International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 6, Issue 3, 2018, PP 11-24
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
DOI: http://dx.doi.org/10.20431/2347-3142.0603002
www.arcjournals.org

The Fischer-Cliford Matrices and Character Table of the Group $2^5: GL(4,2)$

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Abstract: *The purpose of this paper is constructing the Fischer-Clifford matrices and the character tables for the group* $\mathbf{2^5}: \boldsymbol{GL}(\mathbf{4},\mathbf{2})$.

Keywords: *linear groups, group extensions, character table, Clifford theory, inertia groups, Fischer-Clifford matrix.*

1. Introduction

The theory of Clifford-Fischer matrices, which is based on Clifford Theory [1] which was developed by B. Fischer [2]. Let $\bar{G} = 2^5: GL(4,2)$ be the split extension of $N = 2^5$ by $GL(4,2)$ where N is the vector space of dimension 5 over GF(2) on which G acts naturally. The aim of this paper is to construct the character table of $\bar{G}$ by using the technique of Fischer-Clifford matrix $M(g)$ for each class representative g of G and the character tables of the inertia factor groups H_i of the inertia groups $\overline{H_i} = 2^5: H_i$. we use the properties of the Fischer-Clifford matrices discussed in ([3], [4],[5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15]) to compute entries of these matrices. The Fischer-Clifford matrix $M(g)$ will be portioned row-wise into blocks, where each block corresponds to an inertia group $\overline{H_i}$. Now using the columns of character table of the inertia factor H_i of $\overline{H_i}$ which correspond to classes of H_i which fuse to the class [g] in G and multiply these columns by the rows of the Fischer-Clifford matrix $M(g)$ that correspond to $\overline{H_i}$. Thus, we construct the portion of the character table of $\bar{G}$ which is in the block corresponding to $\overline{H_i}$ for the classes of $\bar{G}$ that come from the coset Ng. For more information about this technique see ([3], [4],[5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15]) . The character table of $\bar{G}$ will be divided row-wise into blocks where each block corresponding to an inertia group $\overline{H_i} = N: H_i$. The computations have been carried out with the aid of Maxima [16], MAGMA [17]and GAP [18] , Finally we will follow the notion of Atlas [19].

2. Theory of Fischer-Clifford Matrices

Let $\bar{G} = N: G$ be a split extension of N by G. Then for $\theta \in Irr(N)$, we define $\bar{H} = \{x \in \bar{G}: \theta^x = \theta = I\bar{G}\theta$ and $H=x\in G: \theta g=\theta=IG\theta$ where $IG\theta$ is the stabilizer of θ in the action of G on Irr(N), we have that $I_{\bar{G}}(\theta)$ is a subgroup of $\bar{G}$ and N is normal subgroup in $I_{\bar{G}}(\theta)$. Also $[\bar{G}: I_{\bar{G}}(\theta)]$ is the size of the orbit containing θ . Then it can be shown that $\bar{H} = N: H$, where $\bar{H}$ is the inertia group of θ in $\bar{G}$. The inertia factor $\bar{H}/N \cong H$ can be regarded as the inertia group of θ in the factor group $\bar{G}/N \cong G$. Define θ^g by $\theta^g(n) = \theta(gng^{-1})$for $g \in \bar{G}$, $n \in N$, $\theta^g \in Irr(N)$. We say that θ is extendible to $\bar{H}$ if there exists $\varphi \in Irr(\bar{H})$ such that $\downarrow_N = \theta$. If θ is extendible to $\bar{H}$, then by Gallagher [20], we have $\{\alpha: \alpha \in Irr(\bar{H}), < \alpha \downarrow_N, \theta > \neq 0\} = \{\beta\varphi: \beta \in Irr(\bar{H}/N)\}$. Let $\bar{G}$ has the property that every irreducible character of N can be extended to its inertia group. Now let $\theta_1 = 1_N, \theta_2, \ldots, \theta_t$ be representatives of the orbits of $\bar{G}$ on Irr(N), $\overline{H_i} = I_{\bar{G}}(\varphi_i), 1 \le i \le t, \varphi_i \in Irr(\overline{H_i})$ be an extension of θ_i to $\overline{H_i}$ and $\beta \in Irr(\overline{H_i})$ such that $N \subseteq Ker(\beta)$. Then it can be shown that $Irr(\bar{G}) = \cup_{i=1}^{t}\{(\beta\varphi_i)^{\bar{G}}: \beta \in Irr(\overline{H_i}), N \subseteq Ker(\beta)\} = \cup_{i=1}^{t}\{(\beta\varphi_i)^{\bar{G}}: \beta \in (\overline{H_i}/N)\}$

Hence the irreducible characters of $\bar{G}$ will be divided into blocks, where each block corresponds to an inertia group $\overline{H_i}$. Let $\overline{H_i}$ be the inertia factor group and φ_i be an extension of $\theta_i$to $\overline{H_i}$. Take $\theta_1 = 1_N$ as the identity character of N, then $\overline{H_1} = \bar{G}$and $H_1 \cong G$. Let $X(g) = \{x_1, x_2, \dots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\bar{G}$ from the coset $N\bar{g}$ whose images under the natural homomorphism$\bar{G} \to G$ are in [g] and we take $x_1 = \bar{g}$. We define

