \frac{\mu_f}{\rho_f}, \; v_{nf} = \frac{\mu_{nf}}{\rho_{nf}}, \; \rho_{nf} = (1-\phi)(\rho c_p)_f + \phi\rho_s, \; \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \; \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s, \; \sigma_{nf} = (1-\phi)(\sigma)_f + \phi\sigma_s, \; \frac{k_{nf}}{k_f} = \frac{(k_s+2k_f) - 2\phi(k_f - k_s)}{(k_s+2k_f) + \phi(k_f - k_s)}. \tag{5}$$

In which νf, μf, ρf, kf, σf are the kinematic viscosity, viscosity, density, thermal conductivity and electrical conductivity of the base fluid respectively; ρs, ks, (ρcp)s, σs are the density, thermal conductivity,heat capacitance and electrical conductivity of the nanoparticle respectively; φ is the solid volume fraction parameter of the nanofluid and us is the velocity slip which is assumed to be proportional to the local shear stress as follows

$$u_s = \ell\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{6}$$

is slip length as a proportional constant of the velocity slip.

Introducing the following similarity transformations and making use of Eq.(5)

$$\eta = y\frac{U_0}{v_f}, \; u = U_0F(\eta), \; v = U_0S, \;\; \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}. \tag{7}$$

The equation of continuity (1) is automatically satisfied and equations of momentum (2), and energy (3) reduce to the following dimensionless ordinary differential equations

$$\frac{\partial^2 F}{\partial \eta^2} + S(1-\phi)^{2.5}(1-\phi+\phi\frac{\rho_s}{\rho_f})\frac{\partial F}{\partial \eta} - Ha(1-\phi)^{2.5}(1-\phi+\phi\frac{\sigma_s}{\sigma_f})F = 0. \tag{8}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + SPr\frac{K_f}{K_{nf}}\left(1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\frac{\partial \theta}{\partial \eta} + \frac{Br}{(1-\phi)^{2.5}}\frac{K_f}{K_{nf}}\left(\frac{\partial F}{\partial \eta}\right)^2 + HaBr\frac{K_f}{K_{nf}}(1-\phi+\phi\frac{\sigma_s}{\sigma_f})F^2 = 0. \tag{9}$$

With boundary conditions

$$\begin{aligned} F &= 1 + K\frac{\partial F}{\partial \eta}, \;\; \frac{\partial \theta}{\partial \eta} = Bi(\theta - 1), \; \text{at } \eta = 0, \\ F &= 0, \; \theta = 0, \; \text{as } \eta \to \infty. \end{aligned} \tag{10}$$

Where η is the similarity variable, F and θ represent the dimension less nanofluid velocity and temperature.Pr represents the Prandtl number, Br is the Brinkmann number, S is the suction velocity parameter, K is the slip parameter, Ha is the Hartmann number and Bi is the Biot number. These governing parameters are given as

$$S = \frac{V}{U_0}, \; K = \frac{\ell U_0}{v_f}, \;\; Bi = \frac{hv_f}{U_0K_f}, \;\; Ha = \frac{\sigma_f {B_0}^2 v_f}{\rho_f {U_0}^2}, \;\; Br = \frac{\mu_f {U_0}^2}{K_f(T_f - T_\infty)} \tag{11}$$

The quantities of practical interest in this study are the local skin friction coefficient Cf and the local Nusselt number Nux, which are defined as

$$C_f = \frac{\tau_w}{\rho_f {U_0}^2}, \quad Nu_x = \frac{x q_w}{K_f (T_f - T_\infty)} \tag{12}$$

where τw is the skin friction, and qw is the heat flux through the plate which are given by

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -K_{nf} \left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{13}$$

Making use of Eq.(7) and (5) in (12), we get

$$C_f = \frac{1}{(1-\phi)^{2.5}} F'(0), \quad \frac{Nu_x}{Re_x} = -\frac{k_{nf}}{k_f} \theta'(0). \tag{14}$$

where $Re_x = \frac{U_0 x}{v_f}$ is the local Reynolds number.

