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Simulation of Dengue Disease with Control

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Abstract: *These Dengue is a disease which is transmitted to humans by a female mosquito named 'Aedes Aegypti'. In this paper, a mathematical model has been developed to study the spread of dengue disease. Population of human, mosquitoes and their eggs are taken into consideration. To study the dynamics of the disease a compartmental model involving non-linear ordinary differential equations has been formulated for human, mosquitoes and egg population. Stability analysis has been carried out at dengue free and endemic equilibrium points. Control in terms of preventive measures, spraying insecticides, and killing their eggs which is food for guppy fish are to be taken/applied to humans, mosquitoes and eggs respectively. Numerical Simulation has been carried out to show the impact of control on different compartments.*

Keywords: *Mathematical model, System of non-linear ordinary differential equations, Dengue, Stability, Control.*

1. INTRODUCTION

Dengue is a painful devastating mosquito borne disease caused by the dengue virus name 'Aedes Aegypti'. Aedes Aegypti also known as yellow fever mosquito is a mosquito which can spread dengue, chikungunya, Zika etc. and many other diseases. This mosquito has a white marking on its legs and has marking on upper surface of thorax in the form of lyre. This mosquito which was originated first in Africa is now found in all the tropical and subtropical regions over the world [1].

Of these only female mosquitoes bites which she needs to mature her eggs. Mosquitoes are attracted to the chemical compounds like ammonia, carbon dioxide, lactic acid etc. which are emitted by mammals to find a host. On taking blood meal this mosquito can produce an average of 100 to 200 eggs per batch. During a life time they can produce up to five batches. These number of eggs depends on the size of blood meal. The eggs of this mosquito are smooth, long, oval and about 1 millimeter long. Initially, when laid they appear to be white but in few minutes it turns out to be shiny black. They prefer to lay eggs in tree holes, pots, tanks, water cooler, temporary flood areas and a lot at places where rain water is collected. Development of eggs depends upon time. In warmer climate they take about two days, while in cooler climate they may take upon a week to develop. In dry regions, laid eggs can survive more than years. But once submerged in water, they hatch immediately. These viruses are therefore difficult to control. Infected eggs hatch and becomes an infected mosquito. With regards to control the attack of the diseases, a method to reduce a population of mosquito has been prepared in which a guppy fish is placed in water which eats the mosquito larvae [2].

When a mosquito born with dengue virus bites a person, the virus enters into the skin along with its saliva. It enters into the white blood cells and while moving throughout the body they reproduce inside the cells. This production of virus in the body harms many organs of the body such as liver, bone marrow etc. Normally, these mosquito bites at dawn and dusk time and thus can spread infection at any time of the day. When it bites a person with dengue virus in their blood the mosquito becomes infected [3]. It is not a contagious disease. The symptoms of dengue include fever, headache, joint and muscular pain, nausea, vomiting etc. Similar symptoms as those of common cold, vomiting and diarrhea are found in children, though they prove to be mild initially including high fever but can increase the risk of severity. Particularly, there is no vaccine for this disease [4]. Thus, to protect oneself from the bites of infected mosquitoes make use of mosquito repellents and making use of

spraying insecticides has to be done to kill them. Preventive measures should be taken care off by individuals.

Asmaidi et al. [5] has done their research entitled "A SIR Mathematical Model of Dengue transmission and its simulation" in which he has incorporated temperature as one of the parameter.

This paper has been modelled by considering eggs, mosquitoes and human population as all the three plays an important role in dengue disease. In this paper, human population has been divided into susceptible humans, infected humans, hospitalized humans, recovered humans. Mosquito population has been divided into susceptible mosquitoes and infected mosquitoes. Egg population has been divided into susceptible eggs and infected eggs. Mathematical model for the transmission of dengue disease has been described in Section 2. Section 3, 4 and 5 includes stability, control and simulation for the compartments. Conclusion is described in Section 6.

2. MATHEMATICAL MODEL

Here, we formulate a mathematical model for human-mosquito-egg population. The notations with its description for each parameter is given below Table 1.

Table1. *Notations and its Parametric Values*

Notations	Description	Parametric Values
N_H	Sample Size of Human population	1000
N_M	Sample Size of Mosquitoes population	10000
N_E	Sample Size of mosquitoes Eggs population	100000
$S_H(t)$	Number of Susceptible Humans at some instant of time t	500
$I_H(t)$	Number of Infected Humans at some instant of time t	150
$H_H(t)$	Number of Susceptible Humans at some instant of time t	80
$R_H(t)$	Number of Recovered Humans at some instant of time t	35
$S_M(t)$	Number of Susceptible Mosquitoes at some instant of time t	4000
$I_M(t)$	Number of Infected Mosquitoes at some instant of time t	2500
$S_E(t)$	Number of Susceptible Eggs at some instant of time t	60000
$I_E(t)$	Number of Infected Eggs at some instant of time t	40000
B	New Recruitment Rate	0.0000024
μ_H	Humans Death Rate	0.035
μ_M	Mosquitoes Death Rate	0.05
μ_E	Eggs Death Rate	0.4
α	Disease Induced Death Rate of Humans	0.001
β_H	Transmission rate of humans from susceptible to infection caused by infected mosquitoes	0.14
δ_H	Transmission rate of humans from infected to hospitalized	0.6
η_H	Transmission rate of humans from hospitalized to recovered	0.3
β_M	Transmission rate of mosquitoes from susceptible to infection caused by infected humans	0.6
ρ	Infected Eggs Hatching Rate	0.15
θ	Proportion of infected eggs laid by an infected mosquito	0.1
C_E	Climatic Factor	0.07
γ_M	Oviposition Rate	0.3
u_1	Control rates in terms of preventive measures	$[0,1]$
u_2	Control rates in terms of spraying insecticides	$[0,1]$
u_3	Control rate in terms of guppy fish	$[0,1]$

The transmission diagram of dengue from different compartments is shown in Figure 1.

