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Multi Objective Optimization Techniques: Misconceptions and Clarifications

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Abstract: *There are many techniques for solving Multi-Objective Optimization (MOO) problems. Goal programming, Weighted sum method, lexicographic method, and Ɛ-constraints method are popularly used for solving MOO problems. Under the weighted sum method, a combined objective function is formulated to solve the MOO problems. Correlation, Mean, Median, Average, New Average, Optimal average techniques have been proposed to solve the MOO problems. These techniques have not been formulated appropriately. The application of these techniques has been demonstrated with inferior examples. The solution of these techniques has also not been interpreted appropriately. The misconceptions of these techniques have been clarified in this paper. The solutions of these techniques have also been compared with Sen's MOO technique.*

Keywords: *Multi-Objective Optimization (MOO), Sen's MOO, Mean, Median, Optimal Average Techniques.*

1. Introduction

Many a time's decisions are taken with several considerations. It is not very easy to make an appropriate decision favoring all the considerations. MOO techniques are helpful to address such problems. Sen's MOO [1] was applied by several research workers[2][3] [4][5][6][7][8] [9][10] [11] [12][13] in agriculture for formulating appropriate land use plans for maximizing income, employment and minimizing irrigation, fertilizer**,** CO_2 emissions etc. There are several alternative techniques of Mean, Median, Optimal average etc [14][15][16][17][18][19][20] used to solve the MOO problems. These alternative techniques have not been formulated appropriately. The problem of multi-dimensional aggregation has not been addressed. Applications of these techniques have been demonstrated with inferior examples. The solutions have also not been interpreted correctly. The examples using the MOO techniques have been reproduced here with correct interpretations.

2. Methodology

2.1. Multi-Objective Optimization Methods

The Multi-Objective Optimization problem can be described as:

Optimize Z= [Max. Z_1, Max. Z_2 ......Max. Z_r Min. Z_{r+1}.......Min. Z_s]

Subject to:

$AX = b$ and $X \geq 0$

2.2. Formulation of Multi-Objective Function

All the objective functions to be optimized are combined as detailed below:

$$\text{Max. } Z = \frac{\sum_{j=1}^{r} Zj}{Wj\,,\ mean\,,\ median\ \ or\ optimal\ \ average} - \frac{\sum_{j=r+1}^{s} Zj}{Wj\,,\ mean\,,\ median\ \ or\ optimal\ \ average}$$

Subject to:

$AX = b$ and $X \geq 0$

$w_j > 0$ for j=1,2.......s

Where;

W_j = Individual Optima of J^{th} objective function (Sen's MOO technique)

Mean= Average of absolute values of all the Individual Optima

Median= Median of absolute values of all the Individual Optima

Optimal Average= Average of Least absolute optimal values of maximization &minimization

3. Formulation of Multi-Objective Function

3.1. Sen's MOO Technique

The objective functions may be of different dimensions. The multi-objective function is formulated by weighting each objective function by inverse of individual optima. This make the each objective function dimension free. The deviation of sum of weighted objective functions of maximization and minimization is maximized to obtain compromise solution.

3.2. Mean. Median and Optimal Average Techniques

Under these techniques the multi-objective function is formulated by weighting each objective function by inverse of mean, median and optimal average. However mean, median and optimal average are estimated using individual optima of the objective functions under consideration. If the objective functions are of different dimensions, the estimation of mean, median and optimal average is not logical.

4. Numerical Examples

The applications of these techniques using the examples demonstrated in research papers [15] [16] have been reproduced here.

Example 1:

Max. $Z_1 = (3X_1 - 2X_2)/ (X_1 + X_2 + 1)$

Max. $Z_2 = (9X_1 + 3X_2)/ (X_1 + X_2 + 1)$

Max. $Z_3 = (3X_1 - 5X_2)/ (2X_1 + 2X_2 + 2)$

Min. $Z_4 = (-6X_1 + 2X_2)/ (2X_1 + 2X_2 + 2)$

Min. $Z_5 = (-3X_1 - X_2)/ (X_1 + X_2 + 1)$

Subject to;

$X_1 + X_2 \leq 2$

$9X_1 + X_2 \leq 9$

$X_1, X_2 \geq 0$

Example 2:

$$Max. Z_1 = \frac{(2x_1 + x_2 + 1)(2x_1 + x_2 + 2)}{(3x_1 + 3x_2 + 3)}$$

$$Max. Z_2 = \frac{(6x_1 + 3x_2 + 3)(4x_1 + 2x_2 + 4)}{(2x_1 + 2x_2 + 2)}$$

$$Max. Z_3 = \frac{(8x_1 + 4x_2 + 4)(6x_1 + 3x_2 + 6)}{(5x_1 + 5x_2 + 5)}$$

$$Min. Z_4 = \frac{(10x_1 + 5x_2 + 5)(-8x_1 - 4x_2 - 8)}{(7x_1 + 7x_2 + 7)}$$

$$Min. Z_5 = \frac{(-4x_1 - 2x_2 - 2)(6x_1 + 3x_2 + 6)}{(6x_1 + 6x_2 + 6)}$$

$$Min. Z_6 = \frac{(-2x_1 - x_2 - 1)(4x_1 + 2x_2 + 4)}{(9x_1 + 9x_2 + 9)}$$

Subject to:

$$x_1 + 2x_2 \leq 4$$

$$3x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

5. Solution

All the objective functions of the example 1 have been optimized individually and the solution is presented in following Table1.

