International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 5, Issue 5, 2017, PP 1-7
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
DOI: http://dx.doi.org/10.20431/2347-3142.0505001
www.arcjournals.org

k- Super Lehmer-3 Mean Graphs

S. Somasundaram

Professor
Department of Mathematics
Manonmaniam Sundaranar University, Tirunelveli, India

S.S. Sandhya

Assistant Professor
Department of Mathematics
Sree Ayyappa College for Women, Chunkankadai, India

T.S. Pavithra

Research scholar
Manonmaniam Sundaranar University, Tirunelveli, India

***Corresponding Author:** *T.S. Pavithra, Research scholar Manonmaniam Sundaranar University Tirunelveli, India.*

Abstract: *Let f:V(G)→{1,2,3,……k+p+q-1} be an injunctive function , For a vertex labeling the induced edge labeling f(e=uv) is defined by f(e)=* $\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ *(or)* $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$*, then f is called k-super Lehmer-3 mean labeling if (f(V(G)∪ {f(e):e ϵE(G)}={k,k+1,k+2,……k+p+q-1}. A graph which satisfies this labeling condition is called k-super Lehmer-3 mean graph.*

Keywords: *Lehmer-3 mean graph, Super Lehmer-3 mean graph ,k-Super Lehmer-3 mean graph, Path, Comb, Kite, Crown.*

1. Introduction

A graph considered here are finite, undirected and simple. The vertex set and edge set of a graph are denoted by V(G) and E(G) respectively. For standerd terminology and notations we follow Harrary[1]. S Somasundaram & S.S Sandhya introduced the concept of Harmonic Mean Labeling of Graphs in [2] and its basic results was proved in [3] and [4].We will provide a brief summary of other informations which are necessary for our present investigation.

Definition 1.1

A graph G=(V,E) with P vertices and q edges is called **Lehmer -3 mean graph**. If it is possible to label vertices x ЄV with distinct labels f(x) from 1,2,3,…………q+1 in such a way that when each edge e=uv is labeled with f(e=uv)= $\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ (or) $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$, then the edge labels are distinct. In this case "f " is called Lehmer -3 mean labeling of G.

Definition 1.2

Let f:V(G)→{1,2,…..p+q} be an injective function .For a vertex labeling "f " the induced edge labeling f(e=uv) is defined by ,f(e)= $\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ (or) $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$, then f is called Super Lehmer -3 mean labeling ,if {f (V(G))} ∪ {f(e)/e Є E(G)} ={1,2,3,…..p+q}, A graph which admits Super Lehmer -3 Mean labeling is called **Super Lehmer -3 Mean graph**

Definition 1.3

Let $f:V(G)\rightarrow\{1,2,3,\ldots\ldots k+p+q-1\}$ be an injunctive function , for a vertex labeling the induced edge labeling $f(e=uv)$ is defined by $f(e)=\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ (or) $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$, then f is called k-super Lehmer-3 mean labeling if $(f(V(G)\cup\{f(e):e\in E(G)\}=\{k,k+1,k+2,\ldots\ldots k+p+q-1\}$. A graph which satisfies this labeling condition is called k-super Lehmer-3 mean graph.

2. Main Results

Theorem 2.1

Any path is a k- Super Lehmer-3 mean graph.

Proof :-

Let P_n be a path of n vertices $u_1,u_2,\ldots\ldots u_n$.

We define a function $f:V(G)\rightarrow\{k,k+1,k+2,\ldots\ldots k+p+q-1\}$ by

$f(u_i)=k+(2i-2)$; $i=1,2,3,\ldots.n$

The edges are labeled with

$f(u_iu_{i+1})= k+(2i-1)$; $1\leq i\leq n-1$,

Then we get distinct edge labels in which both $f(V(G)\cup E(G))$ gives the values from $\{k,k+1,k+2,\ldots\ldots k+p+q-1\}$. Thus any path forms a k -super Lehmer 3 mean graph.

Example 2.2

Let us check with k=20 upto 5 vertices we get.