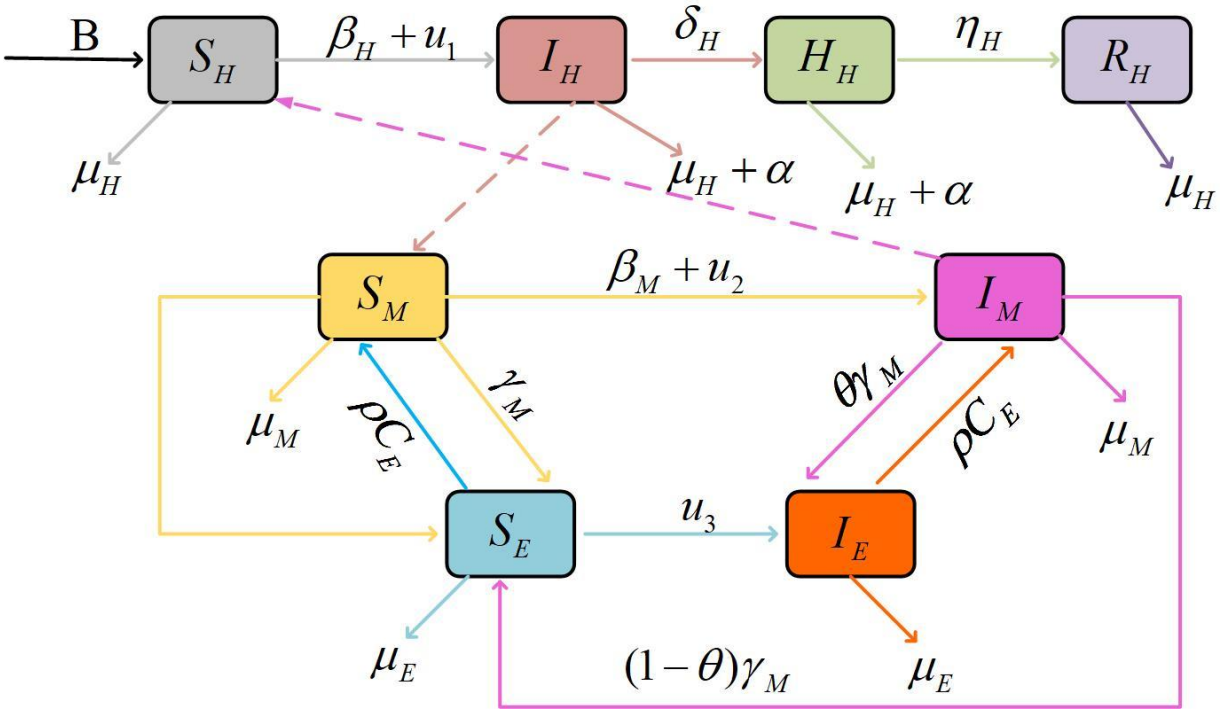

Fig 1.

Infected mosquito (I_M) can infect susceptible egg (S_E) and susceptible human being (S_H). Also, infected human (I_H) can infect susceptible mosquito (S_M). So, the disease can spread by two ways: infected mosquito and infected human. Susceptible human infects human at the rate β_H and so total individuals in the infected human compartment are $\beta_H \frac{I_M}{N_H} S_H$ and the rate of δ_H infected humans are seeking hospitalization so total individuals in the hospitalized compartment (H_H) are $\delta_H I_H$. The treatment given in H_H compartment makes individual free of dengue disease at the rate η_H and so total individuals recovered (R_H) from it are $\eta_H H_H$. Disease induced death occurs in two human compartment namely infected humans and hospitalized humans at the rate α. Infected humans coming in contact with a susceptible mosquito makes them infected by the rate β_M. Total infected mosquito population is $\beta_M \frac{I_H}{N_H} S_M$. Susceptible eggs and infected eggs (I_E) hatches their infected eggs at the rate ρ depending upon the climatic factor C_E. Oviposition rate is described by γ_M and the proportion of infected eggs laid by an infected mosquito is described with the rate θ. μ_E and μ_M denotes the death of eggs or mosquitoes due to unfavorable conditions may be high temperature or high humidity index, while μ_H denotes natural death rate of humans. Control u_1 in terms of preventive measures at human population, u_2 in terms of spraying insecticides for mosquito population, u_3 to curtail number of infected eggs by use of guppy fish which is an aquatic creature.

So, from the above figure 1, we have the following set of nonlinear ordinary differential equations describing the causes and cureness of dengue from one compartment to other.

$$\begin{aligned}
\frac{dS_H}{dt} &= \mathrm{B}N_H\left(1-\frac{N_H}{K_H}\right)-\left(\beta_H\frac{I_M}{N_H}+\mu_H\right)S_H\\
\frac{dI_H}{dt} &= \beta_H\frac{I_M}{N_H}S_H-\left(\delta_H+\mu_H+\alpha\right)I_H\\
\frac{dH_H}{dt} &= \delta_H I_H-\left(\eta_H+\mu_H+\alpha\right)H_H\\
\frac{dR_H}{dt} &= \eta_H H_H-\mu_H R_H\\
\frac{dS_M}{dt} &= -\beta_M\frac{I_H}{N_H}S_M+\rho C_E S_E-\left(\mu_M+\gamma_M\right)S_M\\
\frac{dI_M}{dt} &= \beta_M\frac{I_H}{N_H}S_M+\rho C_E I_E-\left(\theta\gamma_M+(1-\theta)\gamma_M+\mu_M\right)I_M\\
\frac{dS_E}{dt} &= \left(\gamma_M S_M+(1-\theta)\gamma_M I_M\right)\left(1-\frac{N_E}{K_E}\right)-\left(\mu_E+\rho C_E\right)S_E\\
\frac{dI_E}{dt} &= \theta\gamma_M I_M\left(1-\frac{N_E}{K_E}\right)-\left(\mu_E+\rho C_E\right)I_E
\end{aligned} \tag{1}$$

with $S_H+I_H+H_H+R_H\le N_H$, $S_M+I_M\le N_M$, $S_E+I_E\le N_E$

and $S_H,S_M,S_E>0$; $I_H,H_H,R_H,I_M,I_E\ge 0$.

Simplification of equation (1) has been carried out by comparing each population to a total population i.e.

$$S_h=\frac{S_H}{N_H},\ I_h=\frac{I_H}{N_H},\ H_h=\frac{H_H}{N_H},\ R_h=\frac{R_H}{N_H},\ S_m=\frac{S_M}{N_M},\ I_m=\frac{I_M}{N_M},\ S_e=\frac{S_E}{N_E},\ I_e=\frac{I_E}{N_E} \tag{2}$$

To obtain a new set of equations mainly of five dimensions' equation (2) is substituted in equation (1) namely as

$$\begin{aligned}
\frac{dS_h}{dt} &= \mathrm{B}\left(1-\frac{N_H}{K_H}\right)-\left(z_1 I_m+\mu_H\right)S_h\\
\frac{dI_h}{dt} &= z_1 I_m S_h-\left(\delta_H+\mu_H+\alpha\right)I_h\\
\frac{dH_h}{dt} &= \delta_H I_h-\left(\eta_H+\mu_H+\alpha\right)H_h\\
\frac{dI_m}{dt} &= \beta_M I_h\left(1-I_m\right)+\rho C_E z I_E-\left(\mu_M+\gamma_M\right)I_m\\
\frac{dI_E}{dt} &= \frac{\theta\gamma_M I_m}{z}\left(1-\frac{N_E}{K_E}\right)-\left(\mu_E+\rho C_E\right)I_E
\end{aligned} \tag{3}$$

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with $S_h+I_h+H_h+R_h\le 1,\ S_m+I_m\le 1,\ S_e+I_e\le 1$

where $z=\frac{N_E}{N_M}$ and $z_1=N_M\cdot\beta_H$

The feasible region for the above set of equations (3) is

$$\Lambda = \left\{ \left(S_h + I_h + H_h + I_m + I_e\right) / S_h + I_h + H_h + I_m + I_e \le \frac{\mathrm{B}\left(1 - \frac{N_H}{K_H}\right)}{\mu_H}; S_h > 0, I_h, H_h, I_m, I_e \ge 0 \right\}$$

Thus, Dengue free equilibrium point $\left(E_0\right) = \left(S_h, I_h, H_h, I_m, I_e\right) = \left(\frac{\mathrm{B}\left(1 - \frac{N_H}{K_H}\right)}{\mu_H}, 0, 0, 0, 0 \right)$.