$$R(g) = \{(i, y_k): 1 \le i \le t,\ H_i \cap [g] \ne 0, 1 \le k \le r\}$$

and we note that y_k runs over representatives of the conjugacy classes of elements of H_i which fuse into [g] in G. Then we define the Fischer-Clifford matrix M(g) by $M(g) = \left(a^j_{(i,y_k)}\right)$, where $a^j_{(i,y_k)} = \sum_l^t \frac{|C_{\bar{G}}(x_j)|}{|C(y_{lk})|} \varphi_i(y_{lk})$ with columns indexed by X(g) and rows indexed by R(g) and where $\sum_l^t$ is the summation over all l for which $y_{lk} \sim x_j$ in $\bar{G}$. Then the partial character table of $\bar{G}$ on the classes $\{x_1, x_2, \dots, x_{c(g)}\}$ is given by $\begin{bmatrix} C_1(g)M_1(g) \\ C_2(g)M_2(g) \\ \vdots \\ C_t(g)M_t(g) \end{bmatrix}$ where the Fischer-Clifford matrix $M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix}$ is divided into blocks $M(g)$ with each block corresponding to an inertia group $\overline{H_i}$ and $C_i(g)$ is the partial character table of H_i consisting of the columns corresponding to the classes that fuse into [g] in G. We can also observe that the number of irreducible characters of $\bar{G}$ is the sum of the number of irreducible characters of the inertia factors $H_i's$. The group $\bar{G} = 2^5{:}GL(4,2)$ is a split extension with 2^5 abelian and therefore by Mackey's theorem (see [8] - Theorem 4.1.12) , we have each irreducible character of 2^5 can be extended to its inertia group in $\bar{G}$. Hence by the above theoretical outline we can fully determine the character table of $\bar{G} = 2^5{:}GL(4,2)$.

3. THE CONJUGACY CLASSES OF $\bar{G} = 2^5{:}GL(4,2)$

In this section, we will use the method of coset analysis to determine the conjugacy classes of $\bar{G} = 2^5{:}GL(4,2)$. We refer the reader to ([3], [4],[5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15]) for full details and background material regarding the method of coset analysis. Most of the information, which involved the conjugacy classes and permutation characters, were obtained by using direct computations in GAP [18] and MAGMA [17]. The general linear group $GL(4,2)$ of order =20160 is a subgroup of the general linear group $GL(5,2)$. By MAGMA [17], We can generate the group GL(4,2) by the two 5×5 matrices:

$$a = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

We construct the conjugacy class representative of G in terms of 5×5 matrices over GF(2) by GAP [18]. The GL(4,2) has 14 conjugacy classes and under the action of GL(4,2) on 2^5 , we obtain four orbits of lengths 1 , 15, 1 and 15. The point stabilizers for orbits of lengths 1, 15, 1 and 15 are the subgroups GL(4,2) , $2^3{:}PSL(3,2)$, A_8 and $2^3{:}PSL(3,2)$ of indices 1, 15, 1 and 15, respectively in GL(4,2). Let $\mathcal{X}(GL(4,2)|2^5)$ be the permutation character of GL(4,2) on 2^5. The values of $\mathcal{X}(GL(4,2)|2^5)$ on different classes of GL(4, 2) determine the number of k of fixed points of each conjugacy class of GL(4,2) in 2^5. These values of the $k's$ will help us to calculate the conjugacy classes of $2^5{:}GL(4,2)$ which are listed in Table 1. Consequentially, having obtained the values of the $k's$ for the various classes of G, we then need to calculate the values of $f_i's$ corresponding to the various $'s$, where $f_i's$ are the number of orbits $Q_i's$ for $1 \le i \le k$, that fuse together under the action of $C_G(g)$ to form one orbit Δ_j. For this purpose, we used Programme A [8]. For a class representative $dg \in \bar{G}$, where $d \in 2^5$, $g \in GL(4,2)$ and $o(g) = m$, by Theorem 3.3.10 in [15] we have

$$o(dg) = \begin{cases} m & if \quad w = 1_N \\ 2m & otherwise \end{cases}$$

To calculate the orders of the class representative $dg \in \bar{G}$, we used Programme B [8]. If $o(g) = m$ and $w = 1_N$ then $o(dg) = 2m$ otherwise if $\neq 1_N$, then $o(dg) = 2m$. To each class of $\bar{G}$, we have attached some weight m_{ij} which will be used later in computing the Fischer matrices of the extension. These weights are computed by the formula

$$m_{ij} = [N_{\bar{G}}(N\bar{g}_i){:}C_{\bar{G}}(g_{ij})] = |N|\frac{|C_G(g_i)|}{|C_{\bar{G}}(g_{ij})|}$$

Thus, we obtained 50 conjugacy classes for the group $\bar{G} = 2^5{:}GL(4,2)$ and we list these result about the conjugacy classes of $\bar{G}$ in Table 1.

