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Weakly 2-Absorbing Ideals of So-Rings

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Abstract: *A partial semiring is a structure possessing an infinitary partial addition and a binary multiplication, subject to a set of axioms. In this paper we introduce the notion of weakly 2-absorbing ideals in so-rings and studythe conditions under which a weakly 2-absorbing ideal is a 2-absorbing ideal. Also, we obtain various equivalent conditions on the weakly 2-absorbing ideals of Cartesian product of so-rings.*

Keywords: *Ideal, Prime ideal, 2-absorbing ideal, weakly 2-absorbing ideal, commutative so-ring.*

1. Introduction

Partially defined infinitary operations occur in the contexts ranging from integration theory to programming language semantics. The general cardinal algebras studied by Tarski in 1949, Housdorff Topological commutative groups studied by Bourbaki in 1966, $\sum$ - structures studied by Higgs in 1980, sum-ordered partial monoids and sum-ordered partial semirings(so-rings) studied by Arbib, Manes and Benson [2], [4], and streenstrup [10] are some of the algebraic structures of the above type.

G.V.S. Acharyulu [8] and P.V.SrinivasaRao [6] developed the ideal theory for the sum-ordered partial semirings (so-rings). Continuing this study, in [9] & [5] we introduced the notion of 2-absorbing ideals in so-rings and obtained their characteristics in a commutative so-ring. In this paper, we introduce the notion of weakly 2-absorbingidealsin so-rings and obtain its characteristics in so-rings.

2. Preliminaries

In this section we collect some important definitions and results for our use in this paper.

2.1. Definition. [4] *A partial Monoid*is a pair (M, $\sum$) where M is a non-empty set and $\sum$ is a partial addition defined on some, but not necessarily all families $(x_i : i \in I)$ in M subject to the following axioms:

(i) Unary Sum Axiom. If $(x_i : i \in I)$ is a one element family in M and $I = \{j\}$, then $\sum(x_i : i \in I)$is defined and equals x_j.

(ii) Partion-Associativity Axiom. If$(x_i : i \in I)$is a family in M and$(I_j : j \in J)$ is apartition of I, then$(x_i : i \in I)$is summable if and only if$(x_i : i \in I_j)$is summablefor every $j \in J$ and $(\sum(x_i : i \in I_j) : j \in J)$is summable. We write $\sum(x_i : i \in I) = \sum(\sum(x_i : i \in I_j) : j \in J)$.

2.2. Definition. [4] A *Partial Semiring*is a quadruple $(R, \sum, ., 1)$, where $(R, \sum)$ is a partial monoid, $(R, ., 1)$ is a monoid with multiplicative operation '.' and unit 1, and the additive and multiplicative structures obey the following distributive laws: If $\sum(x_i : i \in I)$ is defined in R, then for all y in *R*, $\sum(y.x_i : i \in I)$ and $\sum(x_i.y : i \in I)$ are defined and $y.\sum(x_i : i \in I) = \sum(y.x_i : i \in I)$,

$\sum(x_i : i \in I).y = \sum(x_i.y : i \in I)$.

2.3. Definition. [4] A partial semiring $(R, \sum, ., 1)$ is said to be *commutative* if $xy = yx \; \forall x, y \in R$.

2.4. Definition. [4] The *sum ordering* $\leq$ on a partial monoid $(M, \sum)$ is the binary relation $\leq$ such that $x \leq y$ if and only if there exists a 'h' in M such that $y = x + h$ for $x, y \in M$.

2.5. Definition. [4] A *sum-ordered partial semiring or so-ring*, for short, is a partial semiring in which the sum ordering is a partial order.

2.6. Example. [4] Let D be a set and let the set of all partial functions from D to D be denoted by $Pfn(D, D)$. A family $(x_i : i \in I)$ is summable if and only if for i, j in I, and $i \neq j$, $dom(x_i) \cap dom(x_j) = \phi$. If $(x_i : i \in I)$ is summable then for any d in D

$$d\left(\sum_i x_i\right) = \begin{cases} dx_i, & if \quad d \in dom(x_i) \quad for \quad some \quad (unique) \quad i \in I; \\ & undefined, \quad otherwise. \end{cases}$$

And '.' defined as the usual functional composition, the ordering as the extension of functions and unit defined as the identity defined on D. Then $(Pfn(D, D), \sum, .)$ is a so-ring.