Now, our actual interest lies in calculating the basic reproduction number which is calculated using the next generation matrix method which is defined as FV^{-1} [6] where F and V both are Jacobian matrices of $\Im$ and v evaluated with respect to infected humans, mosquitoes, eggs and hospitalized humans at the point E_0.

Let $X = \left(I_h, H_h, I_m, I_e, S_h\right)$. $\therefore \frac{dX}{dt} = \Im(X) - v(X)$

where $\Im(X)$ denotes the rate of newly recruited and $v(X)$ denotes the rate of derived recruited which is given as

$$\Im(X) = \begin{bmatrix} z_1 I_m S_h \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } v\left(X\right) = \begin{bmatrix} \left(\delta_H + \mu_H + \alpha\right) I_h \\ -\delta_H I_h + \left(\eta_H + \mu_H + \alpha\right) H_h \\ -\beta_M I_h \left(1 - I_m\right) - \rho C_E z I_e + \left(\mu_M + \gamma_M\right) I_m \\ -\frac{\theta \gamma_M I_m}{z}\left(1 - \frac{N_E}{K_E}\right) + \left(\mu_E + \rho C_E\right) I_e \\ -\mathrm{B}\left(1 - \frac{N_H}{K_H}\right) + \left(z_1 I_m + \mu_H\right) S_h \end{bmatrix}$$

Now, the derivative of $\Im$ and v evaluated at a dengue free equilibrium point $\left(E_0\right)$ gives matrices F and V of order 5×5 which is defined as

$$F = \left[\frac{\partial \Im_i(E_0)}{\partial X_j} \right] \quad V = \left[\frac{\partial v_i(E_0)}{\partial X_j} \right] \text{ for } i, j = 1, 2, 3, 4, 5$$

$$\text{So, } F = \begin{bmatrix} 0 & 0 & \frac{z_1 \mathrm{B}\left(N_H - K_H\right)}{\mu_H K_H} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$V = \begin{bmatrix} \delta_H + \mu_H + \alpha & 0 & 0 & 0 & 0 \\ -\delta_H & \eta_H + \mu_H + \alpha & 0 & 0 & 0 \\ -\beta_M & 0 & \mu_M + \gamma_M & -\rho C_E z & 0 \\ 0 & 0 & -\frac{\theta \gamma_M}{z}\left(1 - \frac{N_E}{K_E}\right) & \mu_E + \rho C_E & 0 \\ 0 & 0 & \frac{z_1 \mathrm{B}\left(N_H - K_H\right)}{\mu_H K_H} & 0 & \mu_H \end{bmatrix}$$

where V is non-singular matrix. Thus, the basic reproduction number R_0 which is the spectral radius of matrix FV^{-1} is given as

$$R_0 = \frac{z_1 \mathrm{B}\left(K_H - N_H\right)\left(\mu_E + \rho C_E\right)\beta_M K_E}{\mu_H K_H \left(\delta_H + \mu_H + \alpha\right)\left[\rho C_E \gamma_M \left(N_E\theta + (1-\theta)K_E\right) + \mu_M C_E K_E \rho + \mu_E K_E \left(\mu_M + \gamma_M\right)\right]}$$

On equating set of equations (3) to zero, an endemic equilibrium point $\left(E^*\right)$ is obtained which is as follows:

$E^* = \left(S_h^*, I_h^*, H_h^*, I_m^*, I_e^*\right)$ where

$$S_h^* = \frac{\mathrm{B}\left(K_H - N_H\right)}{K_H\left(z_1 I_m^* + \mu_H\right)}, \quad I_h^* = \frac{z_1 I_m^* S_h^*}{\mu_H + \alpha + \delta_H},$$

$$H_h^* = \frac{I_m^* \delta_H z_1 S_h^*}{K_H\left[\mu_H\left(\mu_H + 2\alpha + \delta_H + \eta_H\right) + \alpha\left(\alpha + \delta_H + \eta_H\right)\delta_H \eta_H\right]},$$

$$I_m^* = \frac{\rho C_E I_e^* z + \beta_M I_h^*}{\beta_M I_h^* + \mu_M + \gamma_M}, \quad I_e^* = \frac{\theta \gamma_M I_m^* \left(K_E - N_E\right)}{z K_E \left(\rho C_E + \mu_E\right)}$$

3. Stability Analysis

In this section, the local and global stability at E_0 and E^* using the linearization method and matrix analysis are to be studied.

3.1 Local Stability

Theorem 3.1.1 (stability at E_0) If $R_0 < 1$ then a system is locally asymptotically stable at dengue free equilibrium point E_0. If $R_0 > 1$ then it is unstable.

Proof: Jacobian Matrix of the system evaluated at point E_0 is

$$J\left(E_0\right) = \begin{bmatrix} -\mu_H & 0 & 0 & -\frac{z_1 \mathrm{B}\left(N_H - K_H\right)}{\mu_H K_H} & 0 \\ 0 & -\left(\delta_H + \mu_H + \alpha\right) & 0 & \frac{z_1 \mathrm{B}\left(N_H - K_H\right)}{\mu_H K_H} & 0 \\ 0 & \delta_H & -\left(\eta_H + \mu_H + \alpha\right) & 0 & 0 \\ 0 & \beta_M & 0 & -\left(\mu_M + \gamma_M\right) & \rho C_E z \\ 0 & 0 & 0 & \frac{\theta \gamma_M}{z}\left(1 - \frac{N_E}{K_E}\right) & -\left(\mu_E + \rho C_E\right) \end{bmatrix}$$

The eigenvalues of the characteristic equation are $\lambda_1 = -\mu_H < 0$, $\lambda_2 = -\left(\eta_H + \mu_H + \alpha\right)$ and $\lambda_3, \lambda_4, \lambda_5$ satisfies the equation $a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$ where

$a_0 = 1 > 0$

$a_1 = \rho C_E + \mu_E + \mu_H + \alpha + \delta_H + \gamma_M > 0$

$$a_2 = \left\{ \begin{array}{l} \rho C_E\left(\mu_H + \mu_M + \delta_H + \alpha + \gamma_M(1-\rho)\right) \\ +\mu_E\left(\mu_H + \mu_M + \alpha + \delta_H + \gamma_M\right) \\ +\mu_H\left(\mu_M + \gamma_M\right) + \left(\mu_M + \gamma_M\right)\left(\alpha + \delta_H\right) \end{array} \right\} + \left[\frac{\mu_H C_E K_H N_E \gamma_M \theta\rho - \beta_M \mathrm{B} K_E z_1 \left(K_H - N_H\right)}{\mu_H K_E K_H} \right] > 0 \text{ which}$$

is obvious.

$$a_3 = \frac{\left(\delta_H + \mu_H + \alpha\right)\left[-C_E K_E \gamma_M \theta\rho + C_E N_E \gamma_M \theta\rho + C_E K_E \mu_M \rho + C_E K_E \gamma_M \rho + \mu_E \mu_M K_E + \mu_E \gamma_M K_E\right]\left[1 - R_0\right]}{K_E}$$

So, if $R_0 < 1$ then $a_3 > 0$.