## 3. NUMERICAL SOLUTION

The non linear boundary value problem represented by equations (8) - (9) and (10) is solved numerically using the Keller box method. In solving the system of non linear ordinary differential equations (8) - (9) together with the boundary condition (10) using the Keller box method the choice of an initial guess is very important. The success of the scheme depends greatly on how much good this guess is to give the most accurate solution. This choice has been made based on the convergence criteria together with the boundary conditions in consideration. As in Cebeci and Pradshaw [45], the values of the wall shear stress, in our case $F^1(0)$ is commonly used as a convergence criteria. This is because in the boundary layer flow calculations the greatest error appears in the wall shear stress parameter. In the present study this convergence criteria is used. In this study a uniform grid of size $\Delta\eta = 0.01$ is chosen to satisfy the convergence criteria of 10−5, which gives about a four decimal places accuracy for most of the prescribed quantities.

## 4. RESULTS AND DISCUSSION

The transformed non linear equations (8) - (9) subjected to the boundary condition (10) was solved numerically using Keller box method, which is described in Cebeci and Bradshaw [45]. The effects of the nanoparticle volume fraction φ, Brinkmann number Br, suction velocity parameter S, slip parameter K, Hartmann number Ha and Biot number Bi on the flow and heat transfer characteristics are determined for three kinds of nanofluids: Silver, Copper, and Aluminium oxide. The Prandtl number Pr is kept constant at 6.2. The values of the magnetic parameter, Hartmann number Ha is considered in the range from $10^{-10}$ to $10^{-13}$ and the nanoparticle volume fraction is varied from 0 to 0.2. Ha = 0 corresponds to the absence of magnetic field and φ = 0 to the regular fluid.

Figures 2 - 6 show the effects of various parameters on the nanofluid velocity, F(η). It is observed that for all parameters, the nanofluid velocity is maximum near the plate surface and decreases gradually to zero at the free stream far away from the plate satisfying the boundary conditions. Fig.2 shows the velocity of the nanofluid for different types of nanoparticles. It is observed that the Ag-water nanofluid attains zero free stream velocity fastest and has smallest velocity boundary layer thickness compared to Cu-water and $Al_2O_3$-water. The reverse holds in the $Al_2O_3$-water nanofluid. This is due to the fact that the surface skin friction is highest in Ag-water and lowest in $Al_2O_3$-water for some fixed value of φ. This is also attributed to the fact that, since the electrical conductivity of Ag-water is high compared to Cu-water and $Al_2O_3$nanofluids and hence the influence of the magnetic field is more observed in the Ag-water nanofluid than the other nanofluids, this is because the presence of the magnetic field or an increment in the value of the magnetic parameter, Ha tends to decrease the velocity of the nanofluid as shown in Fig.3. The effects of nanoparticle volume fraction in the $Al_2O_3$-

water is presented in Fig.4. It is observed that increasing the nanoparticle volume fraction decreases the velocity of the nanofluid and the boundary layer thickness. Thus the velocity reaches the zero free stream velocity at a faster rate when the nanoparticle volume fraction is increased. Fig.5 shows the effects of suction parameter S on the velocity of the nanofluid. It is noted that increasing the suction parameter S leads to a decrease in the nanofluid velocity. This is because as increases more fluid is sucked out of the porous plate, which in turn causes the velocity of the nanofluid to decrease. Fig.6 presents the effects of the slip parameter K on the velocity of the nanofluid. It illustrates that an increase in the slip parameter decreases the nanofluid velocity. This is attributed to the fact that increasing the slip parameter lead to decrease wall shear stress by decreasing the magnitude of the skin friction coefficient,$|F^1(0)|$.