Table 1. Conjugacy Classes of $\bar{G} = 2^5{:}GL(4,2)$

$g_i \in G$	k_i	f_i	$m_{i,j}$	d_i	w	$[x]_{\bar{G}}$	$\|[x]_{\bar{G}}\|$	$\|C_{\bar{G}}(x)\|$
1A	32	1	1	(0,0,0,0,0)	(0,0,0,0,0)	$g_{1,1} = 1a$	1	645120
		1	1	(1,0,0,0,0)	(1,0,0,0,0)	$g_{1,2} = 2a$	1	645120
		15	15	(0,0,0,0,1)	(0,0,0,0,1)	$g_{1,3} = 2b$	15	43008
		15	15	(1,0,0,0,1)	(1,0,0,0,1)	$g_{1,4} = 2c$	15	43008
3A	2	1	16	(0,0,0,0,0)	(0,0,0,0,0)	$g_{2,1} = 3a$	1792	360
		1	16	(1,0,0,0,0)	(1,0,0,0,0)	$g_{2,2} = 6a$	1792	360
5A	2	1	16	(0,0,0,0,0)	(0,0,0,0,0)	$g_{3,1} = 5a$	21504	30
		1	16	(1,0,0,0,0)	(1,0,0,0,0)	$g_{3,2} = 10a$	21504	30
15A	2	1	16	(0,0,0,0,0)	(0,0,0,0,0)	$g_{4,1} = 15a$	21504	30
		1	16	(1,0,0,0,0)	(1,0,0,0,0)	$g_{4,2} = 30a$	21504	30
15B	2	1	16	(0,0,0,0,0)	(0,0,0,0,0)	$g_{5,1} = 15b$	21504	30
		1	16	(1,0,0,0,0)	(1,0,0,0,0)	$g_{5,2} = 30b$	21504	30
7A	4	1	8	(0,0,0,0,0)	(0,0,0,0,0)	$g_{6,1} = 7a$	23040	28
		1	8	(0,0,0,1,0)	(0,1,1,0,0)	$g_{6,2} = 14a$	23040	28
		1	8	(1,0,0,0,0)	(1,0,0,0,0)	$g_{6,3} = 14b$	23040	28
		1	8	(1,0,0,1,0)	(1,1,1,0,0)	$g_{6,4} = 14c$	23040	28
7B	4	1	8	(0,0,0,0,0)	(0,0,0,0,0)	$g_{7,1} = 7b$	23040	28
		1	8	(0,0,0,1,0)	(0,1,1,0,0)	$g_{7,2} = 14d$	23040	28
		1	8	(1,0,0,0,0)	(1,0,0,0,0)	$g_{7,3} = 14e$	23040	28
		1	8	(1,0,0,1,0)	(1,1,1,0,0)	$g_{7,4} = 14f$	23040	28
2A	8	1	4	(0,0,0,0,0)	(0,0,0,0,0)	$g_{8,1} = 2d$	840	768
		1	4	(1,0,0,0,0)	(0,0,0,0,0)	$g_{8,2} = 2e$	840	768
		3	12	(0,0,0,0,1)	(0,1,0,1,0)	$g_{8,3} = 4a$	2520	256
		3	12	(1,0,0,0,1)	(0,1,0,1,0)	$g_{8,4} = 4b$	2520	256
4A	4	1	8	(0,0,0,0,0)	(0,0,0,0,0)	$g_{9,1} = 4c$	20160	32
		1	8	(0,0,0,0,1)	(0,1,1,0,0)	$g_{9,2} = 8a$	20160	32
		1	8	(1,0,0,0,0)	(0,0,0,0,0)	$g_{9,3} = 4d$	20160	32
		1	8	(1,0,0,0,1)	(0,1,1,0,0)	$g_{9,4} = 8b$	20160	32
2B	16	1	2	(0,0,0,0,0)	(0,0,0,0,0)	$g_{10,1} = 2f$	210	3072
		4	8	(0,0,0,0,1)	(0,1,1,0,0)	$g_{10,2} = 4e$	840	768
		3	6	(0,0,0,1,0)	(0,0,0,0,0)	$g_{10,3} = 2g$	630	1024
		1	2	(1,0,0,0,0)	(0,0,0,0,0)	$g_{10,4} = 2h$	210	3072
		4	8	(1,0,0,0,1)	(0,1,1,0,0)	$g_{10,5} = 4f$	840	768
		3	6	(1,0,0,1,0)	(0,0,0,0,0)	$g_{10,6} = 2i$	630	1024
3B	8	1	4	(0,0,0,0,0)	(0,0,0,0,0)	$g_{11,1} = 3b$	4480	144
		3	8	(0,0,0,0,1)	(0,1,0,1,1)	$g_{11,2} = 6b$	13440	48

		1	4	(1,0,0,0,0)	(1,0,0,0,0)	$g_{11,3} = 6c$	4480	144
		3	8	(1,0,0,0,1)	(1,1,0,1,1)	$g_{11,4} = 6d$	13440	48
6A	4	1	8	(0,0,0,0,0)	(0,0,0,0,0)	$g_{12,1} = 6e$	26880	24
		1	8	(0,0,0,0,1)	(0,1,1,0,0)	$g_{12,2} = 12a$	26880	24
		1	8	(1,0,0,0,0)	(0,0,0,0,0)	$g_{12,3} = 6f$	26880	24
		1	8	(1,0,0,0,1)	(0,1,1,0,0)	$g_{12,4} = 12b$	26880	24
4B	8	1	4	(0,0,0,0,0)	(0,0,0,0,0)	$g_{13,1} = 4g$	5040	128
		2	8	(0,0,0,0,1)	(0,0,0,0,0)	$g_{13,2} = 4h$	10080	64
		1	4	(0,0,1,0,0)	(0,0,0,0,0)	$g_{13,3} = 4i$	5040	128
		1	4	(1,0,0,0,0)	(0,0,0,0,0)	$g_{13,4} = 4j$	5040	128
		2	8	(1,0,0,0,1)	(0,0,0,0,0)	$g_{13,5} = 4k$	10080	64
		1	4	(1,0,1,0,0)	(0,0,0,0,0)	$g_{13,6} = 4l$	5040	128
6b	2	1	16	(0,0,0,0,0)	(0,0,0,0,0)	$g_{14,1} = 6g$	26880	24
			16	(1,0,0,0,0)	(0,0,0,0,0)	$g_{14,2} = 6h$	26880	24

Thus, the group $2^5{:}GL(4,2)$ has 50 conjugacy classes.