Hence, $a_1 > 0, a_2 > 0, a_3 > 0$ and $a_1 a_2 - a_3 > 0$ if $R_0 < 1$.

Thus by Routh Hurwitz criteria [7] all the conditions for the stability of $n = 3$ are satisfied.

Therefore, the system is locally asymptotically stable at E_0.

Theorem 3.1.2 (stability at E^*) If $QU - RT > 0$ and $\mu_E - X > 0$ then a system is locally asymptotically stable at an endemic equilibrium point E^*.

Proof: Jacobian Matrix of the system evaluated at point E^* is

$$J\left(E^*\right) = \begin{bmatrix} -\left(P + \mu_H\right) & 0 & 0 & -T & 0 \\ P & -Q & 0 & T & 0 \\ 0 & \delta_H & -S & 0 & 0 \\ 0 & R & 0 & -U & Xz \\ 0 & 0 & 0 & V & -\left(X + \mu_E\right) \end{bmatrix}$$

where

$$P = z_1 I_m^*,\ Q = \delta_H + \mu_H + \alpha,\ R = \beta_M\left(1 - I_m^*\right),\ S = \eta_H + \mu_H + \alpha,$$

$$T = z_1 S_h^*,\ U = \beta_M I_h^* + \mu_M + \gamma_M,\ V = \frac{\theta\gamma_M}{z}\left(1 - \frac{N_E}{K_E}\right),\ X = \rho C_E$$

The eigenvalues of the characteristic equation are $\lambda_1 = -S < 0$ and $\lambda_2, \lambda_3, \lambda_4, \lambda_5$ satisfies the equation $a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$ where

$$a_0 = 1 > 0$$

$$a_1 = \mu_E + X + U + P + Q + \mu_H > 0$$

$$a_2 = \mu_E\mu_H + \mu_E P + \mu_E Q + \mu_E U + \mu_H Q + \mu_H U + \mu_H X + PQ + PU + PX + QU + QX + UX - VXz - RT > 0$$

which is also obvious.

$$a_3 = \mu_E\mu_H Q + \mu_E\mu_H U + \mu_E PQ + \mu_H QX + \mu_H UX - VXz\left(\mu_H + P + Q\right) + PU\left(\mu_E - X\right) + \left(QU - RT\right)\left\{\mu_E + \mu_H + X\right\} + PQU + PQX$$

So, if $QU - RT > 0$ and $\mu_E - X > 0$ then $a_3 > 0$ and $-VXz\left(\mu_H + P + Q\right)$ is very small term as compared to other terms and consist of a very negligible value.

$$a_4 = \mu_E PQU + PQUX + \left(QU - RT\right)\left\{\mu_E\mu_H + \mu_H X\right\} - VQXz\left(\mu_H + P\right)$$

But as $VQXz(\mu_H + P)$ is very small term. So, if $QU - RT > 0$ then $a_4 > 0$.

Thus, $a_1 > 0, a_2 > 0, a_3 > 0, a_4 > 0$ and $a_1 a_2 a_3 > a_3^2 + a_1^2 a_4$ if $QU - RT > 0$.

Hence, all the conditions of Routh Hurwitz criteria for the stability are satisfied for $n = 4$ so the system is locally asymptotically stable at E^*.

3.2 Global Stability

Theorem 3.2.1 (stability at E_0) If $\frac{z_1 B(N_H - K_H)}{\mu K_H} < \frac{\theta\gamma_M}{z}\left(1 - \frac{N_E}{K_E}\right)$ then a system is globally asymptotically stable at dengue free equilibrium point E_0.

Proof: Consider a Lyapunov function

$$L(t) = I_h(t) + H_h(t) + I_m(t) + I_e(t)$$

$$\therefore L'(t) = -(\mu_H + \alpha)(I_h + H_h) - (\mu_E + \rho C_E - \theta C_E z) I_e - (\mu_M + \gamma_M + \beta_M I_h) I_m + \beta_M I_h + I_m\left(z_1 S_h + \frac{\theta\gamma_M}{z}\left(1 - \frac{N_E}{K_E}\right)\right)$$

So, if $z_1 S_h < \frac{\theta\gamma_M}{z}\left(1 - \frac{N_E}{K_E}\right)$ that is $\frac{z_1 B(N_H - K_H)}{\mu K_H} < \frac{\theta\gamma_M}{z}\left(1 - \frac{N_E}{K_E}\right)$ as $E_0 \in \Lambda$ and μ is non negative then $L'(t) < 0$.

And if $I_h = H_h = I_m = I_e = 0$ then $L'(t) = 0$.

Hence, $L'(t) = 0$ is E_0.

So, by LaSalle's Invariance Principle [8], every solution of system (1), with initial conditions in Λ, approaches E_0 as $t \to \infty$.

Thus, a system is globally asymptotically stable at point E_0.

Theorem 3.2.2 (stability at E^*) The system is globally asymptotically stable at endemic equilibrium point E^*.

Proof: Consider a Lyapunov function

$$L(t) = \frac{1}{2}\left[\{S_h(t) - S_h^*(t)\} + \{I_h(t) - I_h^*(t)\} + \{H_h(t) - H_h^*(t)\} + \{I_m(t) - I_m^*(t)\} + \{I_e(t) - I_e^*(t)\}\right]^2$$

$$\therefore L'(t) = \left[\{S_h(t) - S_h^*(t)\} + \{I_h(t) - I_h^*(t)\} + \{H_h(t) - H_h^*(t)\} + \{I_m(t) - I_m^*(t)\} + \{I_e(t) - I_e^*(t)\}\right] \left[S_h' + I_h' + H_h' + I_m' + I_e'\right]$$

On using $B = \mu_H S_H^* + \mu_H I_H^* + \mu_H H_H^* + \mu_M I_m^* + \mu_E I_e^*$ we have

$$L'(t) = -\left[\begin{array}{l}\{(S_h(t) - S_h^*(t)) + (I_h(t) - I_h^*(t)) + (H_h(t) - H_h^*(t)) + (I_m(t) - I_m^*(t)) + (I_e(t) - I_e^*(t))\} \\ \left\{\begin{array}{l}\mu_H(S_h(t) - S_h^*(t)) + \mu_H(I_h(t) - I_h^*(t)) + \mu_H(H_h(t) - H_h^*(t)) + \mu_M(I_m(t) - I_m^*(t)) \\ + \mu_E(I_e(t) - I_e^*(t)) + \alpha(I_h^* + H_h^*) + \eta_H H_h^* + \gamma_M I_m^* + \rho C_E I_e^* + \frac{\theta\gamma_M I_m^*}{z}\left(\frac{N_E}{K_E}\right) \\ + (\mu_H S_h^* + \mu_H I_h^* + \mu_H H_h^* + \mu_H I_m^* + \mu_H I_e^*)\frac{N_H}{K_H}\end{array}\right\}\end{array}\right] < 0$$

Hence, the system is globally asymptotically stable at E^*.