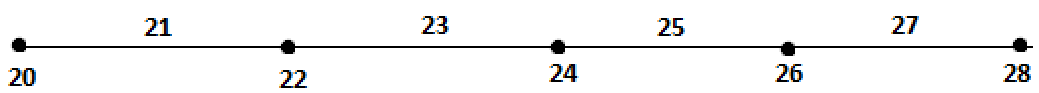

Figure- 1

Theorem 2.3

Any $(P_n\odot K_1)$ is a k-Super Lehmer-3 mean graph.

Proof :-

Let P_n be a path with n vertices K_1 be a pendant vertices from each vertex of the path P_n . let the vertices of P_n be $u_1,u_2,\ldots.u_n$ and that of the pendant vertices be $v_1,v_2,,\ldots.v_n$.

We define a function $f:V(G)\rightarrow\{k,k+1,k+2,\ldots\ldots k+p+q-1\}$ by

$f(u_i)=k+(4i-4)$; $1\leq i\leq n$

$f(v_i)=k+(4i-2)$; $1\leq i\leq n$.

The edge labelings are

$f(u_iu_{i+1})=k+(4i-3)$; $1\leq i\leq n-1$,

$f(u_iv_i)=k+(4i-1)$; $1\leq i\leq n-1$.

Thus the union of vertices of G and edges of G together equals $\{k,k+1,k+2,\ldots\ldots k+p+q-1\}$ which are all distinct and hence forms a k-Super Lehmer-3 mean graph.

Example 2.4

12-Super Lehmer-3 mean labeling pattern on $(P_4\odot K_1)$ is given below.

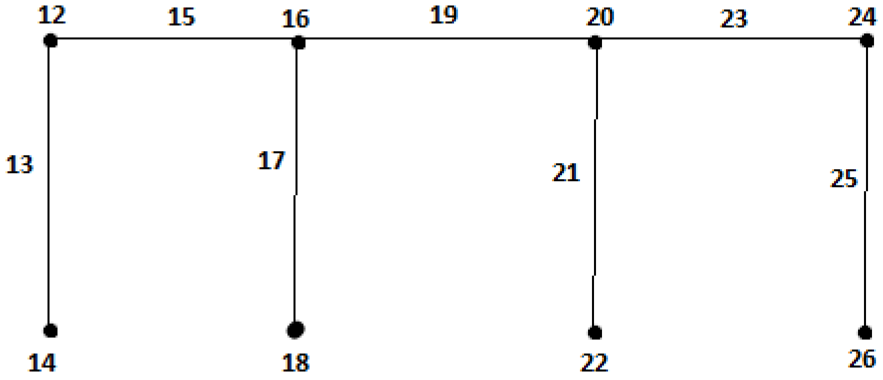

Figure-2

Theorem 2.5

Any graph obtained by attaching C_3 to an end vertex of P_n is a k-Super Lehmer-3 mean graph.

Proof :-

Let G be a graph obtained by attaching C_3 to an end vertex of P_n . let the vertices of P_n be $u_1,u_2,\ldots.u_n$ and the vertices of the cycle C_3 be $u_n vw$.

Define a function $f:V(G)\rightarrow \{k,k+1,k+2,\ldots\ldots k+p+q-1\}$ by

$f(u_i)=k+(2i-2)$; $1\leq i\leq n$,

$f(v)=k+2n$ and

$f(w)= k+(2n+3)$

Thus the edges obtained are all distinct. Also $f(V(G)\cup E(G)) =\{k,k+1,k+2,\ldots\ldots k+p+q-1\}$. Hence this graph G admits a k- Super Lehmer-3 mean graph.

Example 2.6

We obtain a graph by giving the value of k=100 we get.

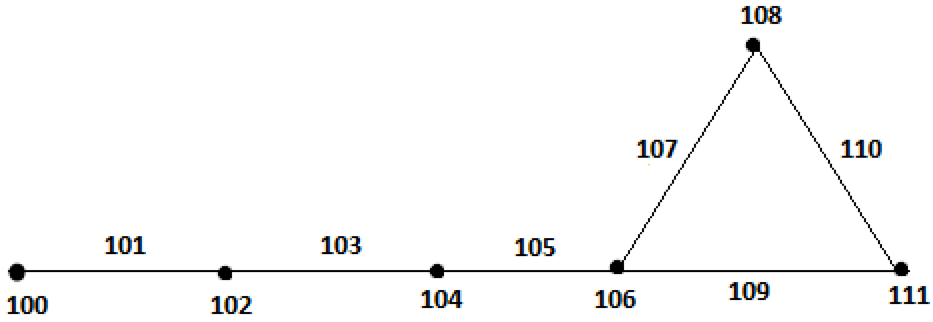

Figure-3

Theorem 2.7

nP_m is a k-Super Lehmer-3 mean graph.