2.7. Example. [4] Let D be a set. A multi-function $x : D \to D$ maps each element in D to an arbitray subset of D. Such multi-functions correspond bijectivelyto the relation $r \subseteq D \times D$, where $(d, e) \in r$ if and only if $e \in dx$. The set of all multi-functions from D to D, denoted by $Mfn(D, D)$, together with $\sum$ defined such that d in D, $d\left(\sum_i x_i\right) = \bigcup_i (dx_i)$, and '.' defined as the usual relational composition. That is, for each d in D and for x, y in $Mfn(D, D)$, $d(x.y) = \bigcup(ey : e \in dx)$, and $d1 = \{d\}$. Then $(Mfn(D, D), \sum, .)$ is a so-ring.

2.8. Definition. [8] Let R be a so-ring. A subset N of R is said to be an *ideal* of R if the following are satisified:

(I_1). If $(x_i : i \in I)$ is a summable family in R and $x_i \in N \; \forall i \in I$ then $\sum(x_i : i \in I) \in N$,

(I_2). If $x \leq y$ and $y \in N$ then $x \in N$,

(I_3). If $x \in N$ and $r \in R$ then $xr, rx \in N$.

2.9. Definition. [7] A proper ideal P of a so-ring R is said to be *weakly prime* if for any a, b of R, $0 \# ab \in P$ imply $a \in P$ or $b \in P$.

2.10. Definition. [9] A proper ideal I of a so-ring R is said to be *2-absorbing* if for any $a, b, c \in R$, $abc \in I$ implies $ab \in I$ or $bc \in I$ or $ac \in I$.

2.11. Remark. [9] Every prime ideal of a so-ring R is a 2-absorbing ideal ofR.

The following is an example of a so-ring R in which a 2-absorbing ideal need not be a prime ideal ofR.

2.12. Example [9] Consider the so-ring $R = \{0, u, v, x, y, 1\}$ with $\sum$ defined on R by

$$\sum(x_i : i \in I) = \begin{cases} x_j, & if \quad x_i = 0 \quad \forall i \neq j \quad for \quad some \quad j, \\ & undefined, \quad otherwise. \end{cases}$$

And '.' defined by the following table:

.	0	u	v	x	y	1
0	0	0	0	0	0	0
u	0	u	0	0	0	u
v	0	0	v	0	0	v
x	0	0	0	0	0	x
y	0	0	0	0	0	y
1	0	u	v	x	y	1

Then the ideal $I = \{0, u, x\}$ is a 2-absorbing ideal, but I is not a prime ideal. Since $v.y = o \in I$, but $v \notin I$ and $y \notin I$.

Throughout this paper, R denotes a commutative so-ring.

3. Weakly 2-Absorbing Ideals

We introduce the notion of weakly 2-absorbing ideals in so-rings as follows:

3.1. Definition. An ideal I of a so-ring R is said to be *weakly 2-absorbing* if for some $a, b, c \in R$ & $0 \neq abc \in I$, then $ab \in I$ or $bc \in I$ or $ac \in I$.

3.2. Remark. Every 2-absorbing ideal I of a so-ring R is a weakly 2-absorbing ideal of R.

The following is an example of a so-ring R in which a weakly 2-absorbing ideal is not a 2-absorbing ideal of R.

3.3. Example. Consider the so-ring Z_8. Take $R := Z_8 \times Z_8$. Then R is a commutative so-ring with respect to c0artesian product operations. Take $I := \{(0,0),(0,4)\}$. Then it can be verified that I is a weakly 2-absorbing ideal of R. Since $(2,0)(2,0)(2,0) = (0,0) \in I$ and $(2,0)(2,0) = (4,0) \notin I$, I is not a 2-absorbing ideal of R.

3.4. Theorem. Let R be a so-ring, I be an ideal of R and $a \in R$. Then the following statements are hold in R:

(i) Suppose $(0:a) \subseteq Ra$, then the ideal Ra is 2-absorbing if and only if it is weakly 2-absorbing

(ii) Suppose $(0:a) \subseteq Ia$, then the ideal Ia is 2-absorbing if and only if it is weakly 2-absorbing.