4. Optimal Control Model

In this section a control function has been implemented to decrease the spread of dengue disease in human population. The objective function along with the optimal control variable is given by

$$J(u_i,\Omega)=\int_0^T\left(\begin{array}{l}A_1S_H{}^2+A_2I_H{}^2+A_3H_H{}^2+A_4R_H{}^2+A_5S_M{}^2+A_6I_M{}^2\\+A_7S_E{}^2+A_8I_E{}^2+w_1u_1{}^2+w_2u_2{}^2+w_3u_3{}^2\end{array}\right)dt \tag{4}$$

where Ω denotes the set of all compartmental variables, $A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8$ denotes non-negative weight constants for the compartments $S_H,I_H,H_H,R_H,S_M,I_M,S_E,I_E$ respectively and w_1,w_2,w_3 are the weight constant for the control variable u_1,u_2,u_3 respectively.

The weights w_1,w_2 and w_3 which are constant parameters for u_1,u_2 and u_3 will standardized the optimal control condition.

Now, we will calculate the value of control variables u_1,u_2 and u_3 from $t=0$ to $t=T$ such that

$$J(u_1(t),u_2(t),u_3(t))=\min\left\{J(u_i^*,\Omega)/u_1,u_2,u_3\in\phi\right\}$$

where ϕ = smooth function on the interval $[0,1]$.

Using, Fleming and Rishel results [9], the optimal control denoted by u_i^* is obtained by collecting all the integrands of the objective function (2) using the lower bounds and upper bounds of the both the control variables respectively.

Using Pontrygin's principle [10], we construct a Lagrangian function consisting of state equation and adjoint variables $A_V=(\lambda_{S_H},\lambda_{I_H},\lambda_{H_H},\lambda_{R_H},\lambda_{S_M},\lambda_{I_M},\lambda_{S_E},\lambda_{I_E})$ which is as follows:

$$\begin{aligned}L(\Omega,A_V)&=A_1S_H{}^2+A_2I_H{}^2+A_3H_H{}^2+A_4R_H{}^2+A_5S_M{}^2+A_6I_M{}^2+A_7S_E{}^2+A_8I_E{}^2+w_1u_1{}^2+w_2u_2{}^2+w_3u_3{}^2\\&+\lambda_{S_H}\left[\mathrm{B}N_H\left(1-\frac{N_H}{K_H}\right)-\left(\beta_H\frac{I_M}{N_H}+\mu_H\right)S_H-u_1S_H\right]\\&+\lambda_{I_H}\left[\beta_H\frac{I_M}{N_H}S_H-(\delta_H+\mu_H+\alpha)I_H+u_1S_H\right]\\&+\lambda_{H_H}\left[\delta_HI_H-(\eta_H+\mu_H+\alpha)H_H\right]\\&+\lambda_{R_H}\left[\eta_HH_H-\mu_HR_H\right]\\&+\lambda_{S_M}\left[-\beta_M\frac{I_H}{N_H}S_M+\rho C_ES_E-(\mu_M+\gamma_M+u_2)S_M\right]\\&+\lambda_{I_M}\left[\beta_M\frac{I_H}{N_H}S_M+\rho C_EI_E-(\theta\gamma_M+(1-\theta)\gamma_M+\mu_M)I_M+u_2S_M\right]\\&+\lambda_{S_E}\left[(\gamma_MS_M+(1-\theta)\gamma_MI_M)\left(1-\frac{N_E}{K_E}\right)-(\mu_E+\rho C_E)S_E-u_3S_E\right]\\&+\lambda_{I_E}\left[\theta\gamma_MI_M\left(1-\frac{N_E}{K_E}\right)-(\mu_E+\rho C_E)I_E+u_3S_E\right]\end{aligned}$$

Now, the partial derivative of the Lagrangian function with respect to each variable of the compartment gives us the adjoint equation such that

$$\dot{\lambda}_{S_H} = -\frac{\partial L}{\partial S_H}$$

$$= -2A_1 S_H + \left(\beta_H \frac{I_M}{N_H} + u_1\right)\left(\lambda_{S_H} - \lambda_{I_H}\right) + \mu_H \lambda_{S_H}$$

$$\dot{\lambda}_{I_H} = -\frac{\partial L}{\partial I_H}$$

$$= -2A_2 I_H + \left(\beta_M \frac{S_M}{N_H}\right)\left(\lambda_{S_M} - \lambda_{I_M}\right) + \delta_H\left(\lambda_{I_H} - \lambda_{H_H}\right) + \left(\mu_H + \alpha\right)\lambda_{I_H}$$

$$\dot{\lambda}_{H_H} = -\frac{\partial L}{\partial H_H}$$

$$= -2A_3 H_H + \eta_H\left(\lambda_{H_H} - \lambda_{R_H}\right) + \left(\mu_H + \alpha\right)\lambda_{H_H}$$

$$\dot{\lambda}_{R_H} = -\frac{\partial L}{\partial R_H}$$

$$= -2A_4 R_H + \mu_H \lambda_{R_H}$$

$$\dot{\lambda}_{S_M} = -\frac{\partial L}{\partial S_M}$$

$$= -2A_5 S_M + \left(\beta_M \frac{I_H}{N_H} + u_2\right)\left(\lambda_{S_M} - \lambda_{I_M}\right) + \gamma_M\left(\lambda_{S_M} - \lambda_{S_E}\right) + \mu_M \lambda_{S_M}$$

$$\dot{\lambda}_{I_M} = -\frac{\partial L}{\partial I_M}$$

$$= -2A_6 I_M + \left(\beta_H \frac{S_H}{N_H}\right)\left(\lambda_{S_H} - \lambda_{I_H}\right) + \theta\gamma_M\left(1 - \frac{N_E}{K_E}\right)\left(\lambda_{S_E} - \lambda_{I_E}\right) + \left(\mu_M + \gamma_M\right)\lambda_{I_M} - \gamma_M \lambda_{S_E}\left(1 - \frac{N_E}{K_E}\right)$$

$$\dot{\lambda}_{S_E} = -\frac{\partial L}{\partial S_E}$$

$$= -2A_7 S_E + \rho C_E\left(\lambda_{S_E} - \lambda_{S_M}\right) + u_3\left(\lambda_{S_E} - \lambda_{I_E}\right) + \mu_E \lambda_{S_E}$$

$$\dot{\lambda}_{I_E} = -\frac{\partial L}{\partial I_E}$$

$$= -2A_8 I_E + \rho C_E\left(\lambda_{I_E} - \lambda_{I_M}\right) + \mu_E \lambda_{I_E}$$

