



Calculation of the Riemann Zeta-Function from the Physical Point of View

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Abstract: The article presents a brief overview of the new direction in the theory of computing of functions represented by divergent series, for example, the Riemann zeta functions based on the use of physical analogy. For this, the notion of a relativistic Turing machine (RMT) is used, which has the same elements as its classic counterpart. The role of its head plays a material particle moving in accordance with the laws of relativistic mechanics along the numerical continuum playing the role of an RMT tape and having a non-Euclidean metric, the character of which depends on the calculation. Within this direction, evidence was obtained of the famous Riemann hypothesis.

Keywords: Riemann zeta-function, divergent series, Turing machine, Riemann hypothesis..

1. INTRODUCTION

Usually, when we mention the Riemann zeta-function, the famous Riemann hypothesis (RH) comes to memory, which says that the real parts of the nontrivial zeros of the zeta-function is $1/2$. By the way, mathematicians have not yet been able to find its evidence (or refutation). This result is so important (it is related to the distribution of prime numbers) that Clay's mathematical institute has included RH in the number of the most important problems of the millennium.

However, in physical applications, the Riemann zeta-function appears much more often without any mention of RH. As an example, perhaps not the best, we mention the problem of regularization of divergent expressions of field theory-the so-called zeta-regularization of S. Hawking [1]. Mathematicians have long been accustomed to the fact that if we represent the final result of the theory in the form of an expression containing the zeta-function, then one does not have to worry, that it, being written in another form, may contain divergence, i.e. be meaningless. This is due to one surprising feature of the zeta-function - to "absorb" infinity into itself, i.e. to ascribe to the expressions, at first sight, divergent, finite values. For example

$$\zeta(0)=1+1+1+1+\dots=-0.5$$

$$\zeta(-1)=1+2+3+4+5+\dots=-1/12$$

However, this fact that did not cause surprise of the mathematicians surprised the non-specialists [2].

2. METHOD OF CALCULATION

An attempt to comprehend the above results was undertaken in [3]. The meaning of the last paper is to represent the calculation of the zeta function as the result of the operation of a certain Turing machine (MT) the role of a tape of which plays a numerical axis, and the role of a head plays some physical particle which is moving in accordance with the equations of motion determined by the divergent expressions on the right-hand side of formulas given above. Since partial sums in the expression for the second formula determine the path traveled by the particle at constant acceleration, it is necessary to introduce into the equations of motion the source of this acceleration, or of gravity according to Einstein's equivalence principle. In other words, for the equations of motion of the particle, one

should choose the equations consistent with the general theory of relativity of Einstein with a suitable source. After solving them, we define the metric on the numerical axis in which the motion of the particle will no longer cause surprise because the final path that a particle will pass in an infinite time will be finite. As was shown in [3] the resulting metric has a singularity which defines so-called horizon of events known from the general theory of relativity [4]. The final expression $(-1/12)$ for the path is obtained if we take into account the curvature of the metric of the numerical axis in accordance with the solution of the Einstein equations. In fairness, it should be noted that the result in this paper differs from the exact one by about 3% due to the fact that instead of the relativistic expression, the nonrelativistic expression was used for acceleration of the particle (in the opposite case, the equations could not be solved).

3. DISCUSSION

From the point of view of the theory of the Turing machine, the result obtained means the inclusion infinity in the number of admissible values of the counting time. Earlier infinite time meant non-computability of the problem. This is true, since summation of a divergent series on an ordinary Turing machine is related to non-computable problems. Therefore, the MT described in the paper refers to the so-called relativistic MT [5].

The introduction of a non-Euclidean metric on the number axis means that the distance S traveled by the material particle before stopping at the horizon point does not coincide with the coordinate difference Δx on the axis between the ending and starting point, as it would be in the case of the Euclidean metric.

To calculate Zeta (0), this analogy can be used in a different form, and it will be necessary to study the motion of a material point on a one-dimensional Einstein manifold with a Ricci-flat metric.

In development of these ideas the calculation of the zeta function of the complex argument [6] was performed and the RH was proved [7]. In addition, the idea was expressed that computation, like motion, can change the geometry of the numerical continuum, and moreover the recognized system of Euclidean postulates should be changed to conform to the above formulas [8].

4. CONCLUSION

We present here a brief overview of the new direction in the theory of computing of functions represented by divergent series. As an example we chose the Riemann zeta functions. Its calculation is based on the use of physical analogy with the motion of relativistic material particle. For this, the notion of a relativistic Turing machine (RMT) is used. The tape of RMT has a non-Euclidean metric, the character of which depends on the calculation. Within this direction, evidence was obtained of the famous Riemann hypothesis.

REFERENCES

- [1] Hawking S. W., Zeta function regularization of path integrals in curved spacetime, *Communications in Mathematical Physics*, 55(2), 133-148 (1977).
- [2] Berman D., Freiberger M., Infinity or $-1/12?$, + plus magazine, Feb. 18, 2014, <http://plus.maths.org/content/infinity-or-just-112>.
- [3] Zayko Y. N., The Geometric Interpretation of Some Mathematical Expressions Containing the Riemann ζ -Function, *Mathematics Letters*, 2(6): 42-46 (2016).
- [4] Landau L. D., Lifshitz E. M., *The Classical Theory of Fields*, (4th ed.), Butterworth-Heinemann, 1975.
- [5] Nemeti I., David G., Relativistic Computers and the Turing Barrier. *Applied Mathematics and Computation*, 178, 118-142(2006).
- [6] Zayko Y. N., Calculation of the Riemann Zeta-function on a Relativistic Computer, *Mathematics and Computer Science*, 2 (2), 20-26 (2017).
- [7] Zayko Y. N., The Proof of the Riemann Hypothesis on a Relativistic Turing Machine, *International Journal of Theoretical and Applied Mathematics*. 3(6), 219-224 (2017).
- [8] Zayko Y. N., The Second Postulate of Euclid and the Hyperbolic Geometry, *International Journal of Scientific and Innovative Mathematical Research (IJSIMR)*, 6(4), 16-20 (2018); <http://dx.doi.org/10.20431/2347-3142.0604003>; arXiv: 1706.08378, 1706.08378v1 [math.GM]).

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