4. The Inertia Factor Group of $\bar{G} = 2^5{:}GL(4,2)$

The action of G on N produce four orbits of lengths 1, 15, 1 and 15. Hence by Brauer'stheorem (see Theorem 5.1.4 in[15]) G acts on Irr(N) with the same number of orbits. The lengths of the these orbits will be $1, r, s, p$where $1 + r + s + p = 32$, with corresponding point stabilizers H_1, H_2, H_3 and H_4 as subgroups of G such that $[G{:}H_1] = 1, [G{:}H_2] = 15$, $[G{:}H_3] = 1$ and $[G{:}H_4] = 15$. Considering the indices of these subgroups in G and investigating the maximal and submaximal subgroups of G, we have the Group $GL(4,2)$ acting on Irr(2^5) produce four inertia factor groups:

(1) $H_1 = GL(4,2)$ of index equal 1,

(2) $H_2 = H_4 = 2^3{:}PSL(3,2)$ of index equal 15,

(3) $H_3 = A_8$of index equal to 1.

Using GAP [18], we can generate the group $H_2 = H_4$ in terms of 5×5 matrices over GF(2) by the follows matrices:

$$a_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, a_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$a_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, a_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

By using these generators in GAP [18], we obtained the conjugacy classes and the character tables for this group.

We can complete the fusion maps by using matrix conjugation into the group (4,2) . This fusion map is listed in table 2.

Table 2. *The Fusion of the group* $H_2 = H_4 = 2^3{:}PSL(3,2)$ *into the group* $G = GL(4,2)$

$[g]_{H_2}$	$\Rightarrow$	$[g]_G$	$[g]_{H_2}$	$\Rightarrow$	$[g]_G$	$[g]_{H_2}$	$\Rightarrow$	$[g]_G$
1a	$\Rightarrow$	1a	2a	$\Rightarrow$	2b	2b	$\Rightarrow$	2b
4a	$\Rightarrow$	4b	2c	$\Rightarrow$	2a	3a	$\Rightarrow$	3b
6a	$\Rightarrow$	6a	4b	$\Rightarrow$	4b	4c	$\Rightarrow$	4a
7a	$\Rightarrow$	7a	7b	$\Rightarrow$	7b			

Also, by using GAP [18], we can generate the group $H_3 = A_8$ in terms of 5×5 matrices over GF(2) by the follows matrices:

$$b_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, b_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

By using these generators in GAP [18], we obtained the conjugacy classes and the character tables for this group.

We can complete the fusion maps by using matrix conjugation in the group (4,2) . This fusion map is listed in table 3.

Table 3. *The Fusion of the group* $H_3 = A_8$ *into the group* $G = GL(4,2)$

$[g]_{H_3}$	$\Longrightarrow$	$[g]_G$	$[g]_{H_3}$	$\Longrightarrow$	$[g]_G$	$[g]_{H_3}$	$\Longrightarrow$	$[g]_G$
1a	$\Longrightarrow$	1a	2a	$\Longrightarrow$	2a	4a	$\Longrightarrow$	4a
2b	$\Longrightarrow$	2b	3a	$\Longrightarrow$	3b	6a	$\Longrightarrow$	6a
3b	$\Longrightarrow$	3a	5a	$\Longrightarrow$	5a	15a	$\Longrightarrow$	15b
15b	$\Longrightarrow$	15a	7a	$\Longrightarrow$	7a	7b	$\Longrightarrow$	7b
6b	$\Longrightarrow$	6b	4b	$\Longrightarrow$	4b			

5. THE FISCHER-CLIFFORD MATRICES OF $\bar{G} = 2^5{:}GL(4,2)$

For each conjugacy class [g] of G with representative$g \in G$, we construct the corresponding Fischer-Clifford matrix $M(g)$of $\bar{G} = 2^5{:}GL(4,2)$. We use the properties of the Fischer-Clifford matrices see ([3], [4],[5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15]) together with fusions of H_2 into H_1 , H_3 into H_1 and H_4 into H_1 (Table 2, Table 3) to compute the entries of these matrices and to construct an algebraic system of linear and non-linear equations with the help of Maxima [16], we can solve these system of equations and compute all the Fischer matrices of $\bar{G}$. The Fischer-Clifford matrix will be partitioned row-wise into blocks, where each block corresponding to an inertia group $\overline{H_i}$. We list the Fischer-Clifford matrices of $\bar{G}$ in Table 4:

Table 4. *The Fischer-Clifford Matrices of* $2^5{:}GL(4,2)$*:*

$M(g)$	$M(g)$
$M(1a) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 15 & 15 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 15 & -15 & 1 & -1 \end{bmatrix}$	$M(3A) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
$M(5A) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(15A) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
$M(15B) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(7A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$
$M(7B) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$	$M(2A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$

$$M(4A)=\begin{bmatrix}1&1&1&1\\1&1&-1&-1\\1&-1&-1&1\\1&-1&1&-1\end{bmatrix}$$

$$M(2B)=\begin{bmatrix}1&1&1&1&1&1\\1&-1&1&-1&1&-1\\1&1&1&-1&-1&-1\\6&0&-2&-6&0&2\\6&0&-2&6&0&-2\\1&-1&1&1&-1&1\end{bmatrix}$$

$$M(3B)=\begin{bmatrix}1&1&1&1\\3&1&-3&-1\\3&-1&3&-1\\1&-1&-1&1\end{bmatrix}$$

$$M(6A)=\begin{bmatrix}1&1&1&1\\1&1&-1&-1\\1&-1&-1&1\\1&-1&1&-1\end{bmatrix}$$

$$M(4B)=\begin{bmatrix}1&1&1&1&1&1\\1&-1&1&1&-1&1\\1&-1&-1&-1&1&1\\2&0&-2&2&0&-2\\2&0&2&-2&0&-2\\1&1&-1&-1&-1&1\end{bmatrix}$$

$$M(6B)=\begin{bmatrix}1&1\\1&-1\end{bmatrix}$$

6. THE CHARACTER TABLE OF $\bar{G}=2^5{:}GL(4,2)$

Now, we have:

(1) The conjugacy classes of $\bar{G}=2^5{:}GL(4,2)$ (Table 1);

(2) The character tables of all the inertia factors (by using GAP [18]);

(3) The fusions of conjugacy classes of the inertia factors into classes of GL(4,2) (Table 2 & Table 3) ;

(4) The Fischer matrices of classes of $\bar{G}=2^5{:}GL(4,2)$ (Table 4);

Thus, we can construct the character table of the $2^5{:}GL(4,2)$ as follows:

Table 5. *The Character Table of* $\bar{G}=2^5{:}GL(4,2)$

Class	1a	2a	2b	2c	3a	6a	5a	10a	15a	30a
Order	1	2	2	2	3	6	5	10	15	30
Size	1	1	15	15	1792	1792	21504	21504	21504	21504
X1	1	1	1	1	1	1	1	1	1	1
X2	7	7	7	7	4	4	2	2	-1	-1
X3	14	14	14	14	-1	-1	-1	-1	-1	-1
X4	20	20	20	20	5	5	0	0	0	0
X5	21	21	21	21	6	6	1	1	1	1
X6	21	21	21	21	-3	-3	1	1	A	A
X7	21	21	21	21	-3	-3	1	1	/A	/A
X8	28	28	28	28	1	1	-2	-2	1	1
X9	35	35	35	35	5	5	0	0	0	0
X10	45	45	45	45	0	0	0	0	0	0
X11	45	45	45	45	0	0	0	0	0	0
X12	56	56	56	56	-4	-4	1	1	1	1
X13	64	64	64	64	4	4	-1	-1	-1	-1
X14	70	70	70	70	-5	-5	0	0	0	0
X15	15	15	-1	-1	0	0	0	0	0	0
X16	45	45	-3	-3	0	0	0	0	0	0
X17	45	45	-3	-3	0	0	0	0	0	0
X18	90	90	-6	-6	0	0	0	0	0	0
X19	105	105	-7	-7	0	0	0	0	0	0
X20	105	105	-7	-7	0	0	0	0	0	0
X21	105	105	-7	-7	0	0	0	0	0	0
X22	120	120	-8	-8	0	0	0	0	0	0
X23	210	210	-14	-14	0	0	0	0	0	0
X24	315	315	-21	-21	0	0	0	0	0	0

X25	315	315	-21	-21	0	0	0	0	0	0
X26	15	-15	1	-1	0	0	0	0	0	0
X27	45	-45	3	-3	0	0	0	0	0	0
X28	45	-45	3	-3	0	0	0	0	0	0
X29	90	-90	6	-6	0	0	0	0	0	0
X30	105	-105	7	-7	0	0	0	0	0	0
X31	105	-105	7	-7	0	0	0	0	0	0
X32	105	-105	7	-7	0	0	0	0	0	0
X33	120	-120	8	-8	0	0	0	0	0	0
X34	210	-210	14	-14	0	0	0	0	0	0
X35	315	-315	21	-21	0	0	0	0	0	0
X36	315	-315	21	-21	0	0	0	0	0	0
X37	1	-1	-1	1	1	-1	1	-1	1	-1
X38	7	-7	-7	7	4	-4	2	-2	-1	1
X39	14	-14	-14	14	-1	1	-1	1	-1	1
X40	20	-20	-20	20	5	-5	0	0	0	0
X41	21	-21	-21	21	6	-6	1	-1	1	-1
X42	21	-21	-21	21	-3	3	1	-1	/A	-/A
X43	21	-21	-21	21	-3	3	1	-1	A	-A
X44	28	-28	-28	28	1	-1	-2	2	1	-1
X45	35	-35	-35	35	5	-5	0	0	0	0
X46	45	-45	-45	45	0	0	0	0	0	0
X47	45	-45	-45	45	0	0	0	0	0	0
X48	56	-56	-56	56	-4	4	-4	4	1	-1
X49	64	-64	-64	64	4	-4	-1	1	-1	1
X50	70	-70	-70	70	-5	5	0	0	0	0

Class	15b	30b	7a	14a	14b	14c	7b	14d	14e
Order	15	30	7	14	14	14	7	14	14
Size	21504	21504	23040	23040	23040	23040	23040	23040	23040
X1	1	1	1	1	1	1	1	1	1
X2	-1	-1	0	0	0	0	0	0	0
X3	-1	-1	0	0	0	0	0	0	0
X4	0	0	-1	-1	-1	-1	-1	-1	-1
X5	1	1	0	0	0	0	0	0	0
X6	/A	/A	0	0	0	0	0	0	0
X7	A	A	0	0	0	0	0	0	0
X8	1	1	0	0	0	0	0	0	0
X9	0	0	0	0	0	0	0	0	0
X10	0	0	B	B	B	B	/B	/B	/B
X11	0	0	/B	/B	/B	/B	B	B	B
X12	1	1	0	0	0	0	0	0	0
X13	-1	-1	1	1	1	1	1	1	1
X14	0	0	0	0	0	0	0	0	0
X15	0	0	1	1	-1	-1	1	1	-1
X16	0	0	A	A	-A	-A	/A	/A	-/A
X17	0	0	/A	/A	-/A	-/A	A	A	-A
X18	0	0	-1	-1	1	1	-1	-1	1
X19	0	0	0	0	0	0	0	0	0
X20	0	0	0	0	0	0	0	0	0
X21	0	0	0	0	0	0	0	0	0
X22	0	0	1	1	-1	-1	1	1	-1
X23	0	0	0	0	0	0	0	0	0
X24	0	0	0	0	-0	0	0	0	0
X25	0	0	0	0	0	0	0	0	0
X26	0	0	1	-1	-1	1	1	-1	-1
X27	0	0	A	-A	-A	A	/A	-/A	-/A
X28	0	0	/A	-/A	-/A	/A	A	-A	-A
X29	0	0	-1	1	1	-1	-1	1	1
X30	0	0	0	0	0	0	0	0	0
X31	0	0	0	0	0	0	0	0	0
X32	0	0	0	0	0	0	0	0	0
X33	0	0	1	-1	-1	1	1	-1	-1
X34	0	0	0	0	0	0	0	0	0
X35	0	0	0	0	0	0	0	0	0
X36	0	0	0	0	0	0	0	0	0
X37	1	-1	1	-1	1	-1	1	-1	1
X38	-1	1	0	0	0	0	0	0	0
X39	-1	1	0	0	0	0	0	0	0