Figures 7 - 12 show the influence of various governing parameters on the nanofluid temperature. It is observed that the temperature decreases from maximum near the plate surface to zero far away from the plate satisfying the free stream conditions. Fig.7 shows the temperature profile for three different nanoparticles; Ag, Cu and $Al_2O_3$. It is observed that the $Al_2O_3$-water nanofluid has smallest temperature near the boundary, thinnest temperature boundary layer thickness and reaches the zero temperature far away from the plate quickly than the Cu-water and Ag-water nanofluids. Fig. 8 shows the effects of the Hartmann number Ha on the temperature of the nanofluid. It is observed that an increase in Ha leads to an increase in the temperature and as a result the thermal boundary layer thickness increases. Usually, an increase in Ha leads to a decrease in the rate of heat transfer at the plate surface which in turn results in an increase the nanofluid temperature. In Fig.9 it is shown that increasing the Biot number Bi inceases the temperature of the nanofluid and the boundary layer thickness, this is a fact attributed to an increase in the convective heating. Fig.10 represents the variation of temperature with respect to the nanoparticle volume fraction φ. The graph depicts that the temperature increases when φ increase. This agrees with the physical behaviour in that when the volume fraction of Aluminium oxide increases the thermal conductivity increases as well, and as a result the thermal boundary layer thickness increases. This observation shows that using nanofluids changes the temperature, thus the use of nanofluids will be of significantly important in the cooling and heating processes. Fig.11 illustrates an increase in Brinkmann number Br increases the temperature and the thermal boundary layer thickness. This is because as Br increases the heat produced by viscous dissipation increases which causes the temperature of the nanofluid to increase. Fig.6 illustrates the effects of the suction parameter S ($S > 0$) on the temperature profile. It is noticed that an increase in S decreases the temperature. This shows that an increase in $S > 0$ means more nanofluid is sucked out of the porous plate leading to a decrease in the temperature and subsequently, a decrease in the thermal boundary layer thickness.

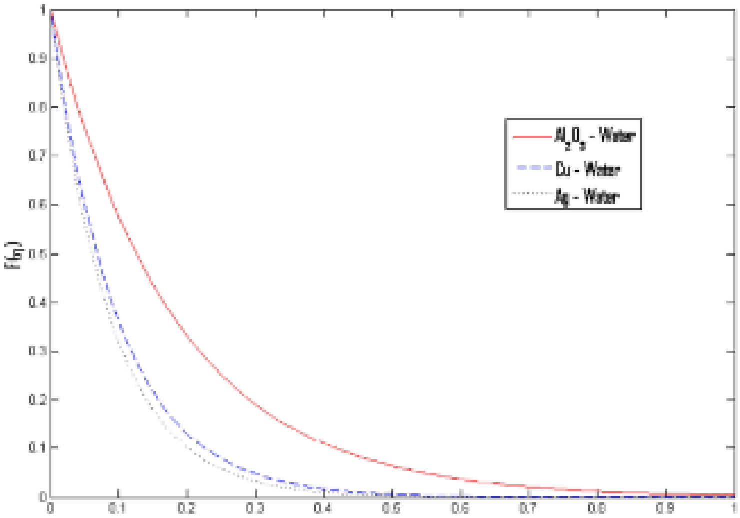

**Fig2.** *Velocity profile of Ag-water, Cu-water and Al2O3-water nanofluids for* $Ha = 10^{-10}$ , $S = 1, K = 0, Bi = Br = 0.1, \varphi = 0.1$.

