International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 6, Issue 4, 2018, PP 1-9
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
DOI: http://dx.doi.org/10.20431/2347-3142.0604001
www.arcjournals.org

Total Chromatic Number of Middle and Total Graph of Path and Sunlet Graph

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Abstract: *A total coloring of a graph G is an assignment of colors to both the vertices and edges of G, such that no two adjacent or incident vertices and edges of G are assigned with the same colors. In this paper, we discussed the theorems some total coloring of middle graph, total graph of path and sunlet graph. Also, we obtained the total chromatic number of middle graph, total graph of path and sun let graph.*

Keywords: *Path; Sunlet graph; Middle graph; Total graph; Total coloring and Total chromatic number.*

1. Introduction

In this paper, we have considered the finite, simple and undirected graphs. Let $G=(V(G),E(G))$ be a graph with the vertex set $V(G)$ and the edge set $E(G)$ respectively. In 1965, the concept of total coloring was introduced by Behzad [1] and in 1967, he [2] came out new ideology that, the total chromatic number of complete graph and complete bi-partite graph. A total coloring of G, is a function $f:S\to C$, where $S\to V(G)\cup E(G)$ and C is a set of colors to satisfies the given conditions.

(i) no two adjacent vertices receive the same colors,

(ii) no two adjacent edges receive the same colors and

(iii) no edges and its end vertices receive the same colors.

The *total chromatic number* $\chi''(G)$ of a graph G is the minimum cardinality k such that G may have a total coloring by k colors. Behzad [1] and Vizing [13] conjectured that for every simple graph G has $\Delta(G)+1\le\chi''(G)\le\Delta(G)+2$, where $\Delta(G)$ is the maximum degree of G. This conjecture is called the Total Coloring Conjecture (TCC). Rosenfeld [9] and Vijayaditya [12] verified the TCC, for any graph G with maximum degree ≤ 3 and Kostochka [6] for maximum degree ≤ 5. In Borodin[4] verified the total coloring conjecture (TCC) for maximum degree ≥ 9 in planar graphs. In 1992, Yap and Chew [14] proved that any graph G has a total coloring with at most $\Delta(G)+2$ colors, if $\Delta(G)\ge|V(G)|-5$. In recent era, total coloring have been extensively studied in different families of graphs. Mohan et.al [7] given the tight bound of Behzad and Vizing conjecture for Corona product of certain classes of graph. Muthuramakrishnan et.al [8] proved that Total Chromatic Number of Line, Middle and Total graph of star and square graph of Bistar graph. Vaidya et.al [11] prove that the total chromatic number of middle graph, total graph, shadow graph of cycle and one point union of cycle. Sudha et.al [10] prove that the lower and upper bound for the total chromatic number of S(n, m) graph. In this paper, we determine the total chromatic number of $M(P_n),T(P_n),M(S_n)$ and $T(S_n)$.

2. Preliminaries

Definition 2.1. A Path in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence.

Definition 2.2. The middle graph [5] of a graph G denoted by $M(G)$ is define as follows, the vertex set of $M(G)$ is $V(G) \cup E(G)$. Two vertices x, y in the vertex set of $M(G)$ are adjacent in $M(G)$ in case one of the following condition holds: (i) x, y are in $E(G)$ and x, y is adjacent in G (ii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

Definition 2.3. The total graph [3] of a graph G, denoted by $T(G)$ is define as, the vertex set of $T(G)$ is $V(G) \cup E(G)$. Two vertices x, y in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one of the following condition holds: (i) x, y are in $V(G)$ and x is adjacent to y in G. (ii) x, y are in $E(G)$ and x, y is adjacent in G. (iii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

Definition 2.4. The n −sunlet graph on $2n$ vertices is obtained by attaching n pendent edges to the cycle C_n and is denoted by S_n.