X40	0	0	-1	1	-1	1	-1	1	-1
X41	1	-1	0	0	0	0	0	0	0
X42	A	-A	0	0	0	0	0	0	0
X43	/A	-/A	0	0	0	0	0	0	0
X44	1	-1	0	0	0	0	0	0	0
X45	0	0	0	0	0	0	0	0	0
X46	0	0	B	-B	B	-B	/B	-/B	/B
X47	0	0	/B	-/B	/B	-/B	B	-B	B
X48	1	-1	0	0	0	0	0	0	0
X49	-1	1	1	-1	1	-1	1	-1	1
X50	0	0	0	0	0	0	0	0	0

Class	14f	2d	2e	4a	4b	4c	8a	4d	8b
Order	14	2	2	4	4	4	8	4	8
Size	23040	840	840	2520	2520	20160	20160	20160	20160
X1	1	1	1	1	1	1	1	1	1
X2	0	3	3	3	3	1	1	1	1
X3	0	2	2	2	2	0	0	0	0
X4	-1	4	4	4	4	0	0	0	0
X5	0	1	1	1	1	-1	-1	-1	-1
X6	0	1	1	1	1	-1	-1	-1	-1
X7	0	1	1	1	1	-1	-1	-1	-1
X8	0	4	4	4	4	0	0	0	0
X9	0	-5	-5	-5	-5	-1	-1	-1	-1
X10	/B	-3	-3	-3	-3	1	1	1	1
X11	B	-3	-3	-3	-3	1	1	1	1
X12	0	0	0	0	0	0	0	0	0
X13	1	0	0	0	0	0	0	0	0
X14	0	2	2	2	2	0	0	0	0
X15	-1	3	-3	-1	1	1	1	-1	-1
X16	-/A	-3	3	1	-1	1	1	-1	-1
X17	-A	-3	3	1	-1	1	1	-1	-1
X18	1	6	-6	-2	2	0	0	0	0
X19	0	-3	3	1	-1	-1	-1	1	1
X20	0	-3	3	1	-1	-1	-1	1	1
X21	0	9	-9	-3	3	1	1	-1	-1
X22	-1	0	0	0	0	0	0	0	0
X23	0	6	-6	-2	2	0	0	0	0
X24	0	-9	9	3	-3	1	1	-1	-1
X25	0	3	-3	-1	1	-1	-1	1	1
X26	1	3	3	-1	-1	1	-1	-1	1
X27	/A	-3	-3	1	1	1	-1	-1	1
X28	A	-3	-3	1	1	1	-1	-1	1
X29	-1	6	6	-2	-2	0	0	0	0

X30	0	-3	-3	1	1	-1	1	1	-1
X31	0	-3	-3	1	1	-1	1	1	-1
X32	0	9	9	-3	-3	1	-1	-1	1
X33	1	0	0	0	0	0	0	0	0
X34	0	6	6	-2	-2	0	0	0	0
X35	0	-9	-9	3	3	1	-1	-1	1
X36	0	3	3	-1	-1	-1	1	1	-1
X37	-1	1	-1	1	-1	1	-1	1	-1
X38	0	3	-3	3	-3	1	-1	1	-1
X39	0	2	-2	2	-2	0	0	0	0
X40	1	4	-4	4	-4	0	0	0	0
X41	0	1	-1	1	-1	-1	1	-1	1
X42	0	1	-1	1	-1	-1	1	-1	1
X43	0	1	-1	1	-1	-1	1	-1	1
X44	0	4	-4	4	-4	0	0	0	0
X45	0	-5	5	-5	5	-1	1	-1	1
X46	-/B	-3	3	-3	3	1	-1	1	-1
X47	-B	-3	3	-3	3	1	-1	1	-1
X48	0	0	0	0	0	0	0	0	0
X49	-1	0	0	0	0	0	0	0	0
X50	0	2	-2	2	-2	0	0	0	0

Class	2f	4e	2g	2h	4f	2i	3b	6b	6c	6d
Order	2	4	2	2	4	2	3	6	6	6
Size	210	840	630	210	840	630	4480	13440	4480	13440
X1	1	1	1	1	1	1	1	1	1	1
X2	-1	-1	-1	-1	-1	-1	1	1	1	1
X3	6	6	6	6	6	6	2	2	2	2
X4	4	4	4	4	4	4	-1	-1	-1	-1
X5	-3	-3	-3	-3	-3	-3	0	0	0	0
X6	-3	-3	-3	-3	-3	-3	0	0	0	0
X7	-3	-3	-3	-3	-3	-3	0	0	0	0
X8	-4	-4	-4	-4	-4	-4	1	1	1	1
X9	3	3	3	3	3	3	2	2	2	2
X10	-3	-3	-3	-3	-3	-3	0	0	0	0
X11	-3	-3	-3	-3	-3	-3	0	0	0	0
X12	8	8	8	8	8	8	-1	-1	-1	-1
X13	0	0	0	0	0	0	-2	-2	-2	-2
X14	-2	-2	-2	-2	-2	-2	1	1	1	1

