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Convolution Theorem for Distributional Fourier-Stieltjes Transform

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Abstract: *In the tremendous expanding knowledge of science, mathematics plays vital role. In the words of Philip, Mathematics is a science of quantity and space. Especially in quantam field theory, field of partial differential equations, Harmonic analysis etc.the notion of generalized functions is very essential.The convolution theorem of the transform plays an important role in digital signal processing.The usefulness of convolution theorem can be best explained by its applications in filtering. This paper is concerned with the generalization of Fourier-Stieltjes transform in the distributional sense. The main aim of this paper is to prove properties of convolution and convolution theorem for Fourier-Stieltjes transform.*

Keywords: *Fourier transform, Stieltjes transform, Fourier-Sieltjes transform, Integral transform, Convolution.*

AMS Mathematics Subject Classification: *(2010) 46F12*

1. Introduction

In mathematics and, in particular, functional analysis, convolution is a mathematical operation on two functions f and g, providing a third function that is typically viewed as a modified version of one of the original functions, giving the area overlap between the two functions as a function of the amount that one of the original functions translated. Convolution is similar to cross-correlation. It has applications that include probability, statistics, computer vision, natural language processing, image and signal processing electrical engineering and differentials equations. Generalizations of convolution have applications in field of numerical analysis, numerical linear algebra and in the design and implementation of finite implse response filters in signal processing.

The integral transforms plays an important role in signal processing. Fourier analysis is one of the frequently used tools in signal processing and many other scientific disciplines. The Stieltjes transform has been widely used in applied mathematics. Here we discuss the properties of convolution and convolution theorem for Fourier-Stieltjes transform which is very applicable. The convential Fourier-Stieltjes transform of complex valued smooth function f(t,x) is defined by the convergent integral.

$$F(s, y) = FS\,\{f(t, x)\} = \int_0^\infty \int_0^\infty f(t,x)e^{-ist}(x+y)^{-p}\,dtdx$$

2. Definitions

2.1. The function Space:The Space FS_α

A function f defined on $0 < t < \infty$, $0 < x < \infty$ is said to be member of FS_α if ϕ (t, x) is smooth for each non-negative integer l, q.

$$\gamma_{k,p,l,q}\phi(t,x) = \sup_I \left| t^k (1+x)^p D_t^l (xD_x)^q \phi(t,x) \right| \tag{2.1.1}$$

$$\leq C_{lq} A^p . p^p \qquad p = 1, 2, 3, \text{------------}$$

Where the constant A and C_{1q} depend on the testing function ϕ.

The space FSα areequiparallel with their natural Housdoff locally topology τ_α. This topology is respectively generated by the total families of semi norms $\{\gamma_{k,p,l,q}\}$ given by (2.1).

2.2.Distributional Fourier-Stieltjes transform of generalized function in FS_α^*

Let FS_α^* is the dual space FS_α. This space FS_α^* consists of continuous linear function on FS_α.

Let $\phi(t,x) \in FS_\alpha^*$, for some s >0 and k > Re p, then distributional Fourier-Stieltjes Transform F(s, y) of

$$FS\{f(t,x)\} = F(s,y) = \langle f(t,x), e^{-ist}(x+y)^{-p} \rangle \tag{2.2.1}$$

Where for each fixed t $(0 < t < \infty)$, x $(0 < x < \infty)$ the right side of above equation has same as an application of $f(t,x) \in FS_\alpha^*$ to $e^{-ist}(x+y)^{-p} \in FS_\alpha$.

2. 3. Fourier-Stieltjes Type Convolution:

Fourier-Stieltjes type convolution is an operation that assigns to each arbitrary pair $f \in FS_\alpha^*$ and $g \in FS_\alpha^*$, the Fourier-Stieltjes type convolution $f * g \in FS_\alpha^*$ defined by

$$\langle f*g, \phi \rangle = \langle f(t,x), \langle g(s,y), \phi(t+s, x+y) \rangle \rangle, \text{ where } \phi \in FS_\alpha \tag{2.3.1}$$

3. Main Results

3.1. Theorem:

If $f(t,x) \in FS_\alpha^*$ and $\phi(t,x) \in FS_\alpha$ then $\phi \to \psi$ is a continuous linear mapping of $FS_\alpha \to FS_\alpha$, where $\psi(s,y) = \langle f(t,x), \phi(t+s, x+y) \rangle$ (3.1.1)

Proof: By induction method we can show that

$$D_{s,y}^{m+n} \psi(s,y) = \langle f(t,x), D_{s,y}^{m+n} \phi(t+s, x+y) \rangle$$