Proof:-

Let n be the number of graphs and m be the number of vertices of each path. Let u_{ij}, $1\leq i\leq n$, $1\leq j\leq m$ be the vertices of nP_m . The edge set is $E=\{u_{ij}u_{ij+1}/1\leq i\leq n, 1\leq j\leq m-1\}$.

Let us define a function $f;V(nP_m))\rightarrow \{k,k+1,k+2,\ldots\ldots k+p+q-1\}$ by

$f(u_{ij})=k+(2m-1)(i-1)+(2j-2)$

Then the edge labels are all distinct such that $(f(V(G) \cup E(G)) = \{k, k+1, k+2, \ldots\ldots k+p+q-1\}$ which is a k-Super Lehmer-3 mean graph.

Example 2.8

50 -Super Lehmer-3 mean labeling of $4P_5$ graph is shown below.

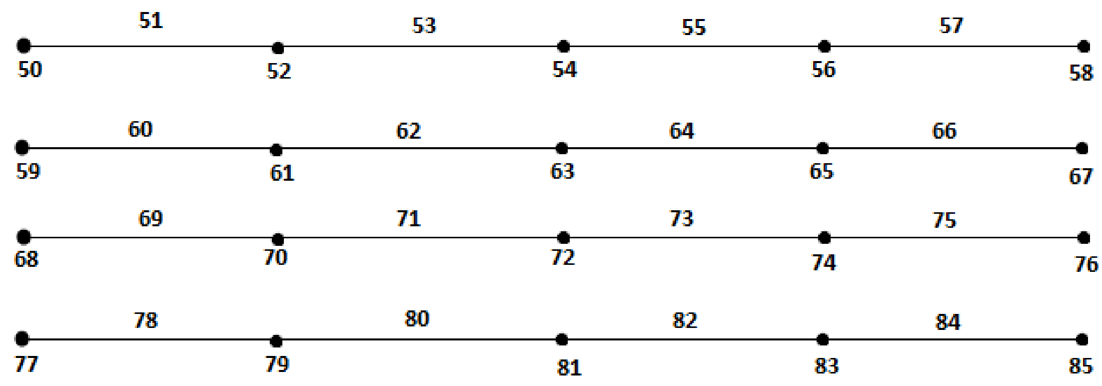

Figure-4

Theorem 2.9

$P_n(P_m \odot k_1)$ is a k-Super Lehmer-3 mean graph.

Proof:-

Let G be a graph obtained from the union of two graphs P_n and $P_m \odot k_1$ consider the vertices of P_n as $u_1, u_2, \ldots\ldots u_n$.and the vertices of the comb $P_m \odot k_1$ be v_l, w_l where $1 \le l \le m$.

We define a function $f:V(G) \rightarrow \{k, k+1, k+2, \ldots\ldots k+p+q-1\}$ by

$f(u_i) = k+(2i-2)$; $1 \le i \le n$

$f(v_l) = k+(2n-2)+(2j-1)$; $j = 1,3,5,\ldots\ldots 2m-1$

$f(w_l) = k+(2n-2)+(2j-1)$; $j = 2,4,6,\ldots\ldots 2m$ where $1 \le l \le m$

Then the edge labels are all distinct such that $(f(V(G) \cup E(G)) = \{k, k+1, k+2, \ldots\ldots k+p+q-1\}$. Thus $P_n(P_m \odot k_1)$ is a k-Super Lehmer-3 mean graph.