X15	7	-1	-1	-7	1	1	3	1	-3	-1
X16	-3	-3	5	3	3	-5	0	0	0	0
X17	-3	-3	5	3	3	-5	0	0	0	0
X18	18	-6	2	-18	6	-2	0	0	0	0
X19	17	1	-7	-17	-1	7	3	1	-3	-1
X20	1	-7	9	-1	7	-9	3	1	-3	-1
X21	-7	1	1	7	-1	-1	3	1	-3	-1
X22	8	-8	8	-8	8	-8	-3	-1	3	1
X23	10	2	-6	-10	-2	6	-3	-1	3	1
X24	3	3	-5	-3	-3	5	0	0	0	0
X25	-21	3	3	21	-3	-3	0	0	0	0
X26	7	1	-1	5	-1	-3	3	-1	3	-1
X27	-3	3	5	-9	-3	-1	0	0	0	0
X28	-3	3	5	-9	-3	-1	0	0	0	0
X29	18	6	2	6	-6	-10	0	0	0	0
X30	17	-1	-7	19	1	-5	3	-1	3	-1
X31	1	7	9	-13	-7	-5	3	-1	3	-1
X32	-7	-1	1	-5	1	3	3	-1	3	-1
X33	8	8	8	-8	-8	-8	-3	1	-3	1
X34	10	-2	-6	14	2	-2	-3	1	-3	1
X35	3	-3	-5	9	3	1	0	0	0	0
X36	-21	-3	3	-15	3	9	0	0	0	0
X37	1	-1	1	1	-1	1	1	-1	-1	1
X38	-1	1	-1	-1	1	-1	1	-1	-1	1
X39	6	-6	6	6	-6	6	2	-2	-2	2
X40	4	-4	4	4	-4	4	-1	1	1	-1
X41	-3	3	-3	-3	3	-3	0	0	0	0
X42	-3	3	-3	-3	3	-3	0	0	0	0
X43	-3	3	-3	-3	3	-3	0	0	0	0
X44	-4	4	-4	-4	4	-4	1	-1	-1	1
X45	3	-3	3	3	-3	3	2	-2	-2	2
X46	-3	3	-3	-3	3	-3	0	0	0	0
X47	-3	3	-3	-3	3	-3	0	0	0	0
X48	8	-8	8	8	-8	8	-1	1	1	-1
X49	0	0	0	0	0	0	-2	2	2	-2
X50	-2	2	-2	-2	2	-2	1	-1	-1	1

Class	6e	12a	6f	12b	4g	4h	4i	4j	4k
Order	6	12	6	12	4	4	4	4	4
Size	26880	26880	26880	26880	5040	10080	5040	5040	10080
X1	1	1	1	1	1	1	1	1	1
X2	-1	-1	-1	-1	-1	-1	-1	-1	-1
X3	0	0	0	0	2	2	2	2	2
X4	1	1	1	1	0	0	0	0	0
X5	0	0	0	0	1	1	1	1	1
X6	0	0	0	0	1	1	1	1	1
X7	0	0	0	0	1	1	1	1	1
X8	-1	-1	-1	-1	0	0	0	0	0
X9	0	0	0	0	-1	-1	-1	-1	-1
X10	0	0	0	0	1	1	1	1	1
X11	0	0	0	0	1	1	1	1	1
X12	-1	-1	-1	-1	0	0	0	0	0
X13	0	0	0	0	0	0	0	0	0
X14	1	1	1	1	-2	-2	-2	-2	-2
X15	1	1	-1	-1	3	-1	-1	3	-1
X16	0	0	0	0	1	1	-3	1	1
X17	0	0	0	0	1	1	-3	1	1
X18	0	0	0	0	2	-2	2	2	-2
X19	-1	-1	1	1	1	1	-3	1	1
X20	1	1	-1	-1	-3	1	1	-3	1
X21	-1	-1	1	1	-3	1	1	-3	1
X22	-1	-1	1	1	0	0	0	0	0
X23	1	1	-1	-1	-2	2	-2	-2	2
X24	0	0	0	0	-1	-1	3	-1	-1
X25	0	0	0	0	3	-1	-1	3	-1
X26	1	-1	-1	1	3	-1	1	-3	1
X27	0	0	0	0	1	1	3	-1	-1
X28	0	0	0	0	1	1	3	-1	-1
X29	0	0	0	0	2	-2	-2	-2	2
X30	-1	1	1	-1	1	1	3	-1	-1
X31	1	-1	-1	1	-3	1	-1	3	-1
X32	-1	1	1	-1	-3	1	-1	3	-1
X33	-1	1	1	-1	0	0	0	0	0
X34	1	-1	-1	1	-2	2	2	2	-2
X35	0	0	0	0	-1	-1	-3	1	1
X36	0	0	0	0	3	-1	1	-3	1

X37	1	-1	1	-1	1	1	-1	-1	-1
X38	-1	1	-1	1	-1	-1	1	1	1
X39	0	0	0	0	2	2	-2	-2	-2
X40	1	-1	1	-1	0	0	0	0	0
X41	0	0	0	0	1	1	-1	-1	-1
X42	0	0	0	0	1	1	-1	-1	-1
X43	0	0	0	0	1	1	-1	-1	-1
X44	-1	1	-1	1	0	0	0	0	0
X45	0	0	0	0	-1	-1	1	1	1
X46	0	0	0	0	1	1	-1	-1	-1
X47	0	0	0	0	1	1	-1	-1	-1
X48	-1	1	-1	1	0	0	0	0	0
X49	0	0	0	0	0	0	0	0	0
X50	1	-1	1	-1	-2	-2	2	2	2

