International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 6, Issue 1, 2018, PP 1-6
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
DOI: http://dx.doi.org/10.20431/2347-3142.0601001
www.arcjournals.org

Power Mean Labeling of Identification Graphs

P.Mercy [1], S. Somasundaram [2]

[1, 2] Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli, India

***Corresponding Author:** P.Mercy, Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli, India

Abstract: *A graph $G = (V, E)$ is called a Power mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,3,\ldots,q+1$ in such way that when each edge $e = uv$ is labeled with*

$$f(e=uv) = \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)} \right)^{\frac{1}{f(u)+f(v)}} \right\rceil$$

or

$$f(e=uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)} \right)^{\frac{1}{f(u)+f(v)}} \right\rfloor$$

so that the resulting edge labels are distinct. Here f is called a Power mean labeling of G. We investigate Power mean labeling for some standard graphs.

Keywords: *Power mean labeling, Power mean graph, union of graphs, union of m copies of cycles.*

AMS subject classification (2010): 05C38, 05C76, 05C78

1. Introduction

The graphs considered here are finite and undirected graphs. Let $G = (V, E)$ be a graph with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian [2] and Acharya et al.[1]. For all other standard terminology and notations we follow Harary [3]. In [6] Somasundaram and Ponraj introduced and studied [9] mean labeling for some standard graphs. Sandhya and Somasundaram [5] introduced Harmonic mean labeling of graphs and Sandhya et al. [4] studied the technique in detail. Somasundaram et al.[7] introduced the concept of Geometric mean labeling of graphs and studied their labeling in [8]. In this paper we define Power mean labeling and investigate some standard graphs for C_n^2, C_m @ P_n, $C_m \circ P_n$, and , C_m *and* C_n sharing a common edge for power mean labeling. We provide illustrative examples to support our study.

2. Definition and Results

Now we introduce the main concept of this paper

Definition 2.1. A graph $G = (V, E)$ with p vertices and q edges is said to be a Power Mean Graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, 3, \ldots, q+1$ is such a way that when each edge $e = uv$ is labeled with

$$f(e=uv) = \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)} \right)^{\frac{1}{f(u)+f(v)}} \right\rceil$$

or

$$f(e=uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)} \right)^{\frac{1}{f(u)+f(v)}} \right\rfloor$$

so that the resulting edge labels are distinct. In this case, f is called Power mean labeling of G.

Remark 2.1. If G is a Power mean labeling graph, then 1 must be a label of one of the vertices of G, since an edge should get label 1.

Remark 2.2. If $p > q + 1$, then the graph $G = (p, q)$ is not a Power mean graph, since it does not have sufficient labels from $\{1, 2, 3, \ldots, q+1\}$ for the vertices of G.

The following results will be used in the edge labelings of some standard graphs to get Power mean labeling.

Proposition 2.1. Let a, b and i be the positive integers with $a < b$. Then

(i) $a < (a^b b^a)^{\frac{1}{a+b}} < b$,

(ii) $i < (i^{1+2}(i+2)^i)^{\frac{1}{2i+2}} < (i+1)$,

(iii) $i < (i^{i+3}(i+3)^i)^{\frac{1}{2i+3}} < (i+2)$,

(iv) $i < (i^{i+4}(i+4)^i)^{\frac{1}{2i+4}} < (i+2)$, *and*

(v) $(1^i i^1)^{\frac{1}{i+1}} = i^{\frac{1}{i+1}} < 2$.

Proof. (i) Since $a^{a+b} = a^a a^b < b^a a^b < b^a b^b = b^{a+b}$, we get the inequality in Proposition 2.1.(i). That is, the Power mean of two numbers lies between the numbers a and b. Thus we infer that if vertices u, v have labels $i, i+1$ respectively, then the edge uv may be labeled i or $i+1$ for Power mean labeling.

(ii) As a proof of this inequality, we see

$$\begin{aligned} i^{i+2}(i+2)^i &< i^2[i(i+2)]^i, \\ &< i^2(i+1)^{2i}, \quad \text{since } i(i+2) < (i+1)^2, \\ &< (i+1)^2(i+1)^{2i}, \\ &= (i+1)^{2i+2}. \end{aligned}$$

This leads to $[(i^{i+2}(i+2)^i)^{\frac{1}{2i+2}}] < i+1$. Therefore, if u, v have labels $i, i+2$ respectively, then the edge uv may be labeled i and $i+1$.

(iii) Next we have

$$\begin{aligned} i^{i+3}(i+3)^i &= i^3[i(i+3)]^i, \\ &< i^3(i+2)^{2i}, \quad \text{since } i(i+3) < (i+2)^2, \\ &< (i+2)^3(i+2)^{2i}, \\ &= (i+2)^{2i+3}. \end{aligned}$$

This leads to $[i^{i+3}(i+3)^i]^{\frac{1}{2i+3}} < (i+2)$. Hence, if u, v have labels $i, i+3$ respectively, then the edge uv may be labeled $i+1$ without ambiguity.