3. Total Coloring of Middle Graph of Path

Theorem 3.1. Let $M(P_n)$ be the middle graph of path of order n. Then $\chi''(M(P_n)) = 5, \quad n \geq 4.$

Proof: Let $V(P_n) = \{v_i : 1 \leq i \leq n\}$ and $E(P_n) = \{e_i = v_i v_{i+1} : 1 \leq i \leq n-1\}$. By the definition of middle graph, each edge $\{e_i = v_i v_{i+1} : 1 \leq i \leq n-1\}$ in P_n is subdivided by the vertices $\{u_i : 1 \leq i \leq n-1\}$ in $M(P_n)$. The vertex set and the edge set $M(P_n)$ is given by $V\big(M(P_{n)})\big) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n-1\}$, where $\{u_i : 1 \leq i \leq n-1\}$ is the vertices of $M(P_n)$ corresponding to the edge $\{v_i v_{i+1} : 1 \leq i \leq n-1\}$ of P_n and

$$E\big(M(P_{n)})\big) = \{v_i u_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} : 1 \leq i \leq n-1\}.$$

Now we define the total coloring f, such that $f: S \to C$ as follows, where $S = V(M(P_n)) \cup E(M(P_n))$ and $C = \{1, 2, 3, 4, 5\}$.

$$f(u_i) = \begin{cases} 3, & if\ i \equiv 0 \ (mod\ 3) \\ 1, & if\ i \equiv 1 \ (mod\ 3) \text{ for } 1 \leq i \leq n-1 \\ 2, & if\ i \equiv 2 \ (mod\ 3) \end{cases}$$

$$f(v_i) = \begin{cases} 1, & if\ i \equiv 1 (mod\ 3) \\ 2, & if\ i \equiv 2 (mod\ 3) \text{ for } 1 \leq i \leq n \\ 3, & if\ i \equiv 0 \ (mod\ 3) \end{cases}$$

$$f(u_i u_{i+1}) = \begin{cases} 4, & if\ i\ is\ odd \\ 5, & if\ i\ is\ even \end{cases} \text{ for } 1 \leq i \leq n-1$$

$$f(v_i u_i) = \begin{cases} 2, & if\ i \equiv 1 \ (mod\ 3) \\ 3, & if\ i \equiv 2 \ (mod\ 3) \text{ for } 1 \leq i \leq n-1 \\ 1, & if\ i \equiv 0 \ (mod\ 3) \end{cases}$$

$$f(u_i v_{i+1}) = \begin{cases} 1, & if\ i \equiv 1 \ (mod\ 3) \\ 2, & if\ i \equiv 2 \ (mod\ 3) \text{ for } 1 \leq i \leq n-1 \\ 3, & if\ i \equiv 0 \ (mod\ 3) \end{cases}$$

It is clear that the above rule of total coloring, the graph $M(P_n)$ is properly total colored with 5colors. Hence the total chromatic number of the middle graph of $M(P_n)$, $\chi''(M(P_n)) = 5$.

Illustration 3.2: Consider the middle graph of a path P_6

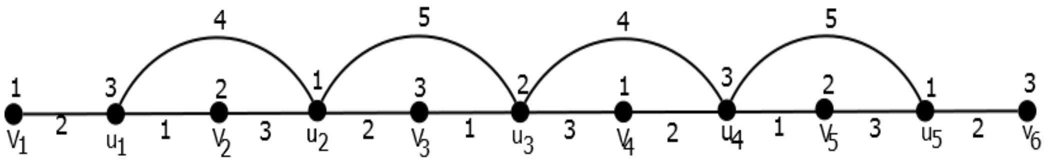


Figure 1. Middle graph of path $\boldsymbol{P_6}$

By applying the above rule of total coloring pattern as given in the theorem 3.1 the colors {1, 2, 3, 4, 5} to the vertices and edges are received as shown in Fig 1. Thus the total chromatic number of middle graph of path is 5.

4. Total Coloring of Total Graph of Path

Theorem 4.1. Let $T(P_n)$ be the total graph of path of order n. Then $\chi''(T(P_n)) = 5, \quad n \geq 3.$

Proof: Let $V(P_n) = \{v_i : 1 \leq i \leq n\}$ and $E(P_n) = \{e_i = v_i v_{i+1} : 1 \leq i \leq n-1\}$. By the definition of total graph, each edge $\{e_i = v_i v_{i+1} : 1 \leq i \leq n-1\}$ in P_n is subdivided by the vertices $\{u_i : 1 \leq i \leq n-1\}$ in $T(P_n)$. The vertex set and the edge set of $T(P_n)$ is given by $V(T(P_n)) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n-1\}$, where $\{u_i : 1 \leq i \leq n-1\}$ is the vertices of $T(P_n)$ corresponding to the edge $\{v_i v_{i+1} : 1 \leq i \leq n-1\}$ of P_n and

$$E(T(P_n)) = \begin{cases} \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \\ \{v_i u_i : 1 \leq i \leq n\} \cup \{u_i v_{i+1} : 1 \leq i \leq n-1\} \end{cases}$$