Table1. *Solution of single objective optimization*

Item	Individual Optimization				
	Max. Z_1	Max. Z_2	Min. Z_3	Min. Z_4	Min. Z_5
X_1, X_2	1, 0	1, 0	1, 0	1, 0	1, 0
Z_1	3/2	3/2	3/2	3/2	3/2
Z_2	9/2	9/2	9/2	9/2	9/2
Z_3	3/4	3/4	3/4	3/4	3/4
Z_4	-3/2	-3/2	-3/2	-3/2	-3/2
Z_5	-3/2	-3/2	-3/2	-3/2	-3/2

The Table 1 clearly indicates that all of the above objective functions have a unique solution (values of decision variables X_1 & X_2). There are no conflicts amongst objectives. Hence, the above example is not appropriate to assess the suitability of MOO techniques. The example was solved using MOO techniques and the solution is given in Table 2.

Table2. *Solution of Multi-Objective Optimization*

Item	Multi-Objective Optimization			
	Z_{mean}	Z_{median}	$Z_{optimal\ average}$	Sen's MOO
X_1, X_2	1, 0	1, 0	1, 0	1, 0
Value of modified objective function	5	6.5	8.67	5
Z_1	3/2	3/2	3/2	3/2
Z_2	9/2	9/2	9/2	9/2
Z_3	3/4	3/4	3/4	3/4
Z_4	-3/2	-3/2	-3/2	-3/2
Z_5	-3/2	-3/2	-3/2	-3/2

The solutions of all the MOO techniques presented in Table 2 are unique. The values of decision variables and individual objective functions are all same. This reveals that all the techniques are equally efficient to solve MOO problems. However, the Values of modified objective functions were not all equal. This is due to formulation of a combined objective function using different methods. The basic purpose of MOO is to achieve all the real objectives (Z_1, Z2, Z3, Z4 and Z_5) only.

Example 2 was also solved for achieving all the objectives individually as well as simultaneously. The solution of single objective optimization and multi-objective optimization is presented in Tables 3 and 4.

Table3. *Solution of single objective optimization*

Item	Individual Optimization					
	Max. Z_1	Max. Z_2	Max. Z_3	Min. Z_4	Min. Z_5	Min. Z_6
X_1, X_2	2, 0	2, 0	2, 0	2, 0	2, 0	2, 0
Z_1	3.33	3.33	3.33	3.33	3.33	3.33
Z_2	30	30	30	30	30	30
Z_3	24	24	24	24	24	24

Z_4	-28.571	-28.571	-28.571	-28.571	-28.571	-28.571
Z_5	-10	-10	-10	-10	-10	-10
Z_6	-2.22	-2.22	-2.22	-2.22	-2.22	-2.22

Table4. *Solution of Multi-Objective Optimization*

Item	Multi-Objective Optimization			
	Z_{mean}	Z_{median}	$Z_{optimal\ average}$	Sen's MOO
X_1, X_2	2, 0	2, 0	2, 0	2, 0
Value of modified objective function	32	6.369	35.359	15
Z_1	3.33	3.33	3.33	3.33
Z_2	30	30	30	30
Z_3	24	24	24	24
Z_4	-28.571	-28.571	-28.571	-28.571
Z_5	-10	-10	-10	-10
Z_6	-2.22	-2.22	-2.22	-2.22

Table 3 clearly indicate that all the objective functions have unique optimal solution. Therefore this example is also not suitable to evaluate the multi-objective optimization techniques. The solution of multi-objective optimization techniques is given in Table 4. There is unique solution of all the multi-objective optimization techniques. All the methods are equally efficient. Similar to example 1, the values of multi-objective functions are not the same for all the multi-objective functions due to differences in their formulations.

6. Conclusion

The alternative MOO techniques of mean, median and optimal average have not been formulated appropriately to tackle the problem of multi-dimensional aggregation. The examples used to explain these techniques are inferior. The solutions of mean, median and optimal average techniques have also not been interpreted appropriately. However, the Sen's MOO technique is simple, efficient and free from multi-dimensional aggregation problem.

References

[1] Sen Chandra, A new approach for multi-objective rural development planning, *The Indian Economic Journal*, Vol.30, (4), 91-96, 1983.