Class	4l	6g	6h
Order	4	6	6
Size	5040	26880	26880
X1	1	1	1
X2	-1	0	0
X3	2	-1	-1
X4	0	1	1
X5	1	-2	-2
X6	1	1	1
X7	1	1	1
X8	0	1	1
X9	-1	1	1
X10	1	0	0
X11	1	0	0
X12	0	0	0
X13	0	0	0
X14	-2	-1	-1
X15	-1	0	0
X16	-3	0	0
X17	-3	0	0
X18	2	0	0
X19	-3	0	0
X20	1	0	0

X21	1	0	0
X22	0	0	0
X23	-2	0	0
X24	3	0	0
X25	-1	0	0
X26	-1	0	0
X27	-3	0	0
X28	-3	0	0
X29	2	0	0
X30	-3	0	0
X31	1	0	0
X32	1	0	0
X33	0	0	0
X34	-2	0	0
X35	3	0	0
X36	-1	0	0
X37	1	1	-1
X38	-1	0	0
X39	2	-1	1
X40	0	1	-1
X41	1	-2	2
X42	1	1	-1
X43	1	1	-1
X44	0	1	-1
X45	-1	1	-1
X46	1	0	0
X47	1	0	0
X48	0	0	0
X49	0	0	0
X50	-2	-1	1

Explanation of Character Value Symbols:

$A=\frac{-1-\sqrt{-15}}{2},\quad B=\frac{-1-\sqrt{-7}}{2}$

REFERENCES

Clifford A. H., "Representations induced in an invariant subgroup" Ann. of Math., pp. 533 – 550, (1973).

Fischer B., "Clifford matrices" Representation theory of finite groups and finite-dimensional Lie algebras, pp. 1 – 16, (1991).

Prins A. L.,"The Fischer-Clifford Matrices And Character Table of The maximal Subgroup $2^9{:}(L_3(4){:}S_3)$ Of $U_6(2){:}S_3$ ", Bull. Iranian Math. Soc., Vol. 42, No. 5, pp. 1179-1195, (2016).

Prins A. L. and Fray R. L., "The Fischer-Clifford Matrices Of The Inertia Group $2^7{:}O_6^-(2)$ Of A Maximal Subgroup $2^7{:}Sp_6(2)$ In $Sp_8(2)$", International Journal of Group Theory, Vol.2 No.3 , pp. 19-38, (2013).

Prins A. L. and Fray R. L., "The Fischer-Clifford Matrices Of An Extension Group Of The form $2^7{:}(2^5{:}S_6)$", International Journal of Group Theory, Vol. 3 No.2, pp. 21-39, (2014).

Basheer B. M., "Clifford-Fischer Theory Applied to Certain Groups Associated with Symplectic Unitary and Thompson Groups" PhD Thesis, University of KwaZulu-Natal, Pietermaitzburg, (2012).

Basheer B. M., ''On A Group of The Form $2^{14}{:}Sp(6,2)$", QuaestionesMathematicae, pp 1-13, (2015).

Ali F., "Fischer-Clifford Theory for Split and Non-Split Group Extensions" PhD Thesis, University of Natal, Pietermaritzburg, (2001).

Ali F., "The Fischer-Clifford Matrices And Chracter Table Of The Splite Extension $2^6{:}S_8$", Hacettepe Journal of Mathematics and Statistics, Vol. 43 No.2, pp. 153-171, (2014).

Moori J., "On the Groups G+ and G of the form 210:M22 and 210:M22", PhD Thesis, University of Birmingham, (1975).

Moori J. and SeretloT.,"On An Inertia Factor Group Of $2^8{:}O_8^+(2)$", Italian Journal of Pure and Applied Mathematics No. 33, pp. 241-254, (2014).

Moori J. and SeretloT.,"On a Maximal Subgroup Of $O_{10}^+(2)$", Southeast Asian Bulletin of Mathematics Vol. 39, pp. 249-263, (2015).

R.L. Fray and A.L. Prins,"A Group Of The Form $2^7{:}S_8$ As An Inertia Group Of The Affine Subgroup $2^7{:}Sp_6(2)$Of $Sp_8(2)$", Internaional Journal Of Modern Mathematical Scince, Vol.13 No.3 (2015), pp.330-351.

Seretlo T., "Fischer-Clifford Matrices and Character Tables of Certain Groups Associated with Simple Groups O+8 (2), HS and Ly", PhD Thesis, University of KwaZulu-Natal, Pietermaitzburg, (2012).

Mpono Z. E., "Fischer Clifford Theory and Character Tables of Group Extensions", PhD Thesis, University of Natal, Pietermaritzburg, (1998).

Maxima, A Computer Algebra System. Version 5.18.1; 2009. Available: http://maxima.sourceforge.net.

Magma program, version 2.20-9. Available: http://magma.maths.usyd.edu.au/magma/.

The GAP Group, GAP – Groups, Algorithms, and Programming, Version 4.8.6; 2016. Available: http://www.gap-system.org.

Wilson R. A. et al.," Atlas of finite group representations". Available: http://brauer.maths.qmul.ac.uk/Atlas/v3.

Gallagher P. X.," Group characters and normal Hall subgroups", Nagoya Math. J. **21**, pp. 223 – 230, (1962).

Citation: *R. I. Elkhatib, " The Fischer-Cliford Matrices and Character Table of the Group* $2^5{:}GL(4,2)$ *", International Journal of Scientific and Innovative Mathematical Research, vol. 6, no. 3, p. 11-24, 2018., http://dx.doi.org/10.20431/2347-3142.0603002*