Now we define total coloring f, such that $f : S \to C$ as follows, where $S = V(T(P_n)) \cup E(T(P_n))$ and $C = \{1, 2, 3, 4, 5\}$

$$f(v_i) = \begin{cases} 1, & if\ i \equiv 1\ (mod\ 3) \\ 2, & if\ i \equiv 2\ (mod\ 3) \\ 3, & if\ i \equiv 0\ (mod\ 3) \end{cases} \text{ for } 1 \leq i \leq n$$

$$f(u_i) = \begin{cases} 3, & if\ i \equiv 1\ (mod\ 3) \\ 1, & if\ i \equiv 2\ (mod\ 3) \\ 2, & if\ i \equiv 0\ (mod\ 3) \end{cases} \text{ for } 1 \leq i \leq n-1$$

$$f(u_i u_{i+1}) = \begin{cases} 4, & if\ i\ is\ odd \\ 5, & if\ i\ is\ even \end{cases} \text{ for } 1 \leq i \leq n-1$$

$$f(v_i v_{i+1}) = \begin{cases} 4, & if\ i\ is\ odd \\ 5, & if\ i\ is\ even \end{cases} \text{ for } 1 \leq i \leq n-1$$

$$f(v_i u_i) = \begin{cases} 2, & if\ i \equiv 1\ (mod\ 3) \\ 3, & if\ i \equiv 2\ (mod\ 3) \\ 1, & if\ i \equiv 0\ (mod\ 3) \end{cases} \text{ for } 1 \leq i \leq n-1$$

$$f(u_i v_{i+1}) = \begin{cases} 1, & if\ i \equiv 1\ (mod\ 3) \\ 2, & if\ i \equiv 2\ (mod\ 3) \\ 3, & if\ i \equiv 0\ (mod\ 3) \end{cases} \text{ for } 1 \leq i \leq n-1$$

It is clear that the above rule of total coloring, the graph$T(P_n)$ is properly total colored with 5colors. Hence the total chromatic number of the total graph of path graph $T(P_n)$, $\chi''(T(P_n) = 5.$

Illustration 4.2: Consider the total graph of a path P_6

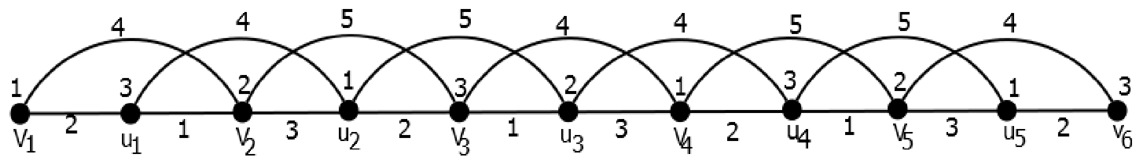


Figure 2. *Total graph of path* $\boldsymbol{P_6}$

By applying the above rule of total coloring pattern as given in the theorem 4.1, the colors {1, 2, 3, 4, 5} to the vertices and edges are received as shown in Fig 2. Thus the total chromatic number of total graph of path is 5.

5. Total Coloring of Middle Graph of Sunlet Graph

Theorem 5.1. Let $M(S_n)$ be the middle graph of sun let graph S_n. Then

$$\chi''(M(S_n)) = \begin{cases} 7, & if\ n\ is\ even \\ 8, & if\ \ n\ is\ odd \end{cases}$$