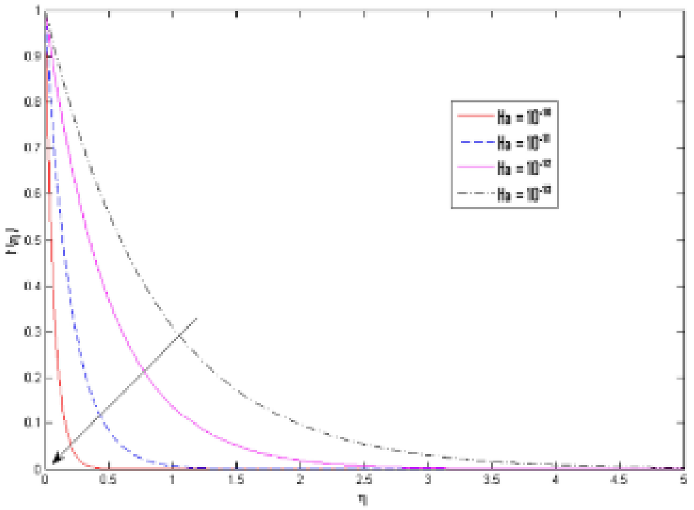

**Fig3.** *Effects of Ha on velocity profile with Al2O3-water nanofluid for Ha* $10^{-10}$*,S =1,K =0,BiBr = 0.1,φ = 0.1.*

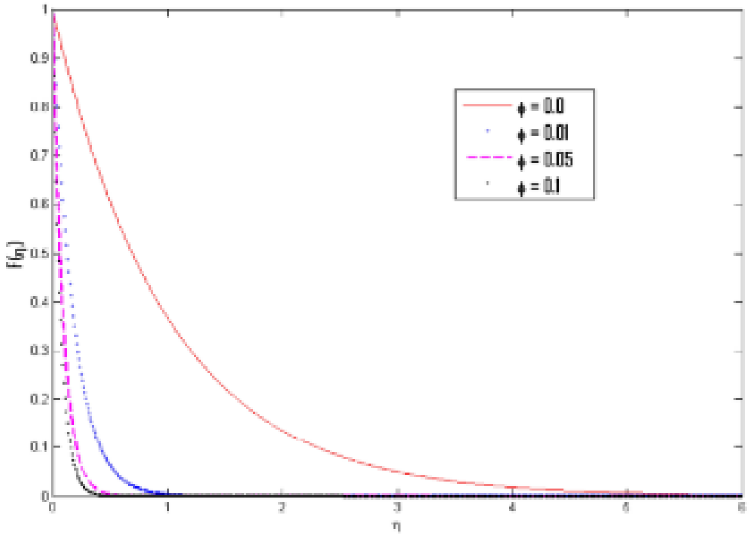

**Fig4.** *Effects of φ on velocity profile with Al2O3-water nanofluid for Ha =* $10^{-10}$ *, S = 1, K = 0, Bi = Br = 0.1.*

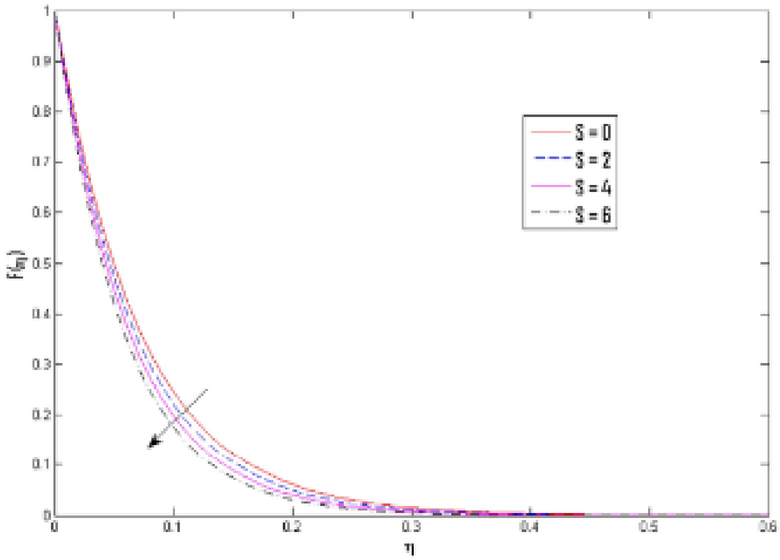

**Fig5.** *Effects of S on velocity profile with Al2O3-water nanofluid for Ha =* $10^{-10}$ *, K = 0, Bi = Br = 0.1, φ = 0.1.*