Proof. Let $a \in R$. (i) Assume that $(0:a) \subseteq Ra$. Suppose Ra is a weakly 2-absorbing ideal of R. Let $r, s, t \in R$ such that $rst \in Ra$. Suppose $rst \neq 0$. Since Ra is weakly 2-absorbing, we have $rs \in Ra$ or $st \in Ra$ or $rt \in Ra$. Suppose $rst = 0$. Then $r(s+a)t = rst + rat = rat = rta \in Ra$. Therefore $r(s+a)t \in Ra$. If $r(s+a)t \neq 0$. Then $r(s+a) \in Ra$ or $(s+a)t \in Ra$ or $rt \in Ra$ (Since Ra is weakly 2-absorbing). That implies $rs \in Ra$ or $st \in Ra$ or $rt \in Ra$. If $r(s+a)t = 0$. Then $rst + rat = 0$. That implies $rat = 0$. That implies $rta = 0$. That implies $rt \in (0:a) \subseteq Ra$. That implies $rt \in Ra$. Hence Ra is a 2-absorbing ideal of R. By Remark – 3.2., if Ra is a 2-absorbing ideal of R then Ra is a weakly 2-absorbing ideal of *R*.

(ii) Assume that $(0:a) \subseteq Ia$. Suppose Ia is a weakly 2-absorbing ideal of R. Let $r, s, t \in R$ such that $rst \in Ia$. Suppose $rst \neq 0$. Since Ia is weakly 2-absorbing, we have $rs \in Ia$ or $st \in Ia$ or $rt \in Ia$. Suppose $rst = 0$. Then 0 $r(s+a)t = rst + rat = rat = rta \in Ia$.Therefore $r(s+a)t \in Ia$. If $r(s+a)t \neq 0$.Then $r(s+a) \in Ia$ or $(s+a)t \in Ia$ or $rt \in Ia$ (Since Ia is weakly 2-absorbing). That implies $rs \in Ia$ or $st \in Ia$ or $rt \in Ia$. If $r(s+a)t = 0$. Then $rst + rat = 0$. That implies $rat = 0$. That implies $rta = 0$. That implies $rt \in (0:a) \subseteq Ia$. That implies $rt \in Ia$. Hence Ia is a 2-absorbing ideal of R. By Remark – 3.2., if Ia is a 2-absorbing ideal of R then Ia is a weakly 2-absorbing ideal of *R*.

3.5. Theorem. IfI and J are weakly prime ideals of a so-ring R, then $I \bigcap J$ is a weakly 2-absorbing ideal of R.

Proof.Let $0 \neq abc \in I \bigcap J$ for some $a,b,c \in R$. i.e., $0 \neq abc \in I$ & $0 \neq abc \in J$. Since I,J are weakly prime ideals of R, we have either $a \in I$ or $bc \in I$ & either $a \in J$ or $bc \in J$. Suppose $bc = 0$ then $bc \in I \bigcap J$, there is nothing to prove (Since $0 \in I \bigcap J$). So assume that $bc \neq 0$. Then either $a \in I$ or $0 \neq bc \in I$ & either $a \in J$ or $0 \neq bc \in J$. That implies $a \in I$ or $b \in I$ or $c \in I$ & $a \in J$ or $b \in J$ or $c \in J$ (SinceI, Jare weakly prime ideals). That implies $ab \in I \bigcap J$ or $bc \in I \bigcap J$ or $ac \in I \bigcap J$. Hence $I \bigcap J$ is a weakly 2-absorbing ideal of R.

3.6. Definition. Let I be a weakly 2-absorbing ideal of a so-ring R and $a,b,c \in R$. We say that (a,b,c) is a *triple-zero*if $abc = 0$, $ab \notin I$, $bc \notin I$ and $ac \notin I$.

3.7. Theorem. Let I be a weakly 2-absorbing ideal of a so-ring R and suppose that (a,b,c) is a triple-zero of Ifor some $a,b,c \in R$. Then

(i) $abI = bcI = acI = \{0\}$

(ii) $aI^2 = bI^2 = cI^2 = \{0\}$.

Proof.(i) In a contrary way suppose that $abI \neq \{0\}$. i.e., $abi \neq 0$ for some $i \in I$. That implies $ab(c+i) \neq 0$. Since $ab \notin I$ and $ab(c+i) \neq 0$, we have $a(c+i) \in I$ or $b(c+i) \in I$ (Since I is weakly 2-absorbing). i.e., $ac \in I$ or $bc \in I$, a contradiction to the fact that (a,b,c) is a triple-zero. So our assumption is wrong. Hence $abI = \{0\}$. Similarly we can prove that $bcI = \{0\}$, $acI = \{0\}$.