Proof: Let $\{u_1, u_2, u_3, \ldots\ldots, u_n\}$ and $\{v_1, v_2, v_3, \ldots\ldots, v_n\}$ be the vertices of sunlet graph and let $\{e_1, e_2, e_3, \ldots\ldots, e_n\}$ and $\{e_1', e_2', e_3', \ldots\ldots, e_n'\}$ be the edges of the sunlet graph, where $e_i = v_i v_{i+1}$ for $1 \le i \le n-1$ be the edges, e_n is the edge $v_n v_1$ and e_i' is the edges $v_i u_i$ for $(1 \le i \le n)$. By the definition of middle graph, each edges $\{e_i' = v_i u_i : 1 \le i \le n\}$, $\{e_i = v_i v_{i+1} : 1 \le i \le n-1\}$ and $\{v_n v_1\}$ in $M(S_n)$ are subdivided by the vertices $\{v_i' : 1 \le i \le n\}$ and $\{u_i' : 1 \le i \le n\}$ in $M(S_n)$. Therefore, the vertex set and the edge set is given by $V\big(M(S_{n)}\big) = \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\} \cup \{v_i' : 1 \le i \le n\} \cup \{u_i' : 1 \le i \le n\}$, where u_i' and v_i' represents the edges $e_i' (1 \le i \le n)$ and e_i $(1 \le i \le n-\ 1)$ respectively,

$$E\big(M(S_{n)}\big) = \begin{cases} (v_i v_i' : 1 \le i \le n) \cup (u_i' v_i' : 1 \le i \le n) \cup (v_i' u_{i+1}' : 1 \le i \le n-1) \cup \\ (v_i' v_{i+1} : 1 \le i \le n-1\ ) \cup (v_i u_i' : 1 \le i \le n) \cup (v_n' u_1') \cup (v_n' v_1) \cup \\ (u_i u_i' : 1 \le i \le n) \cup (v_i' v_{i+1}' : 1 \le i \le n-1) \cup (v_n' v_1') \end{cases}$$

We define the total coloring f, such that $f: S \to C$ as follows, where $S = V(M(S_n)) \cup E(M(S_n))$ and $C = \{1, 2, 3, 4, 5, 6, 7, 8\}$. The total coloring is obtained by coloring these vertices and edges as follows:

Case (i): When n is even

$$f(u_i) = 7; \quad f(u_i') = 6 \quad \text{for } 1 \le i \le n$$

$$f(v_i) = \begin{cases} 1, & if\ i \equiv 1(mod\ 2) \\ 3, & if\ i \equiv 0(mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i') = \begin{cases} 2, & if\ i \equiv 1(mod\ 2) \\ 4, & if\ i \equiv 0(mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i v_i') = \begin{cases} 4, & if\ i \equiv 1(mod\ 2) \\ 2, & if\ i \equiv 0(mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i' v_{i+1}) = \begin{cases} 1, & if\ i \equiv 1(mod\ 2) \\ 3, & if\ i \equiv 0(mod\ 2) \end{cases} \text{for } 1 \le i \le n-1$$

$$f(v_n' v_1) = 3;$$

$$f(u_i' v_i') = \begin{cases} 3, & if\ i \equiv 1(mod\ 2) \\ 1, & if\ i \equiv 0(mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i' u_{i+1}') = 7; \ f(v_n' u_1') = 7; \quad \text{for } 1 \le i \le n-1$$

$$f(v_i' v_{i+1}') = \begin{cases} 5, & if\ i \equiv 1(mod\ 2) \\ 6, & if\ i \equiv 0(mod\ 2) \end{cases} \quad \text{for } 1 \le i \le n-1$$

$$f(v_i u_i') = \begin{cases} 2, & if\ i \equiv 1(mod\ 2) \\ 4, & if\ i \equiv 0(mod\ 2) \end{cases} \quad \text{for } 1 \le i \le n$$

$$f(v_n' v_1') = 6; \ f(u_i u_i') = 5 \quad \text{for } 1 \le i \le n$$

It is clear that the above rule of total coloring, the graph $M(S_n)$ is properly total colored with 7colors. Hence the total chromatic number of the middle graph of sunlet graph S_n, $\chi''(M(S_n) = 7$ for n is even.