For showing $\psi(t,x) \in FS_\alpha$, consider

$$= \sup_I \left| s^k (1+y)^p D_s^l (yD_y)^q \langle f(t,x), \phi(t+s, x+y) \rangle \right|$$

$$\leq C \max_{\substack{0\leq n\leq r_2 \\ 0\leq m\leq r_1}} \sup_I \left| s^k (1+y)^p D_s^l (yD_y)^q t^k (1+x)^q D_s^m (xD_x)^q \phi(t+s, x+y) \right|$$

Where r_1 and r_2 are non-negative integers depending on f.

$$\leq C \max_{\substack{0\leq n\leq r_2 \\ 0\leq m\leq r_1}} \sup_I \left| s^k (1+y)^p t^k (1+x)^q D_{s,y}^{l+q} D_{s,y}^{m+n} \phi(t+s, x+y) \right|$$

$$\leq C \max_{\substack{0\leq n\leq r_2 \\ 0\leq m\leq r_1}} \gamma_{k,p,l+m,q+n}(\phi) \quad (3.1.2)$$

Thus $\psi \in FS_\alpha$ continuity follows from (3.1.2) and hence theorem.□

Properties of Fourier-Stieltjes Type Convolution

3.2. Theorem:

If $f(t,x) \in FS_\alpha^*$ and $g \in D_+(I)$ then $g \to f*g$ is continuous linear map from D_+ into E_+, where $(f*g)(s,y) = \langle f(t,x), g(s-t, y-x) \rangle$ .sagggd

Proof: It is easy to prove that $f*g)$ is smooth and the mapping is linear.

For its continuity,

$$| D_{s,y}^{k_1+k_2}(f*g)(s,y) | = | \langle f(t,x), D_{s,y}^{k_1+k_2}\{g(s-t,y-x)\} \rangle |$$
$$\leq C \max_{\substack{0\leq q+k_2\leq r_2 \\ 0\leq l+k_1\leq r_1}} \sup_I | D_{l,x}^{l+q} D_{s,y}^{k_1+k_2}\{g(s-t,y-x)\} |$$

Since $g \in D_+(I)$, continuity follows from above inequality. We call $(f*g)(s,y) = \langle f(t,x), g(s-t,y-x) \rangle$ as Fourier-Stieltjes type regularization.□

3.3. Theorem:

Convolution operation in (3.2) commutes with shifting scaling operator S i.e. $S(f*g) = f*(S(g))$

Proof: Consider

$$\langle S(f*g), \phi(t,x) \rangle = \langle f*g, \phi(t+s, x+y) \rangle$$

$$= \langle f, \langle g, \phi(s-t, y-x) \rangle \rangle \qquad (3.3.1)$$

Now $\langle f*S(g), \phi(t,x) \rangle = \langle f, \langle S(g), \phi(t+s, x+y) \rangle \rangle$

$$= \langle f, \langle g, \phi(s-t, y-x) \rangle \rangle \qquad (3.3.1)$$

Therefore from (3.3.1) and (3.3.2), we write

$$S(f*g) = f*(S(g))$$

3.4. Theorem:

$f \in D_+^*$ and $FS\{(u,v)\} = f(s,y)$, and $y \in \Omega_f$ and $g \in D_+^*$, $FS\{g(t,x)\} = G(s,y)$, s and y $\in \Omega_g$ and $\Omega_f \cap \Omega_g$ is not empty, then $f*g$ exists in the sense of FS-type convolution in FS_α^* where the strip of definition is the intersection of $\Omega_f \cap \Omega_g$ with real axis. Moreover $FS(f*g) = FS(f).FS(g)$

Proof: Using theorem (3.3) it can be easily shown that $f*g \in FS_\alpha^*$.

Further as $K(t,x,s,y) = e^{-ist}(x+y)^{-p} \in FS_\alpha$ for each fixed s and y

$$FS(f*g) = \langle f*g, e^{-ist}(x+y)^{-p} \rangle$$

$$= \langle f(u,v), \langle g(t,x), e^{-is(t+u)}(v+y)^{-p}(x+y)^{-p} \rangle \rangle$$

$$= \langle f(u,v), e^{-isu}(v+y)^{-p} \rangle \langle g(t,x), e^{-ist}(x+y)^{-p} \rangle$$

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$$= FS\{f(u,v\}.FS\{g(t,x)\}$$

$$= F(s,y).G(s,y)$$

Hence the theorem.

4. Conclusions

This paper is concerned with the generalization of Fourier-Stieltjes transform in the distributional sense. In this paper the Fourier-Stieltjes type convolution and its properties are proved.

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