(ii) In a contrary way suppose that $aI^2 \neq \{0\}$. i.e., $ai_1i_2 \neq 0$ for some $i_1, i_2 \in I$. That implies $a(b+i_1)(c+i_2) = ai_1i_2 \neq 0 \in I$ (Since by (i), $abI = bcI = acI = \{0\}$). Since Iis a weakly 2-absorbing ideal of R, either $a(b+i_1) \in I$ or $a(c+i_2) \in I$ or $(b+i_1)(c+i_2) \in I$. i.e., either $ab \in I$ or $bc \in I$ or $ac \in I$, a contradiction to the fact that (a,b,c) is a triple-zero. So our assumption is wrong. Hence $aI^2 = \{0\}$. Similarly we can prove that $bI^2 = cI^2 = \{0\}$.

4. Weakly 2-Absorbing Ideals in Cartesian Products

4.1. Theorem. Let R_1 and R_2 be so-rings and I be a proper ideal of R_1. Then the following conditions are equivalent:

(i) I is a weakly 2-absorbing ideal of $R = R_1 \times R_2$,

(ii) $I \times R_2$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$.

Proof.(i) $\Rightarrow$ (ii): Suppose I is a weakly 2-absorbing ideal of R. Let $(a_1, a_2), (b_1, b_2), (c_1, c_2) \in R = R_1 \times R_2$ such that $0 \neq (a_1, a_2)(b_1, b_2)(c_1, c_2) \in I \times R_2$. Then $0 \neq (a_1b_1c_1, a_2b_2c_2) \in I \times R_2$. Therefore $0 \neq a_1b_1c_1 \in I$. Since I is a weakly 2-absorbing ideal of R, $a_1b_1 \in I$ or $b_1c_1 \in I$ or $a_1c_1 \in I$. If $a_1b_1 \in I$ then $(a_1, a_2)(b_1, b_2) \in I \times R_2$. If $b_1c_1 \in I$ then $(b_1, b_2)(c_1, c_2) \in I \times R_2$. If $a_1c_1 \in I$ then $(a_1, a_2)(c_1, c_2) \in I \times R_2$. Hence $I \times R_2$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$.

(ii) $\Rightarrow$ (i): Suppose $I \times R_2$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$. Let $0 \neq abc \in I$ for some $a,b,c \in R$. Then for each $0 \neq r \in R_2$, we have $0 \neq (a,1)(b,1)(c,r) \in I \times R_2$. Since $I \times R_2$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$, $(a,1)(b,1) \in I \times R_2$ or $(b,1)(c,r) \in I \times R_2$ or $(a,1)(c,r) \in I \times R_2$. That implies $ab \in I$ or $bc \in I$ or $ac \in I$. Hence I is a weakly 2-absorbing ideal of R.

4.2. Theorem. Let $R = R_1 \times R_2$ where R_1 and R_2 are so-rings. Let I be a proper ideal of R_1 and J be a proper ideal of R_2. Then the following statements are equivalent:

(i) $I \times J$ is a weakly 2-absorbing ideal of R,

(ii) ($J = R_2$ and I is a weakly 2-absorbing ideal of R_1) or (J is a prime ideal of R_2 and I is a prime ideal of R_1).