Case (ii): When n is odd

$$f(v_i) = 7; \quad f(u_i') = 4; \quad f(u_i) = 8 \text{ for } 1 \le i \le n$$

$$f(v_i') = \begin{cases} 1, & if\ i \equiv 1(mod\ 2) \\ 2, & if\ i \equiv 0(mod\ 2) \end{cases} \text{for } 1 \le i \le n-1$$

$$f(v_n') = 3$$

$$f(v_i' v_{i+1}') = \begin{cases} 6, & if\ i \equiv 1(mod\ 2) \\ 7, & if\ i \equiv 1(mod\ 2) \end{cases} \quad \text{for } 1 \le i \le n-1$$

$$f(v_n^{'}v_1^{'}) = 8; \quad f(v_i v_i^{'}) = 4 \quad \text{for } 1 \le i \le n$$

$$f(v_i^{'}v_{i+1}) = 5; \quad f(v_n^{'}v_1) = 5 \quad \text{for } 1 \le i \le n-1$$

$$f(u_i^{'}v_i^{'}) = \begin{cases} 2, & if \ \ i \ \equiv 1(mod\ 2) \\ 1, & if \ \ i \ \equiv 1(mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i^{'}u_{i+1}^{'}) = 3 \quad \text{for } 1 \le i \le n-1$$

$$f(u_i u_i^{'}) = 5\,; \quad f(v_i u_i^{'}) = 8 \ \text{ for } 1 \le i \le n, f(v_n^{'}u_1^{'}) = 1;$$

It is clear that the above rule of total coloring, the graph $M(S_n)$ is properly total colored with 8colors. Hence the total chromatic number of the middle graph of sunlet graph $M(S_n)$, $\chi''(M(S_n) = 8$, for n is odd.

Illustration 5.2: Consider the middle graph of a sun let graph S_6

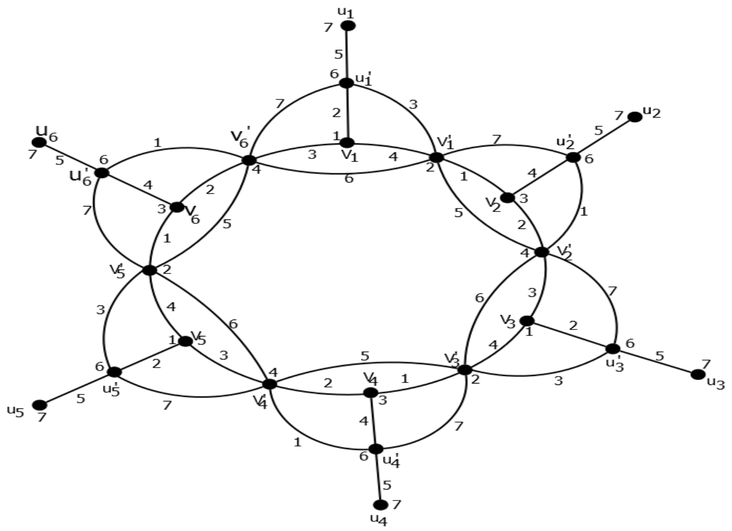


Figure 3. *Middle graph of a sun let graph* $\boldsymbol{S_6}$

By applying the above rule of total coloring pattern as given in the theorem 5.1, of case (i) the colors {1, 2, 3, 4, 5, 6, 7} to these vertices and edges are received as shown in Fig 3. Thus the total chromatic number of middle graph of sunlet graph is 7 for n is even.