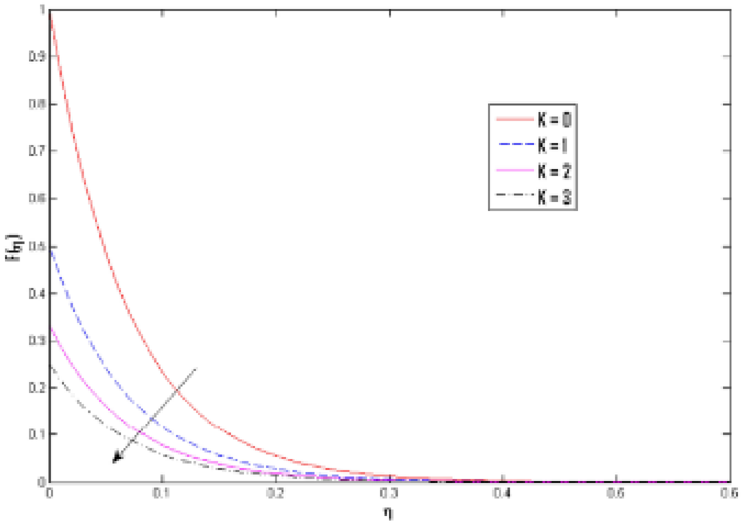

**Fig6.** *Effects of K on velocity profile with Al2O3-water nanofluid for* $Ha = 10^{-10}$ , $S = 1$, $Bi = Br = 0.1$, $\varphi = 0.1$.

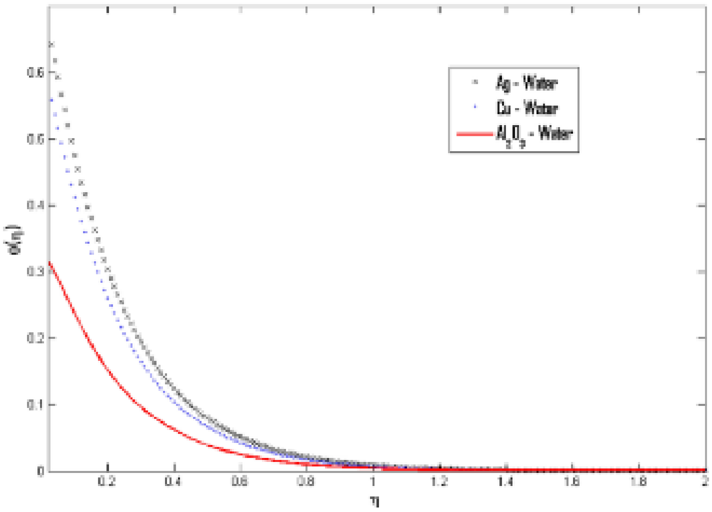

**Fig7.** *Temperature profile of Ag-water, Cu-water and Al2O3-water nanofluids for* $Ha = 10^{-10}$, $S = 1$, $K = 0$, $Bi = Br = 0.1$, $\varphi = 0.1$.