Example 2.10

A 25-Super Lehmer-3 mean labeling of $P_4(P_6 \odot k_1)$ is

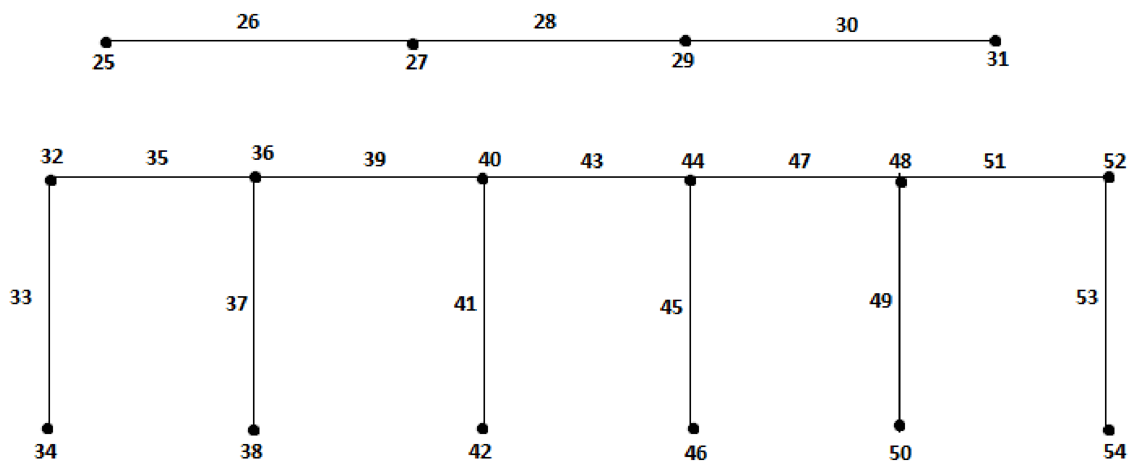

Figure-5

Theorem 2.11

P_n union graph obtained by attaching C_3 to an end vertex of P_m is a k-Super Lehmer-3 mean graph .

Proof :-

Let G be a graph with vertices be $u_1,u_2,\ldots..u_n$, $v_1,v_2,\ldots.v_m,w,x$ respectively.

A function defined by $f:V(G)\rightarrow \{k,k+1,k+2,\ldots\ldots k+p+q-1\}$ by

$f(u_i)=k+(2i-2)$; $1\le i\le n$

$f(v_j)=k+(2n-2)+(2j-1)$; $1\le j\le m$

$f(w)=k+(2n-2)+(2m+1)$

$f(x)=k+(2n-2)+(2m+4)$

Thus the edge labels obtained are all distinct so that $(f(V(G)\cup E(G)) =\{k,k+1,k+2,\ldots k+p+q-1\}$. Hence P_n union graph obtained by attaching C_3 to an end vertex of P_m is a k -Super Lehmer-3 mean graph .

Example 2.12

100 -Super Lehmer-3 mean labeling is

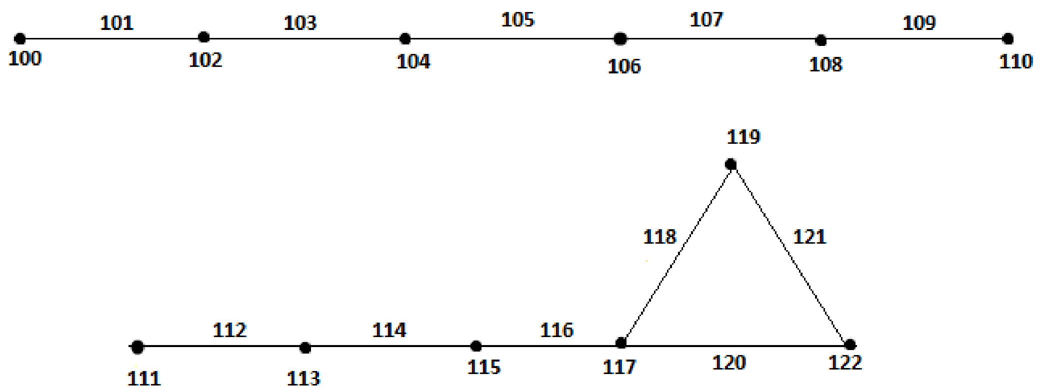

Figure-6

Theorem 2.13

The union of comb and a graph obtained by attaching C_3 to an end vertex of P_m is a k-Super Lehmer-3 mean graph.