The necessary conditions for Lagrangian function L to be optimal for control are

$$\frac{\partial L}{\partial u_1} = 2w_1 u_1 - S_H\left(\lambda_{S_H} - \lambda_{I_H}\right) = 0 \tag{5}$$

$$\frac{\partial L}{\partial u_2} = 2w_2 u_2 - S_M\left(\lambda_{S_M} - \lambda_{I_M}\right) = 0 \tag{6}$$

$$\frac{\partial L}{\partial u_3} = 2w_3 u_3 - S_E\left(\lambda_{S_E} - \lambda_{I_E}\right) = 0 \tag{7}$$

On solving equation (5), (6) and (7) we get,

$$u_1 = \frac{S_H\left(\lambda_{S_H} - \lambda_{I_H}\right)}{2w_1}, u_2 = \frac{S_M\left(\lambda_{S_M} - \lambda_{I_M}\right)}{2w_2} \text{ and } u_3 = \frac{S_E\left(\lambda_{S_E} - \lambda_{I_E}\right)}{2w_3}$$

Hence, the required optimal control condition is obtained as

$$u_1^* = \max\left(a_1, \min\left(b_1, \frac{S_H\left(\lambda_{S_H} - \lambda_{I_H}\right)}{2w_1}\right)\right)$$

$$u_2^* = \max\left(a_2, \min\left(b_2, \frac{S_M\left(\lambda_{S_M} - \lambda_{I_M}\right)}{2w_2}\right)\right)$$

$$u_3^* = \max\left(a_3, \min\left(b_3, \frac{S_E\left(\lambda_{S_E} - \lambda_{I_E}\right)}{2w_3}\right)\right)$$

where a_1, a_2, a_3 = lower bounds and b_1, b_2, b_3 = upper bounds of the control variables u_1, u_2 and u_3 respectively.

5. NUMERICAL SIMULATION

Using the data given in Table 1, it is observed that 25% population gets infected by dengue disease, however approximately only 2% disease is transmitted from dengue free equilibrium point to the existence. In this section, we will analyze and study the effect of control on each compartment numerically.

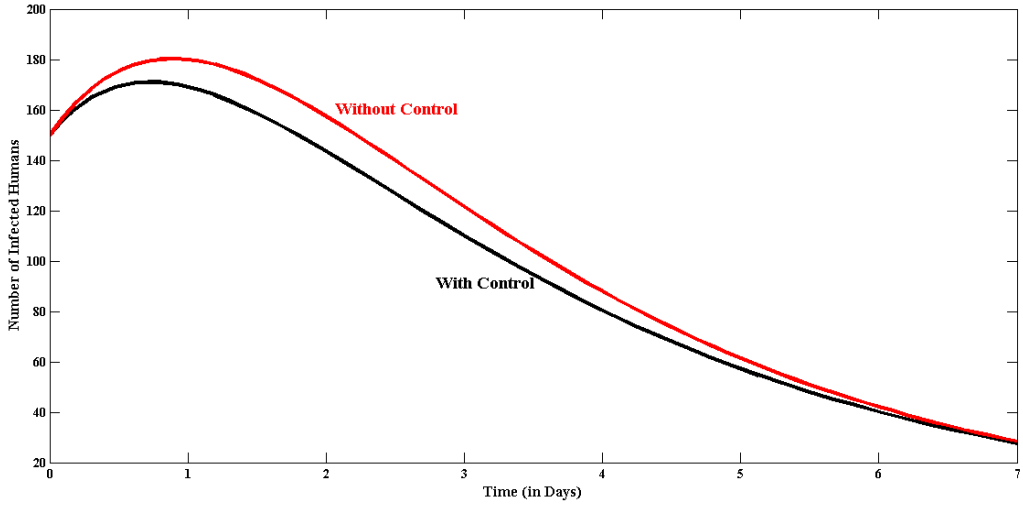

Figure 2. *Control in terms of preventive measures on I_H compartment*

Fig. 2 depicts that when control is not applied infected humans increases from 150 to 180 but when control in terms of preventive measures are taken by them the individual decrease to 170 approximately within a day.

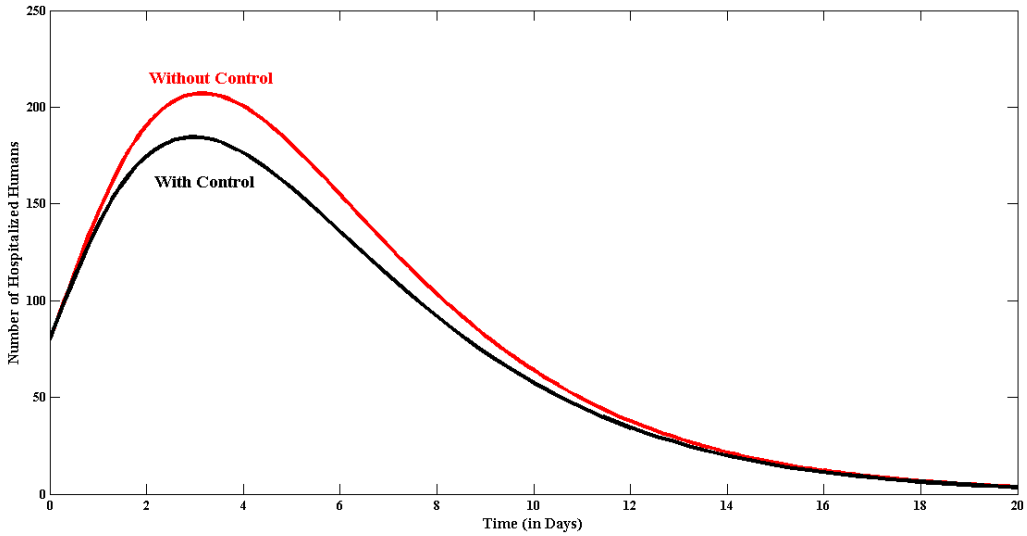

Figure 3. *Control in terms of preventive measures on H_H compartment*

From Fig. 3 it can be seen that when an individual gets medical treatment along with preventive measures, they recover from the diseases faster comparatively.