Illustration 5.3: Consider the middle graph of a sun let graph S_5

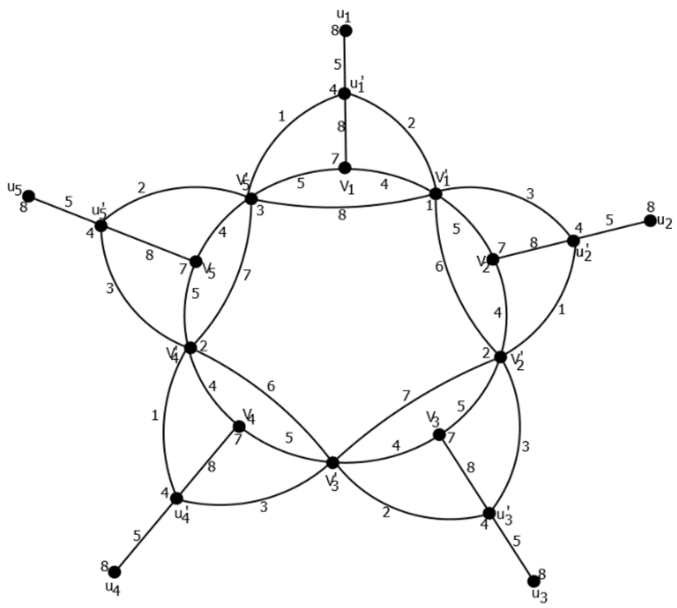


Figure 4. *Middle graph of a sun let graph* $\boldsymbol{S_5}$

By applying the above rule of total coloring pattern as given in the theorem 5.1, of case (ii) the colors$\{1, 2, 3, 4, 5, 6, 7, 8\}$ to these vertices and edges are received as shown in Fig 4. Thus the total chromatic number of middle graph of sunlet graph is 8 for n is odd.

6. Total Coloring of Total Graph of Sunlet Graph

Theorem 6.1. Let $T(S_n)$ be the total graph of sunlet graph S_n. Then

$$\chi''(T(S_n)) = \begin{cases} 7, & if\ n\ is\ even \\ 8, & if\ \ n\ is\ odd \end{cases}$$

Proof: Let $\{u_1, u_2, u_3, \dots\dots, u_n\}$ and $\{v_1, v_2, v_3, \dots\dots, v_n\}$ be the vertices of the sunlet graph and let $\{e_1, e_2, e_3, \dots\dots, e_n\}$ and $\{e_1', e_2', e_3', \dots\dots, e_n'\}$ be the edges of the sunlet graph, where $e_i = v_i v_{i+1}$: $1 \le i \le n-1$ be the edges, e_n is the edge $v_n v_1$ and e_i' is the edges $v_i u_i (1 \le i \le n)$. By the definition of total graph, each edges $\{e_i' = v_i u_i : 1 \le i \le n\}$, $\{e_i = v_i v_{i+1} : 1 \le i \le n-1\}$ and $\{v_n v_1\}$ in $T(S_n)$ are subdivided by the vertices $\{v_i': 1 \le i \le n\}$ and $\{u_i': 1 \le i \le n\}$ in $T(S_n)$. Therefore, the vertex set and the edge set is given by $V(T(S_{n)})) = \{v_i: 1 \le i \le n\} \cup \{u_i: 1 \le i \le n\} \cup \{v_i': 1 \le i \le n\} \cup \{u_i': 1 \le i \le n\}$, where u_i' and v_i' represents the edges e_i' $(1 \le i \le n)$ and e_i $(1 \le i \le n-\ 1)$ respectively,

$V(T(S_{n)})) = \{v_i: 1 \le i \le n\} \cup \{u_i: 1 \le i \le n\} \cup \{v_i': 1 \le i \le n\} \cup \{u_i': 1 \le i \le n\}$, where u_i' and v_i' represents the edges e_i' and e_i $(1 \le i \le n)$ respectively,

$$E(T(S_{n)})) = \begin{cases} (v_i v_i': 1 \le i \le n) \cup (u_i' v_i': 1 \le i \le n) \cup (v_i' u_{i+1}': 1 \le i \le n-1) \cup \\ (v_i' v_{i+1}: 1 \le i \le n-1\ ) \cup (v_i u_i': 1 \le i \le n) \cup (v_n' u_1') \cup (v_n' v_1) \cup \\ (v_n' v_1') \cup (u_i u_i': 1 \le i \le n) \cup (v_i' v_{i+1}': 1 \le i \le n-1) \cup (u_i v_i: 1 \le i \le n) \end{cases}$$

We define total coloring f, such that$f: S \to C$ as follows, where $S = V(T(S_n)) \cup E(T(S_n))$ and $C = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Case (i): When n is even

$$f(u_i) = 7; \qquad f(u_i') = 6: \ \text{for } 1 \le i \le n$$

$$f(v_i) = \begin{cases} 1, & if\ \ i\ \equiv 1 (mod\ 2) \\ 3, & if\ \ i\ \equiv 0 (mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i') = \begin{cases} 2, & if\ \ i\ \equiv 1 (mod\ 2) \\ 4, & if\ \ i\ \equiv 0 (mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i v_i') = \begin{cases} 4, & if\ \ i\ \equiv 1 (mod\ 2) \\ 2, & if\ \ i\ \equiv 0 (mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i' v_{i+1}) = \begin{cases} 1, & if\ \ i\ \equiv 1 (mod\ 2) \\ 3, & if\ \ i\ \equiv 0 (mod\ 2) \end{cases} \text{for } 1 \le i \le n-1$$