Proof:-

Let G be the union of graphs. Let its vertices be $u_1,u_2,\ldots.u_n$, $v_1,v_2,\ldots.v_n,w_1,w_2,\ldots w_m,x,y$ respectively.

Let us define a function $f:V(G)\rightarrow \{k,k+1,k+2,\ldots\ldots k+p+q-1\}$ by

$f(u_i)=k+(4i-4)$; $1\le i\le n$

$f(v_i)=k+(4i-2)$; $1\le i\le n$

$f(w_j)=k+(4n-2)+(2j-1)$; $1\le j\le m$

$f(x)=k+(4n-2)+(2m+1)$

$f(y)=k+(4n-2)+(2m+4)$

The edge labels are all different and $(f(V(G)\cup E(G)) =\{k,k+1,k+2,\ldots\ldots k+p+q-1\}$. Thus G forms a k-Super Lehmer-3 mean graph.

Example 2.14

1000-Super Lehmer-3 mean labeling of G is given below.

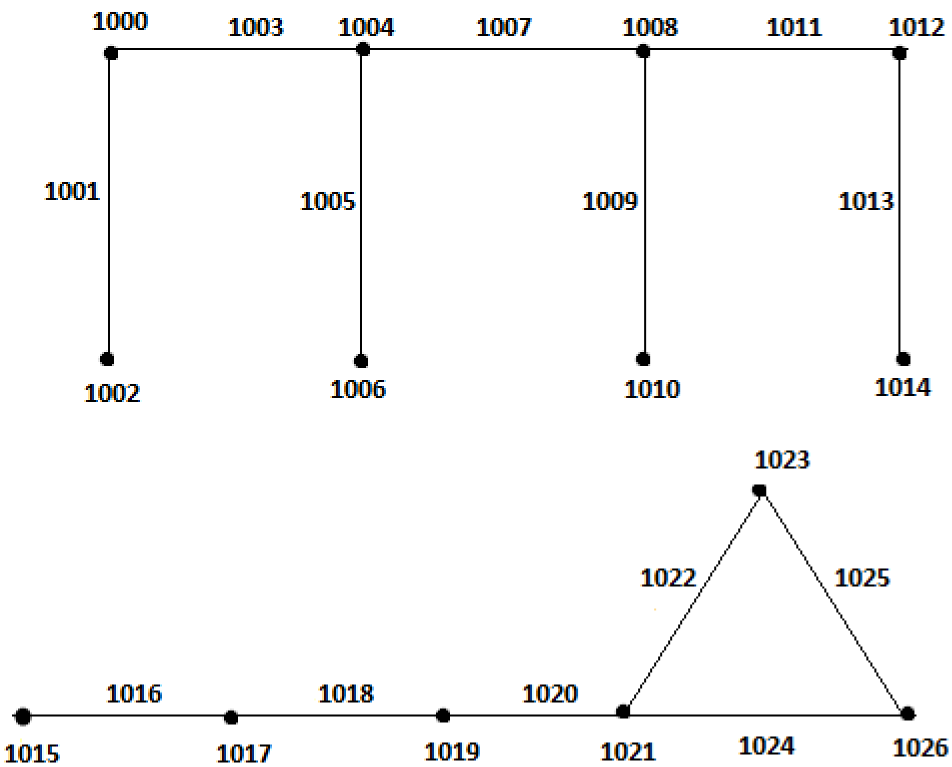

Figure-7

Theorem 2.15

$nP_m \cup (P_l \Theta k_1)$ is a k- Super Lehmer-3 mean graph.

Proof:-

Let G be a graph of nP_m, $P_l \Theta k_1$ union graphs. Let the vertices be u_{ij} where $1 \leq i \leq n$, $1 \leq j \leq m$, v_p, w_p where $1 \leq p \leq l$.