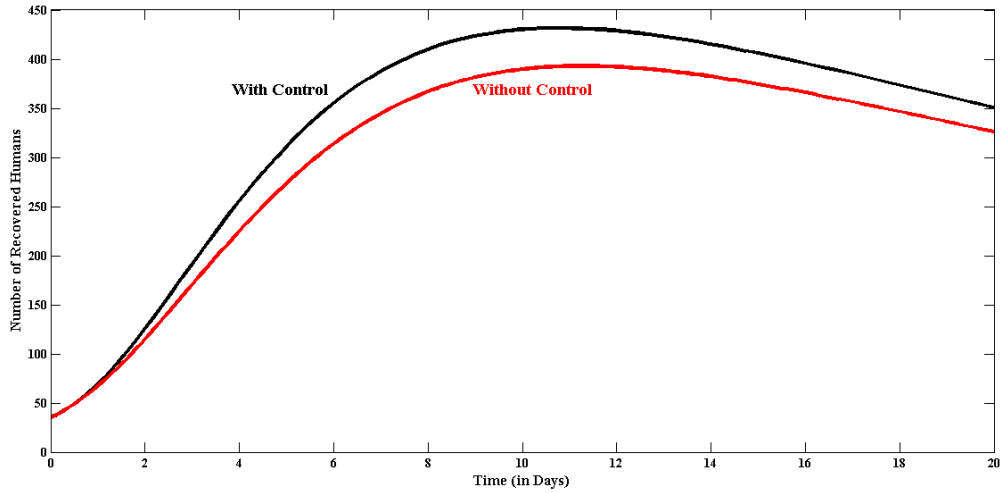

Figure 4. *Control in terms of preventive measures on* R_H *compartment*

Fig. 4 shows that individual affected from dengue recovers soon if the ways to control are adopted by them. In fact, it seems to be more fruitful ass it increases the recovered individual from 390 to 430 in the same duration of 10 days.

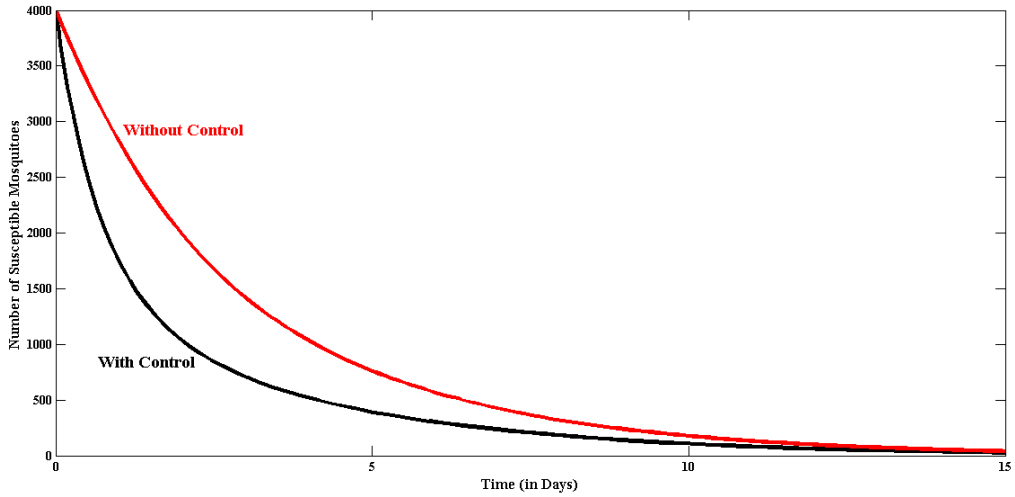

Figure 5. *Control in terms of spraying insecticides on* S_M *compartment*

Fig. 5 shows that a mosquito responsible for dengue disease are removed from the community in a small duration of time when insecticides are used.

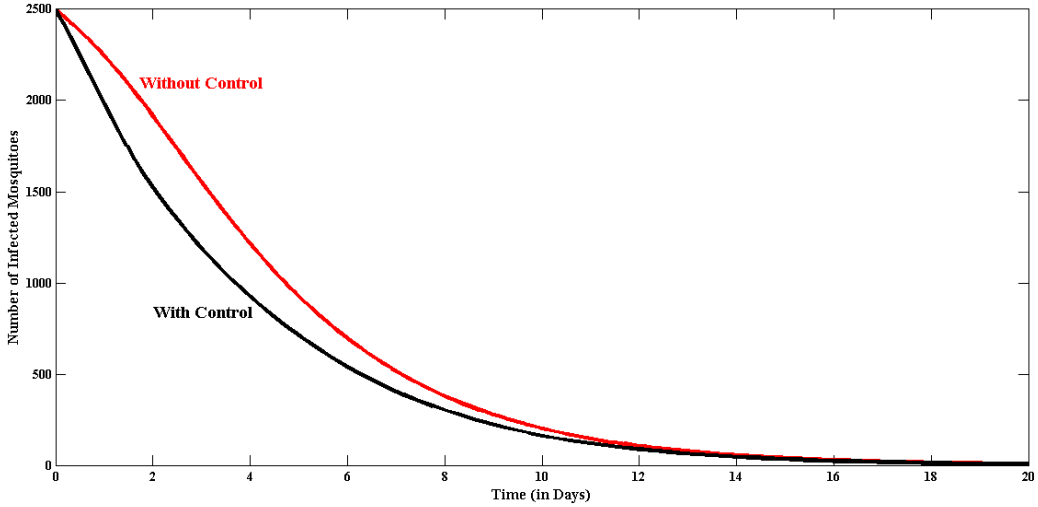

Figure 6. *Control in terms of spraying insecticides on* I_M *compartment*

Fig. 6 depicts that in the initial stage itself infected mosquitoes decreases in comparison to when control in terms of spraying insecticides is not done. But approximately after 12 days, no effect of control is observed which means that the mosquitoes have lost their life span and has died out.

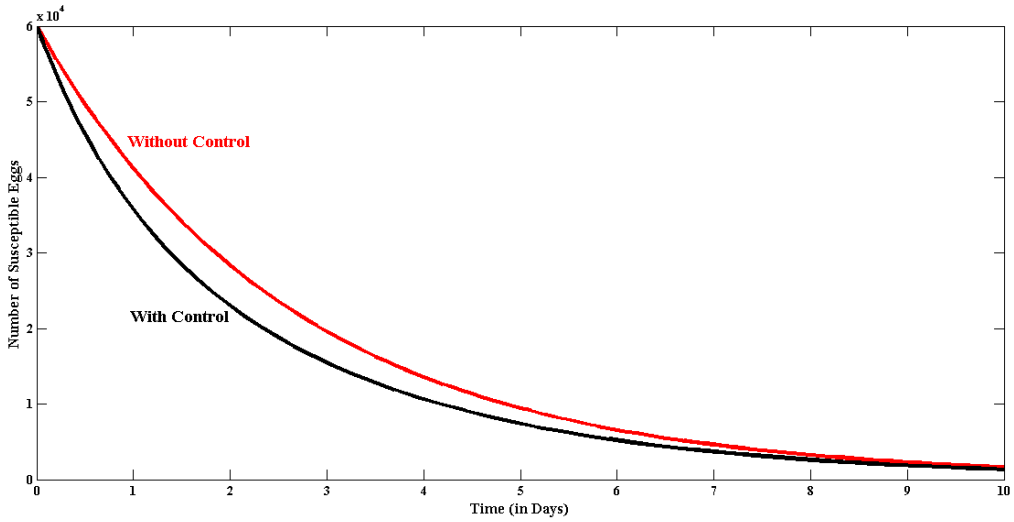

Figure 7. *Control in terms of guppy fish on* S_E *compartment*

Fig. 7 shows that the eggs laid by a susceptible mosquito proves to be susceptible which can be decreased as this is the food of guppy fish. If this eggs are eaten by the fishes, we can have our surrounding free from mosquitoes.