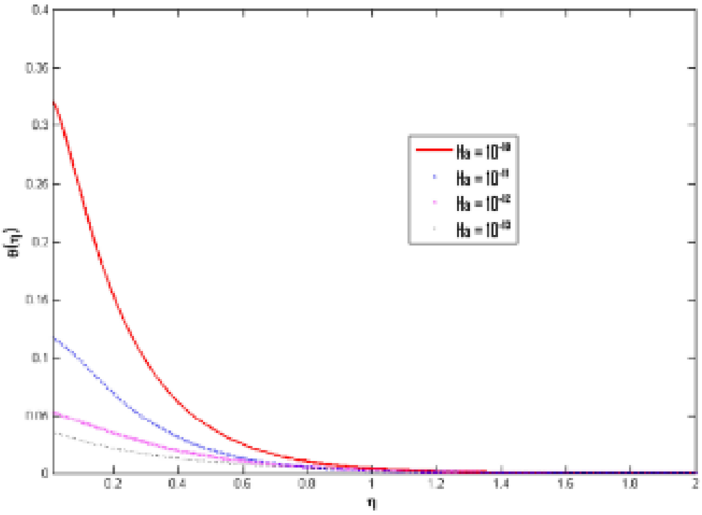

**Fig8.** *Effects of Ha on temperature profile with Al2O3-water nanofluid for* $Ha = 10^{-10}$, $S = 1$, $K = 0$, $Bi = Br = 0.1$, $\varphi = 0.1$.

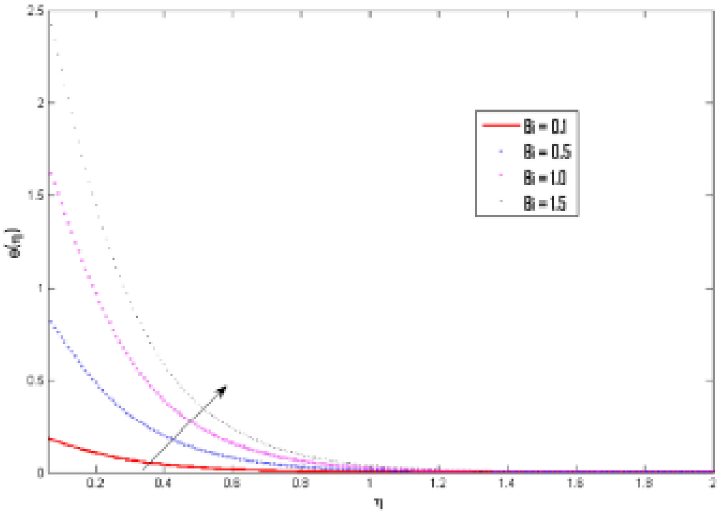

**Fig9.** *Effects of Bi on temperature profile with Al2O3-water nanofluid for* $Ha = 10^{-10}$, $S = 1$, $K = 0$, $Br = 0.1$, $\varphi = 0.1$.

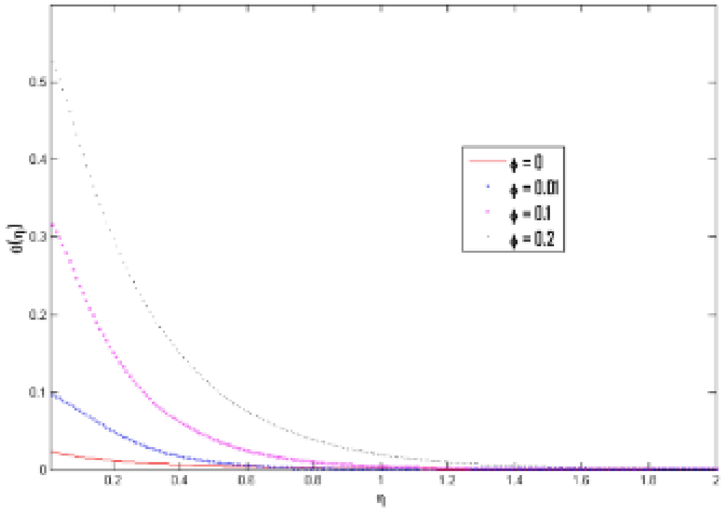

**Fig10.** *Effects of* $\varphi$ *on temperature profile with Al2O3-water nanofluid for* $Ha = 10^{-10}$, $S = 1$, $K = 0$, $Bi = Br = 0.1$