$$f(v_n' v_1) = 3; \qquad f(u_i v_i) = 6\ \text{ for } 1 \le i \le n$$

$$f(u_i' v_i') = \begin{cases} 3, & if\ \ i\ \equiv 1 (mod\ 2) \\ 1, & if\ \ i\ \equiv 0 (mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i' u_{i+1}') = 7 \quad \text{for } 1 \le i \le n-1; \qquad f(v_n' u_1') = 7;$$

$$f(v_i' v_{i+1}') = \begin{cases} 5, & if\ \ i\ \equiv 1 (mod\ 2) \\ 6, & if\ \ i\ \equiv 0 (mod\ 2) \end{cases} \text{for } 1 \le i \le n-1$$

$$f(v_i u_i') = \begin{cases} 2, & if\ \ i\ \equiv 1 (mod\ 2) \\ 4, & if\ \ i\ \equiv 0 (mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_n' v_1') = 6; \ f(u_i u_i') = 5\ \text{ for } 1 \le i \le n$$

It is clear that the above rule of total coloring, the graph $T(S_n)$ is properly total colored with 7colors. Hence the total chromatic number of the total graph of sun let graph $T(S_n)$, $\chi''(T(S_n) = 7$, for n is even.

Case (ii): When n is odd

$$f(v_i) = 7;\ \ f(u_i^{'}) = 4;\quad f(u_i) = 8 \text{ for } 1 \le i \le n$$

$$f(v_i^{'}) = \begin{cases} 1, & if\ \ i\ \equiv 1(mod\ 2) \\ 2, & if\ \ i\ \equiv 0(mod\ 2) \end{cases} \text{for } 1 \le i \le n-1$$

$$f(v_n^{'}) = 3;\quad f(u_i v_i) = 6 \quad \text{for } 1 \le i \le n$$

$$f(v_i^{'} v_{i+1}^{'}) = \begin{cases} 6, & if\ \ i\ \equiv 1(mod\ 2) \\ 7, & if\ \ i\ \equiv 0(mod\ 2) \end{cases} \text{for } 1 \le i \le n-1$$

$$f(v_n^{'} v_1^{'}) = 8;\qquad f(v_i v_i^{'}) = 4 \quad \text{for } 1 \le i \le n$$

$$f(v_i^{'} v_{i+1}) = 5 \quad \text{for} \quad 1 \le i \le n-1\,;\qquad f(v_n^{'} v_1) = 5$$

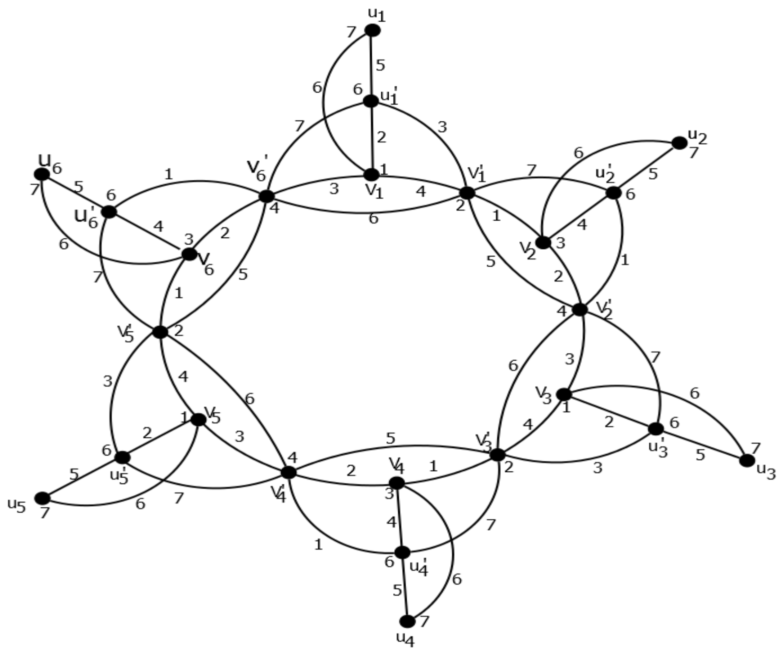
$$f(u_i^{'} v_i^{'}) = \begin{cases} 2, & if\ \ i\ \equiv 1(mod\ 2) \\ 1, & if\ \ i\ \equiv 0(mod\ 2) \end{cases} \text{for } 1 \le i \le n$$