Proof.(i) $\Rightarrow$ (ii): Suppose $I \times J$ is a weakly 2-absorbing ideal of R. If $J = R_2$ then $I \times R_2$ is a weakly 2-absorbing ideal of R. Then by theorem 4.2., I is a weakly 2-absorbing ideal of R_1. Suppose $J \neq R_2$. We have to prove that J is a prime ideal of R_2 and I is a prime ideal of R_1. Let $a, b \in R_2$ such that $ab \in J$, and let $0 \neq i \in I$. Then $0 \neq (i,1)(1,a)(1,b) = (i, ab) \in I \times J$. Now $(1,a)(1,b) = (1,ab) \notin I \times J$ (Since $1 \notin I$). Since $I \times J$ is a weakly 2-absorbing ideal of R, either $(i,1)(1,a) = (i,a) \in I \times J$ or $(i,1)(1,b) = (i,b) \in I \times J$. That implies either $a \in J$ or $b \in J$. Hence J is a prime ideal of R_2. Similarly let $c, d \in R_1$ such that $cd \in I$, and let $0 \neq j \in J$. Then $0 \neq (c,1)(d,1)(1, j) = (cd, j) \in I \times J$. Now $(c,1)(d,1) = (cd,1) \notin I \times J$ (Since $1 \notin J$). Since $I \times J$ is a weakly 2-absorbing ideal of R, either $(c,1)(1, j) = (c, j) \in I \times J$ or $(d,1)(1, j) = (d, j) \in I \times J$. That implies either $c \in I$ or $d \in I$. Hence I is a prime ideal of R_1.

(ii) $\Rightarrow$ (i): Suppose $J = R_2$ and I is a weakly 2-absorbing ideal of R_1 or J is a prime ideal of R_2 and I is a prime ideal of R_1. We have to prove that $I \times J$ is a weakly 2-absorbing ideal of R. Suppose $J = R_2$ & I is a weakly 2-absorbing ideal of R_1, by theorem 4.2., $I \times R_2$ is a weakly 2-absorbing ideal of R. i.e., $I \times J$ is a weakly 2-absorbing ideal of R. SupposeJ is a prime ideal of R_2 & I is a prime ideal of R_1. Let $0 \neq (a_1, b_1)(a_2, b_2)(a_3, b_3) \in I \times J$ for some $a_1, a_2, a_3 \in R_1$ and $b_1, b_2, b_3 \in R_2$. Then $a_1 \in I$ or $a_2 \in I$ or $a_3 \in I$ and $b_1 \in J$ or $b_2 \in J$ or $b_3 \in J$. Thus $(a_1, b_1)(a_2, b_2) \in I \times J$ or $(a_2, b_2)(a_3, b_3) \in I \times J$ or $(a_1, b_1)(a_3, b_3) \in I \times J$. Hence $I \times J$ is a weakly 2-absorbing ideal of R.

4.3. Theorem. Let R_1, R_2 be a so-rings such that R_2 has no nonzero divisors. Let I be a proper ideal of R_1 and J be an ideal of R_2. Then the following statements are equivalent:

(i) $I \times J$ is a weakly 2-absorbing ideal of $R = R_1 \times R_2$,

(ii) I is a weakly prime ideal of R_1 and $J = \{0\}$ is a prime ideal of R_2.

Proof.(i) $\Rightarrow$ (ii): Suppose $I \times J$ is a weakly 2-absorbing ideal of R. Suppose $J = \{0\}$. We have to prove that $J = \{0\}$ is a prime ideal of R_2. Let $ab \in J = \{0\}$ for some $a, b \in R_2$. Let $0 \neq i \in I$, we have $0 \neq (i,1)(1,a)(1,b) = (i, ab) \in I \times J$. Also we have $(1,a)(1,b) = (1,ab) \notin I \times J$ (Since $1 \notin I$). Since $I \times J$ is a weakly 2-absorbing ideal of R, either $(i,1)(1,a) = (i,a) \in I \times J$ or $(i,1)(1,b) = (i,b) \in I \times J$. That implies either $a \in J$ or $b \in J$. Hence $J = \{0\}$ is a prime ideal of R_2. Now we have to prove that I is a weakly prime ideal of R_1 that is not a prime ideal. Suppose $0 \neq ab \in I$ for some $a, b \in R_1$. We have $0 \neq (a,1)(b,1)(1,0) = (ab,0) \in I \times \{0\}$. Since $(a,1)(b,1) = (ab,1) \notin I \times \{0\}$ & $I \times \{0\}$ is a weakly 2-absorbing ideal of R, either $(a,1)(1,0) = (a,0) \in I \times \{0\}$ or $(b,1)(1,0) = (b,0) \in I \times \{0\}$. That implies either $a \in I$ or $b \in I$. Hence I is a weakly prime ideal of R_1.