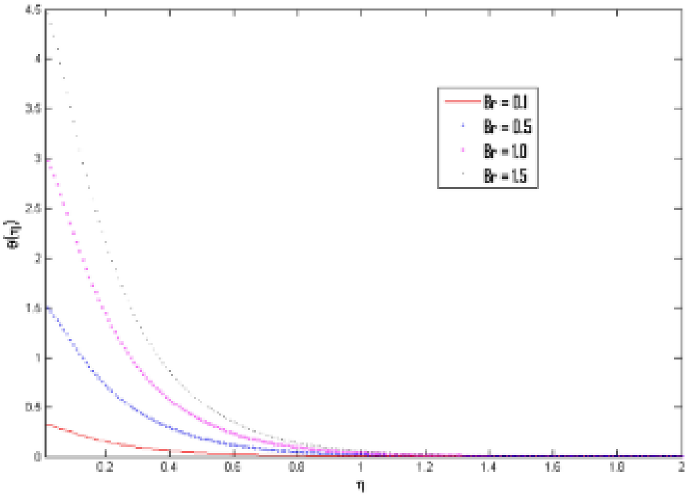

**Fig11.** *Effects of Br on temperature profile with Al2O3-water nanofluid for* $Ha = 10^{-10}$, $S = 1$, $K = 0$, $Bi = 0.1$, $\varphi = 0.1$.

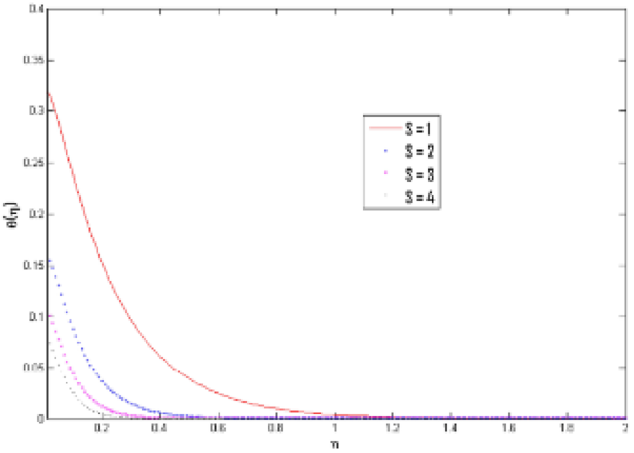

**Fig12.** *Effects of S on temperature profile with Al2O3-water nanofluid for* $Ha = 10^{-10}$ *, K = 0, Bi = Br = 0.1, φ = 0.1.*

## 5. CONCLUSION

This paper investigated, the effects of magnetic field on the boundary layer flow of *Al2O3*-water, Cu-water and Ag-water nanofluids past over a semi infinite permeable moving surface with partial slip in the presence of convective heating. The governing momentum and energy equations were reduced in to a non dimensional equations and solved numerically using Keller box method by taking in to consideration the enhanced electrical conductivity of the conventional base fluid due to the presence of the nanoparticles. The effects of Ha, φ, S, K, Bi, and Br on the flow and heat transfer characteristics are determined and discussed in detail. It is shown that as expected, increasing the value of the magnetic parameter leads to a decrease in the velocity and an increase in the temperature of the nanofluid. The velocity of the nanofluid decreases with an increase in the nanoparticle volume fraction φ while the temperature of the nanofluid increases with an increase in φ. This is attributed to the fact that an increase in φ increases the skin friction and decreases the rate of heat transfer on the plate surface. Moreover, an increase in both Bi and Br increases temperature and the thermal boundary layer thickness of the nanofluid. The velocity and temperature of the nanofluid decrease as the value of the suction parameter $S > 0$ increases. An increase in the slip parameter K leads to a decrease in the velocity of the nanofluid. The results presented here in justify the physics of the problem, however, experimental data are yet to be found to further validate the formulation of this problem. It is hoped that work such as this will encourage further work in area involving electrically conducting nanofluids and the authors will highlight the interaction between the electrical conductivity of both the conventional base fluid and the nanoparticle in the presence of a magnetic field.

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