(iv) Now

$$\begin{aligned} i^{i+4}(i+4)^i &= i^4[i(i+4)]^i, \\ &< i^4(i+2)^{2i}, \quad \text{since } i(i+4) < (i+2)^2, \\ &< (i+2)^4(i+2)^{2i}, \\ &= (i+2)^{2i+4}. \end{aligned}$$

Therefore 1

$$[i^{i+4}(i+4)^{i}]^{\frac{1}{2i+4}} < i+2.$$

Hence if u, v have labels $i, i+4$ respectively, then the edge uv may be labeled $i+1$.

(v) Now

$$\begin{aligned} 2^{i+1} &= (i+1)^{i+1}, \\ &= 1 + {}^{(i+1)}C_1 + \cdots + {}^{(i+1)}C_{i+1}, \\ &\geq 1 + 1 + \cdots + (i+2) \text{ terms}, \\ &\geq i+2 > i. \end{aligned}$$

Therefore $(1^i\, i^1)^{\frac{1}{i+1}} = i^{\frac{1}{i+1}} < 2$. Thus we observe that if u, v are labeled $1, I$ respectively, then the edge uv may be labeled 1 or 2.

3. Identification of Two Graphs

In this section we study the power mean labeling of some identification graphs.

Theorem 3.1. *Let $G_1 = (p_1, q_1)$ and $G_2 = (p_2, q_2)$ be any two graphs with power mean labeling f and g respectively. Let u and v be the vertices of G_1 and G_2 respectively, such that $f(u) = g(v) = q_1$. Then the graph $(G_1)_f * (G_2)_g$ obtained from G_1 and G_2 by identifying the vertices u and v is a power mean graph.*

Proof. Obviously $(G_1)_f * (G_2)_g$ has $p_1 + p_2 - 1$ vertices and $q_1 + q_2$ edges. Let the vertex set of G_1 be $V(G_1) = \{u, u_i : 1 \leq i \leq p_1 - 1\}$ and that of G_2 be $V(G_2) = \{v, v_i : 1 \leq i \leq p_2 - 1\}$.

Define a function

$$h : V((G_1)_f * (G_2)_g) \to \{1, 2, 3, \ldots, q_1 + q_2 - 1\}$$

by $h(u_i) = f(u_i), 1 \leq i \leq p_1 - 1$, $h(v_i) = q_1 + g(v_i), 1 \leq i \leq p_2 - 1$. Then edge labels of G_1 are $1, 2, 3, \ldots, q_1$ and edge labels of G_2 are $q_1 + 1, q_2 + 2, q_3 + 3, \ldots, q_1 + q_2$. Hence $(G_{1_f} * G_{2_g})$ is a power mean graph.

3.1 Power mean labeling for C_n^2

In this section, we prove the power mean labeling of common vertices between two cycles with n number of vertices and illustrate with examples.

Theorem 3.2. *The graph $C_n^{(2)}$ is a Power mean graph.*

Proof. Let u be the central vertex of $C_n^{(2)}$. Let the vertices of first cycle be $u_1, u_2, u_3, \ldots, u_n$ and the vertices of second cycle be $w_1, w_2, w_3, \ldots, w_n$. Each cycle is a Power mean graph. Let f be the corresponding Power mean labeling of the cycle. Take $G_1 = G_2 = C_n$ then $(G_1)_f * (G_2)_f = C_n$ then $(G_1)_f * (G_2)_f = C_n^{(2)}$. By Theorem 3.1, $C_n^{(2)}$ is a Power mean graph.

Example 3.1. A Power mean labeling of $C_6^{(2)}$ is given in Figure 3.1.

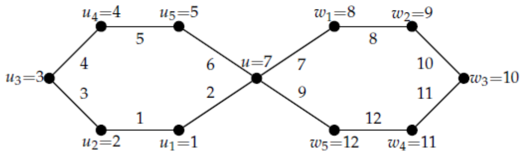

Figure 3.1: C_6^2

Example 3.2. A power mean labeling of $C_3^{(2)}$ is given in Figure 3.2

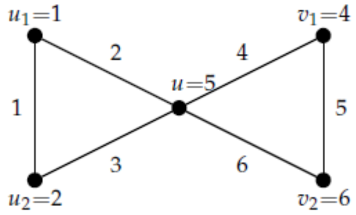

Figure 3.2: C_3^2

3.2 Power mean labeling for Two cycles C_n and C_m sharing a common edge

In this section, we prove the power mean labeling of common edge between two cycles with different number of vertices and provide an example.