(ii) $\Rightarrow$ (i): Suppose I is a weakly prime ideal of R_1 that is not a prime ideal & $J = \{0\}$ is a prime ideal of R_2. We have to prove that $I \times \{0\}$ is a weakly 2-absorbing ideal of R. Let $0 \neq (a,b)(c,d)(e, f) = (ace, bdf) \in I \times \{0\}$. Since I is a weakly prime ideal of R_1, we may assume that $a \in I$. Since R_2 has no nonzero divisors, we may assume that $d = 0$. Therefore $(a,b)(c,d) = (a,b)(c,0) = (ac,0) \in I \times \{0\}$. Hence $I \times \{0\}$ is a weakly 2-absorbing ideal of R.

4.4. Theorem. Let $R = R_1 \times R_2 \times R_3$ where R_1, R_2, R_3 are so-rings. Let I_1 be a proper ideal of R_1, I_2 be an ideal of R_2, and I_3 be an ideal of R_3 such that $I = I_1 \times I_2 \times I_3 \neq \{(0,0,0)\}$. Then the following statements are equivalent:

(i) $I = I_1 \times I_2 \times I_3$ is a weakly 2-absorbing ideal of R,

(ii) $I = I_1 \times R_2 \times R_3$ and I_1 is a weakly 2-absorbing ideal of R_1 or $I = I_1 \times I_2 \times R_3$ such that I_1 is a prime ideal of R_1 and I_2 is a prime ideal of R_2 or $I = I_1 \times R_2 \times I_3$ such that I_1 is a prime ideal of R_1 and I_3 is a prime ideal of R_3.

Proof. (i) $\Rightarrow$ (ii): Suppose $I = I_1 \times I_2 \times I_3$ is a weakly 2-absorbing ideal of R. Since I is a weakly 2-absorbing ideal of R, I_1 is a weakly 2-absorbing ideal of R_1. If $I_2 = R_2$ and $I_3 = R_3$, then $I = I_1 \times R_2 \times R_3$. Suppose $I_2 \neq R_2 \,\&\, I_3 = R_3$. i.e., $I = I_1 \times I_2 \times R_3$. Now we have to prove that I_1 is a prime ideal of R_1 and I_2 is a prime ideal of R_2. Let $a,b \in R_1$ such that $ab \in I_1$ and $c,d \in R_2$ such that $cd \in I_2$. Then $0 \neq (a,1,1)(1,cd,1)(b,1,1) = (ab,cd,1) \in I$. Now $(a,1,1)(b,1,1) = (ab,1,1) \notin I$ (Since $I = I_1 \times I_2 \times R_3$ and $1 \notin I_2$). SinceI is a weakly 2-absorbing ideal of R, we have either $(a,1,1)(1,cd,1) = (a,cd,1) \in I$ or $(1,cd,1)(b,1,1) = (b,cd,1) \in I$. That implies either $a \in I_1$ or $b \in I_1$. Hence I_1 is a prime ideal of R_1. Similarly $0 \neq (ab,1,1)(1,c,1)(1,d,1) = (ab,cd,1) \in I$. Now $(1,c,1)(1,d,1) = (1,cd,1) \notin I$ (Since $I = I_1 \times I_2 \times R_3 \,\&\, 1 \notin I_1$). SinceI is a weakly 2-absorbing ideal of R, we have either $(ab,1,1)(1,c,1) = (ab,c,1) \in I$ or $(ab,1,1)(1,d,1) = (ab,d,1) \in I$. That implies either $c \in I_2$ or $d \in I_2$. Hence I_2 is a prime ideal of R_2. Finally assume that $I_2 = R_2$ and $I_3 \neq R_3$ (i.e., $I = I_1 \times R_2 \times I_3$). By applying the above arguement, we conclude that I_1 is a prime ideal of R_1 and I_3 is a prime ideal of R_3.

(ii) $\Rightarrow$ (i): Suppose I is one of the given three forms. Then by theorem 4.2., $I = I_1 \times I_2 \times I_3$ is a weakly 2-absorbing ideal of R.

5. Conclusion

In this paper we introduced the notion of weakly 2-absorbing ideals in so-rings and provided a counter example that proves the class of weakly 2-absorbing ideals is strictly wider than the class of all 2-absorbing ideals. Also we obtained the conditions under which a weakly 2-absorbing ideal is a 2-absorbing ideal. We considered this notation of weakly 2-absorbing ideals in the Cartesian product of so-rings and obtained various equivalent conditions on the weakly 2-absorbing ideals of Cartesian product of so-rings.

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