[2] Sen C., Integrated multi period rural development plan for Dwarahat block, Almora (U.P.): A multi-objective programming Approach. Ph.D. thesis, G. B. P. University of Agriculture and Technology, Pantnagar, Uttrakhand, India, 1982.

[3] Sen C. and D.K. Painuli., An integrated land use planning for minimizing soil erosion in the hills of Uttar Pradesh. Indian Journal of Soil Conservation, 12(1), 1-4, 1984.

[4] Sen Chandra and P.P. Dubey, Resource use planning in Agriculture with single and Multi-objective programming approaches (A comparative study). *Journal of Scientific Research*, Vol. 44, 75-81, 1994.

[5] Gangwar L. S.,Technological Advancements and its Implications on Sustainable Agriculture- A Case Study of Nainital Tarai of Uttar Pradesh. PhD thesis Department of Agricultural Economics, Institute of Agricultural Sciences, Banaras Hindu University, Varanasi, India, 1994.

[6] Kushwaha, S. and J.E. Ochi, Economic Simulation to Alternative Resource Use for Sustainable Agriculture in Nigeria-Sen's MOP Approach. *Journal of Scientific Research* Vol.46 (1), 105-114, 1996.

[7] Singh Vipin Kumar,Agricultural Development and Environmental Pollution- A Case Study of District Varanasi. Ph.D. thesis, Department of Agricultural Economics, Institute of Agricultural Sciences, Banaras Hindu University, Varanasi, India, 2002.

[8] Sinha Hanish Kumar, Effect of Air Pollution on Peri-Urban Agriculture in Varanasi. PhD thesis, Department of Agricultural Economics, Institute of Agricultural Sciences,Banaras Hindu University, Varanasi, India, 2002.

[9] Singh P. K.,Multilevel multi-objective planning for Agriculture for district Mau, U.P. PhD thesis, Dept. of Agricultural Economics, Institute of Agricultural Sciences, Banaras Hindu University, Varanasi, India, 2005.

[10] Kumar Hraday, Economic Analysis of Fresh Water Aquaculture in Maharajganj district of Eastern Uttar Pradesh. PhD thesis. Department of Agricultural Economics, Institute of Agricultural Sciences, Banaras Hindu University, Varanasi, India, 2012.

[11] Gautam Kusumakar, Natural and Human Resource Use Planning for Vindhyan Region of Eastern Uttar Pradesh. PhD thesis, Department of Agricultural Economics, Institute of Agricultural Sciences, Banaras Hindu University, Varanasi, India, 2013.

[12] Kumari Maina, O.P.Singh and Dinesh Chand Meena, Optimizing Cropping Pattern in Eastern Uttar Pradesh Using Sen's Multi-Objective Programming Approach. *Agricultural Economics Research Review* Vol. 30 (2), 285-291, 2017.

[13] Maurya Mukesh Kumar, Livelihood Security of Farmers in Eastern Uttar Pradesh- an economic Analysis. PhD thess, Department of Agricultural Economics, Institute of Agricultural Sciences, Banaras Hindu University, Varanasi, India, 2018.

[14] Sulaiman N.A. and Hamadameen, Abdul-Qader O., Optimal Transformation Technique to Solve Multi - Objective Linear Programming Problem (MOLPP), *Journal of Kirkuk University – Scientific Studies* , vol.3, (2), 158-168, 2008.

[15] Sulaiman N. A. and Abdulrahim B. K., Using Transformation Technique To Solve Multi-Objective Linear Fractional Programming Problem. *IJRRAS*, Vol.14, (3), 559-567, 2013.

[16] Nejmaddin A. Suleiman, Maher A. Nawkhass,Transforming and Solving Multi-objective Quadratic Fractional Programming Problems by Optimal Average of Maximin & Minimax Techniques. *American Journal of Operational Research*, 3(3): 92-98, 2013.

[17] Nejmaddin A. Sulaiman, Gulnar W. Sadiq & Basiya K. Abdulrahim, New Arithmatic average technique to solve Multi-Objective Linear Fractional Programming Problem and its comparison with other techniques. *IJRRAS* Vol.18 (2), 122-131, 2014.

[18] Samsun Nahar, Md. Abdul Alim, A New Statistical Averaging Method to Solve Multi-Objective Linear Programming Problem. *International Journal of Science and Research*, Vol. 6 (8), 623-629, 2015.

[19] Akhtar Huma, Modi Geeta and Duraphe Sushma, Transforming and Optimizing Multi-Objective Quadratic Fractional Programming Problem. *International Journal of Statistics and Applied Mathematics*, Vol. 2, (1) 01-05, 2017.

[20] Akhtar Huma, Geeta Modi and Sushma Duraphe, An Appropriate Approach for Transforming and Optimizing Multi-Objective Quadratic Fractional Programming Problem. *International Journal of Mathematics Trends and Technology* Vol. 50 (2), 80-83, 2017.

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