Theorem 3.3. *Two cycles C_n and C_m sharing a common edge admit Power mean labeling.*

Proof. *Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of cycle C_n. Let $w_1, w_2, w_3, \ldots, w_m$ be the vertices of cycle C_m .Let G be the graph sharing a common edge of the two cycles. Without loss generality, assume that $e = v_{n-1}v_n$ is the common edge between C_n and C_m . Define a function $f : V(G) \longrightarrow \{1,2,3,\ldots,q+1\}$ as*

(i) $f(v_i) = i$, $1 \le i \le n-1$.

(ii) $f(v_n) = n + 1$.

(iii) $f(w_i) = n + i$, $2 \le i \le n-1$, and

(iv) $f(v_{n-1}) = f(w_n)$ and $f(v_n) = f(w_1)$

By Proposition 2 . 1 .(i) and (v), the edges are labeled.

(i) $E(v_i v_{i+1}) = i + 1, 1 \le i \le n-2$

(ii) $E(v_{n-1} v_n) = n$

(iii) $E(v_n v_1) = 1$

(iv) $E(w_{i-1} w_i) = n + i$, $2 \le i \le n$, and

(v) $E(w_{m-1} w_m) = n + 1$.

As the edges are distinct, the graph G is a Power mean graph.

Example 3.3. A graph G sharing a common edge between the cycles C_5 and C_8 is explained in Figure 3.3.

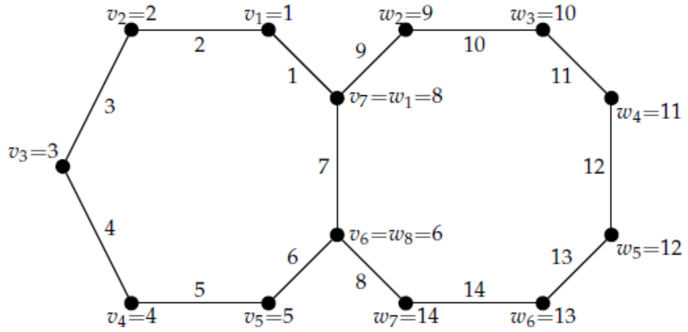

Figure 3.3: C_7 *and* C_8

3.3 Power mean labeling for Dragon $C_m@P_n$

Dragon: A dragon is a graph formed by joining an end vertex of a path P_n with a vertex of the cycle C_m. It is denoted by $C_m@P_n$.

Here, we prove the power mean labeling of dragon and provide an illustrative example.

Theorem 3.4. *A Dragon $C_m@P_n$ is a Power mean graph.*

Proof. Let $G = C_m@P_n$ be the given graph. Let $u_1, u_2, u_3, \ldots, u_m$ be the vertices of cycle C_m. Let $w_1, w_2, w_3, \ldots, w_n$ be the vertices of path P_n. Here $u_m = w_1$. Define a function *f*: $V(C_m@P_n) \longrightarrow \{1, 2, 3, \ldots, q+1 = 2(n+1)\}$ as

(i) $f(u_i) = i \ , \ 1 \le i \le m$

(ii) $f(w_{i+1}) = m + i \ , \ 1 \le i \le n-1$

(iii) $f(u_m) = f(w_1)$.

We get the edge labels as

(i) $E(u_i u_{i+1}) = i + 1; 1 \le i \le m-1$

(ii) $E(w_i w_{i+1}) = m + i; 1 \le i \le n-1$

(iii) $E(u_m u_1) = 1$

By Proposition 2.1.(i) and (v), the edge labels are distinct. The Dragon $C_m@P_n$ is a Power mean graph.

Example 3.4. The graph Dragon $C_m@P_n$ is given in Figure 3.4.

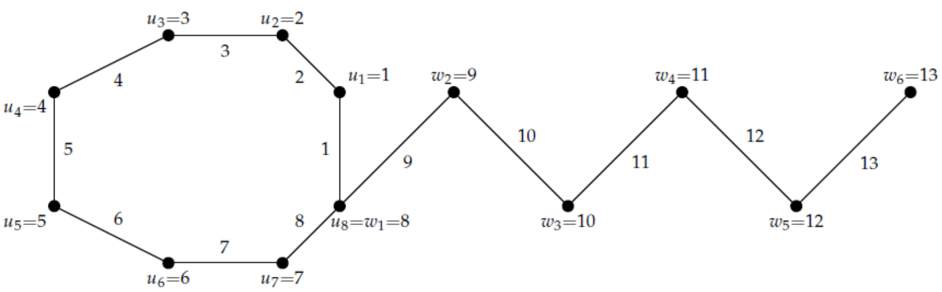

Figure 3.4: Dragon $C_m@P_n$

3.4 Power mean labeling for $C_m \circ P_n$

In this section, we establish the power mean labeling of the graph G. It is obtained by identifying a pendant vertex of P_n and a vertex of C_m, and illustrate with an example.