We define a function $f: V(G) \rightarrow \{k, k+1, k+2, \ldots\ldots k+p+q-1\}$ by

$f(u_{ij}) = k+(2m-1)(i-1)+(2j-2)$

$f(v_p) = k+(2m-1)(n-1)+(2m-2)+(2s-1)$; $s=1,3,5,7,\ldots.2l-1$

$f(w_p) = k+(2m-1)(n-1)+(2m-2)+(2s-1)$; $s=2,4,6,8,\ldots..2l$ and $1 \leq p \leq l$

Then the edge labels are all distinct. Thus $(f(V(G) \cup E(G)) = \{k, k+1, k+2, \ldots\ldots k+p+q-1\}$ and hence $nP_m \cup (P_l \Theta k_1)$ is a k-Super Lehmer-3 mean graph.

Example 2.16

72 -Super Lehmer-3 mean labeling of G is given below.

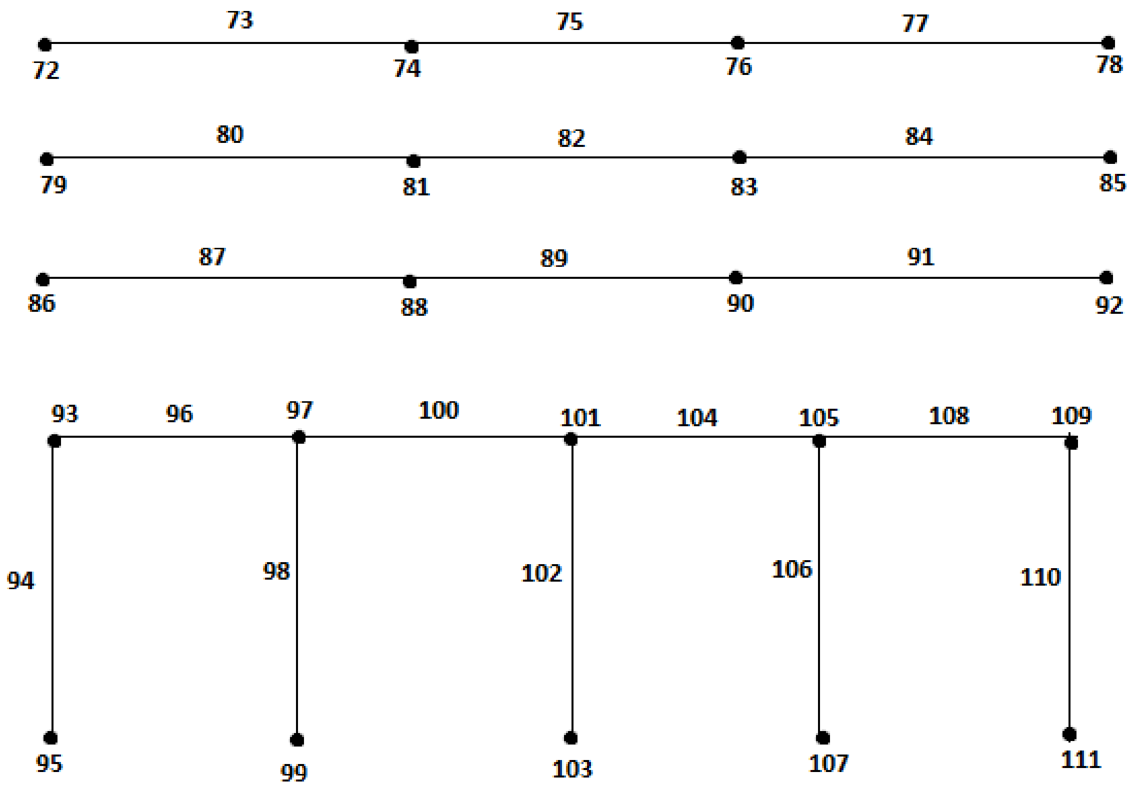

Figure-8

Remark 2.17

$nP_m \cup kite$ is a k-Super Lehmer-3 mean graph.

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Citation: *S. Somasundaram et al., " k- Super Lehmer-3 Mean Graphs ", International Journal of Scientific and Innovative Mathematical Research, vol. 5, no. 5, p. 7, 2017., http://dx.doi.org/10.20431/2347-3142.0505001*