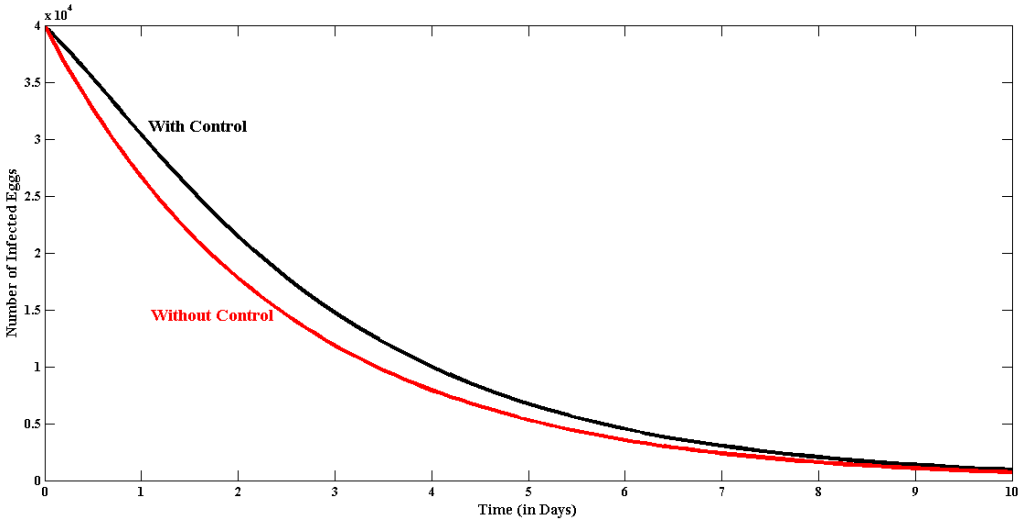

Figure 8. *Control in terms of guppy fish on* I_E *compartment*

Fig. 8 depicts that the susceptible eggs which are not eaten by guppy fish will surely become infective and will give birth to an infected mosquito which is harmful for the individuals in the society. But it can be decreased by the control which is observed from the figure.

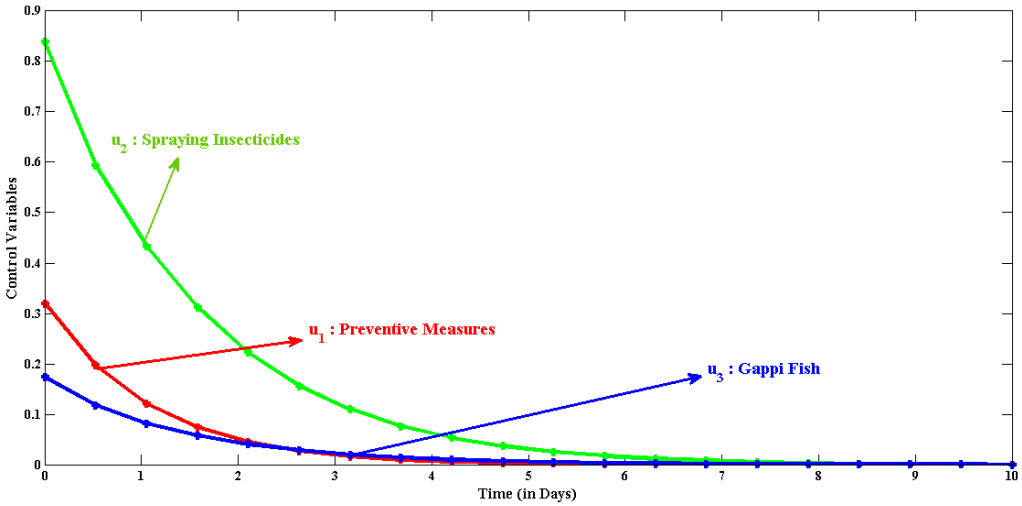

Figure 9. *Control Variables versus Time (in days)*

Fig. 9 shows that each control has a vital role in protecting humans from dengue disease. It is seen that 31% preventive measures, 83% use of insecticides, 18% Guppy fish are required in the initial days to stop oneself from becoming the victim of this disease.

6. CONCLUSION

In this paper, a mathematical model has been constructed to study the transmission of dengue disease. It has been established that this number of dengue affected individual can be reduced by using control on them as well on mosquitoes and their eggs. Two equilibrium points have been found: Dengue free equilibrium point and endemic equilibrium point. At both these points system proves to be locally asymptotically stable and globally asymptotically stable. Results for different compartments have been calculated numerically which interprets that to reduce overall severity of the dengue disease, increase in use of guppy fish at initial stage of mosquito cycle, increase spray of insecticides for the last stage of mosquito cycle and preventive measures at the end of humans is required.

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REFERENCES

[1] https://en.wikipedia.org/wiki/Aedes Aegypti

[2] http://www.denguevirusnet.com/life-cycle-of-aedes-aegypti.html

[3] https://en.wikipedia.org/wiki/Dengue_fever

[4] https://www.webmd.com/a-to-z-guides/dengue-fever-reference#1

[5] Asmaidi, A., Sianturi, P., and Nugrahani, E. H., A SIR Mathematical Model of Dengue Transmission and its Simulation, Indonesian Journal of Electrical Engineering and Computer Science. 12(11), 7920-7926, (2014).

[6] Diekmann, O., Heesterback, J.A.P., Roberts, M.G., The construction of next generation matrices for compartmental epidemic models, Journal of the Royal Society Interface, 7(47), 873-885, (2010).

[7] http://web.abo.fi/fak/mnf/mate/kurser/dynsyst/2009/R-Hcriteria.pdf

[8] LaSalle, J. P., The Stability of Dynamical Systems, Society for Industrial and Applied Mathematics, Philadelphia, Pa, (1976). https://doi.org/10.1137/1.9781611970432

[9] Fleming, W. H., Rishel, R. W., Deterministic and Stochastic optimal control (1). Springer Science and Business Media, (2012).

[10] Pontriagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V., Mishchenko, E. F., (1986). "The Mathematical Theory of Optimal Process", *Gordon and Breach Science Publishers*, NY, USA, 4-5.

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