Theorem 3.5. *Let G be a graph obtained by identifying a pendant vertex of P_n and a vertex of C_m. The graph G admits a Power mean graph.*

Proof. Let $u_1, u_2, u_3, \ldots, u_m$ be the vertices of C_m and $v_1, v_2, v_3, \ldots, v_n$ be the vertices of P_n. Here we may take $u_m = v_1$.

Define a function *f*: $V(G) \longrightarrow \{1, 2, 3, \ldots, q+1\}$ by

(i). $f(u_i) = i \ ; \ 1 \le i \le m.$

(ii). $f(v_{i+1}) = m + i \ , \ 1 \le i \le n-1.$

By Proposition 2.1.(i) and (v), the edges are labeled

(i) $E(u_i u_{i+1}) = i + 1; 1 \le i \le m-1$

(ii) $E(u_m u_1) = 1$

(iii) $E(v_i v_{i+1}) = m + i; 1 \le i \le n-1$

As the edges are distinct, the graph $C_m \circ P_n$ is a Power mean graph.

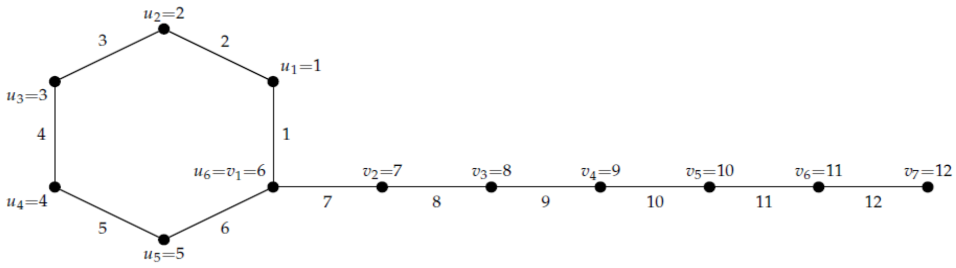

Figure 3.5: $C_6 \circ P_7$

4. Conclusion

In this paper we have proved that C_n^2, $C_m @ P_n$, $C_m \circ^{\circ} P_n$,and, C_m and C_n sharing a common edge are amenable for Power mean labeling. Also illustrative examples are provided.

References

[1] Gallian.J. A, A dynamic Survey of graph labeling, The electronic Journal of Combinatories, **17 DS6**, 2012.

[2] Acharya. B. D, Arumugam. Sand Rosa. A, Labeling of Discrete Structures and Applications, Narosa Publishing House, New Delhi, 2008, pp. 1-14..

[3] Harary. F, Graph Theory, Narosa Publishing House, New Delhi, 1988.

[4] Somasundaram. Sand Ponraj. R Mean Labelings of Graphs, National Academy Science Letters, vol. 26, (2003), pp. 210–213.

[5] Somasundaram. S and Ponraj. R, Some Results on Mean Graphs, Pure and Applied Mathematika Sciences, vol.53, (2003), pp. 29–35.

[6] Sandhya S.S and Somasundaram. S, Harmonic Mean Labeling for Some Special Graphs, International Journal of Mathematics Research, vol. 5 no. 1,(2013), pp 55–64.

[7] Sandhya, S.S., Somasundaram, S and Ponraj, R, Some More Results on Harmonic Mean Graphs, Journal of Mathematics Research, vol. 4, no. 1, (2012), pp. 21–29.

[8] Somasundaram. S,, Vidhyarani. P and Ponraj. R, Geometric Mean Labeling of Graphs, Bullettin of Pure and Applied Sciences, vol.30E, no. 2, (2011), pp. 153–160.

[9] Somasundaram. S, Vidhyarani. P and Ponraj. R, Some Results on Geometric Mean Graphs, International Journal of Mathematical Forum, vol. 7**,** no.28, (2012), pp., 1381–1391.

Authors' Biography

Ms. P. Mercy is a Research Scholar in Mathematics at Manonmaniam Sundaranar University, Tirunelveli. She works in Graph theory, particularly at present, in Power Mean Labeling of Graphs with the guidance of Prof.S. Somasundaram**.**

Dr.S.Somasundaram is a Professor of Mathematics at Manonmaniam Sundaranar University, Tirunelveli. His research interests include Harmonic Analysis, Fixed point theory and Graph theory. He has widely published in national and international journals.

Citation: *P.Mercy, S. Somasundaram, " Power Mean Labeling of Identification Graphs ", International Journal of Scientific and Innovative Mathematical Research, vol. 6, no. 1, p. 1-6, 2018., http://dx.doi.org/ 10.20431/2347-3142.0601001*