$$f(v_i^{'} u_{i+1}^{'}) = 3\ for\ 1 \le i \le n-1$$

$$f(u_i u_i^{'}) = 5;\quad f(v_n^{'} u_1^{'}) = 1;\qquad f(v_i u_i^{'}) = 8 \quad \text{for} \quad 1 \le i \le n-1$$

It is clear that the above rule of total coloring, the graph$T(S_n)$ is properly total colored with 8colors. Hence the total chromatic number of the total graph of sun let graph$T(S_n)$, $\chi''(T(S_n) = 8$, for n is odd.

Illustration 6.2: Consider the total graph of a sun let graph $T(S_5)$



F**igure 5.** *Total graph of a sun let graph* $\boldsymbol{T(S_6)}$

By applying the above method of total coloring pattern as given in the theorem 6.1 of case (i) the colors{1, 2, 3, 4, 5, 6, 7} to these vertices and edges are received as shown in Fig 5 . Thus the total chromatic number of total graph of sunlet graph is 7 for n is odd.

Illustration 6.3: Consider the total graph of a sun let graph $T(S_5)$

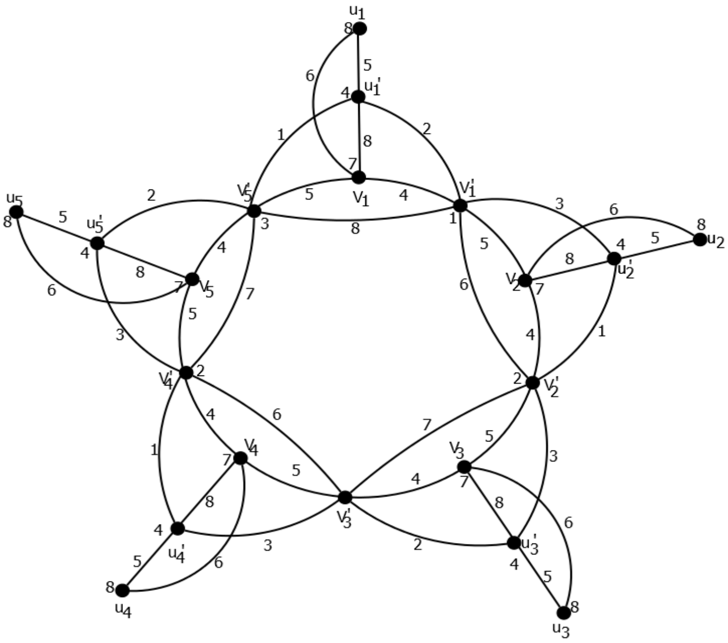


Figure 6. *Total graph of a sun let graph* $T(S_5)$

By applying the above method of total coloring pattern as given in the theorem 6.1, of case (ii) the colors{1, 2, 3, 4, 5, 6, 7, 8} to these vertices and edges are received as shown in Fig 6. Thus the total chromatic number of total graph of sunlet graph is 8 for n is odd.

7. Conclusion

The total coloring of the middle and total graph of path and sunlet graph are discussed in this paper and found the total chromatic numbers to be

1. $\chi''(M(P_n)) = 5, n \geq 4.$
2. $\chi''(T(P_n)) = 5, n \geq 3.$
3. $\chi''(M(S_n)) = \begin{cases} 7, n \text{ is even} \\ 8, n \text{ is odd} \end{cases}$
4. $\chi''(T(S_n)) = \begin{cases} 7, n \text{ is even} \\ 8, n \text{ is odd} \end{cases}$

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Citation: *D. Muthuramakrishnan, G. Jayaraman, " Total Chromatic Number of Middle and Total Graph of Path and Sunlet Graph ", International Journal of Scientific and Innovative Mathematical Research, vol. 6, no. 4, p. 1-9, 2018., http://dx.doi.org/10.20431/2347-3142.0604001*