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Continued Fractions of Different Quotients

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Abstract: *The use of continued fractions as an important tool in number theory began with 17th century results of Schwenter, Huygens and Wallis and came to maturity with the work of Euler in 1737 and the subsequent use of continued fractions as a number theoretic tool by Lagrange, Legendre, Gauss, Galois and their successors. In this paper, we will find the continued fraction representation of quotients of different powers of consecutive numbers between 2 and 10.*

Keywords: *Continued fractions, quotients, rational approximations, Euclidean algorithm*.

1. Introduction

The efficient process of finding the best rational approximations of any real number x is the continued fraction expansion of that number. In general, a simple continued fraction is an expression of the form

$$[a_0, a_1, \ldots\ldots] = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ldots\ldots}}. \qquad \ldots(1.1)$$

where the letters a_0, a_1, a_2, ……denote independent variables and may be interpreted as real or complex numbers, functions etc. If the number of terms is finite, we write

$$[a_0, a_1, \ldots\ldots, a_n] = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ldots\ldots + \cfrac{1}{a_n}}} \qquad \ldots(1.2)$$

If the number of terms is infinite, we write

$$[a_0, a_1, \ldots\ldots] = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ldots\ldots}} \qquad \ldots(1.3)$$

For a finite continued fraction $[a_0, a_1, a_2, \ldots, a_n]$ and a positive integer $k \le n$, the k-th remainder is defined as the continued fraction

$$r_k = [a_k, a_{k+1}, \ldots\ldots, a_n]. \qquad \ldots(1.4)$$

Any rational number can be represented as a finite continued fraction. If $x = a / b$ is a rational number, then the method for obtaining the continued fraction of x is nothing else than the Euclidean algorithm for computing the greatest common divisor of a and b:

$$a = a_0 b + r_0, \quad 0 \le r_0 \le b, \quad x_1 = b / r_0,$$

$$b = a_1 r_0 + r_1, \quad 0 \le r_1 \le r_0, \quad x_2 = r_0 / r_1,$$

$$r_0 = a_2 r_1 + r_2, \quad 0 \le r_2 \le r_1, \quad x_3 = r_1 / r_2,$$

….. …..

Therefore, on the other hand, since the Euclidean algorithm always stops, the continued fraction of a rational number is always finite. Hence, it is obvious that a finite continued fraction represents a rational number[1].

In view of unwieldiness of this notation, various authors have proposed other ways of writing continued fractions, for example

$$b_0 + \frac{a_1|}{|b_1} + \frac{a_2|}{|b_2} + \ldots\ldots + \frac{a_n|}{|b_n} \qquad \text{(Pringsheim, [2])} \qquad \ldots\ldots(1.5)$$

$$b_0 + \frac{a_1}{b_1} + \frac{a_2}{b_2} + \ldots\ldots + \frac{a_n}{b_n} + \ldots\ldots \qquad \text{(Muller, [3])} \qquad \ldots\ldots(1.6)$$

$$b_0 + \frac{a_1}{b_1 +} \; \frac{a_2}{b_2 +} \ldots\ldots + \frac{a_n}{b_n +} \ldots\ldots \qquad \text{(Rogers, [4])} \qquad \ldots\ldots(1.7)$$

where the fraction $\frac{a_n}{b_n}$ is called the n[th] partial quotient of the continued fraction; a_n and b_n are the coefficients of the continued fraction; b_1, b_2,…..its partial denominators; a_1,a_2……its partial numerators.

The study of finite continued fractions, began in its explicit form in the latter decades of the 16[th] century with a paper by Bombelli written when the concepts and notations of algebra were first being laid down in Italy and France such expressions play a natural role in connection with the integrated application of the Euclidean algorithm and some mathematical historians have claimed to have found similar usages in Hindu or even Greek mathematics. Infinite continued fractions were first considered by Lord Broucker, first president of the Royal Society.

The use of continued fractions as an important tool in number theory began with 17[th] century results of Schwenter, Huygens and Wallis and came to maturity with the work of Euler in 1737 and the subsequent use of continued fractions as a number theoretic tool by Lagrange, Legendre, Gauss, Galois and their successors. Continued fraction expansions involving functions of a complex variable rather than simply numbers were introduced by Euler and became an important tool in the approximation of special classes of analytic functions in the work of Euler, Lambert and Lagrange.

Ramanujan's continued fraction of order 5 is given by,

$$R_1(q) = 1 + \frac{q}{1+}\frac{q^2}{1+}\frac{q^3}{1+}\ldots\ldots\ldots \qquad \ldots\ldots(1.8)$$

Ramanujan's continued fraction of order 6 is

$$R(q) = 1 + \frac{q(1+q)}{1+}\;\frac{q^2(1+q^2)}{1+}\;\frac{q^3(1+q^3)}{1+}\ldots\ldots\ldots \qquad \ldots\ldots(1.9)$$

Ramanujan [5] gave the following result

$$1 + \frac{q(1+q)}{1+}\;\frac{q^2(1+q^2)}{1+}\;\frac{q^3(1+q^3)}{1+}\ldots\ldots\ldots = \frac{(q^3,q^3;q^6)_\infty}{(q,q^5;q^6)_\infty} \qquad \ldots\ldots(1.10)$$

Andrews [6] gave the following continued fraction

$$R(q) = 1 + \frac{q+q^2}{1+}\;\frac{q^4}{1+}\;\frac{q^3+q^6}{1+}\;\frac{q^8}{1+}\ldots\ldots\ldots \qquad \ldots\ldots(1.11)$$

2. Main Results

Let us consider the natural numbers from 2 to 10. In this paper, we will find the continued fraction representation of quotients of different powers of consecutive numbers between 2 and 10.

2.1. Quotients of 3 and 2 with powers of 2 to 10

Let A_1, A_2, A_3, ….., A_9 represent the quotients of the numbers 3 and 2 raised to the same powers starting from power 2 to power 10.

$$A_1 = 2+\frac{1}{4} = [2,4]. \qquad \text{.....(2.1)}$$

$$A_2 = 3+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}} = [3,2,1,2]. \qquad \text{.....(2.2)}$$

$$A_3 = 5+\frac{1}{16} = [5,16]. \qquad \text{.....(2.3)}$$

$$A_4 = 7+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3}}}} = [7,1,1,4,3]. \qquad \text{.....(2.4)}$$

$$A_5 = 11+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{2}}}}}} = [11,2,1,1,3,1,2]. \qquad \text{.....(2.5)}$$

$$A_6 = 17+\cfrac{1}{11+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3}}}}} = [17,11,1,1,1,3]. \qquad \text{.....(2.6)}$$

$$A_7 = 25+\cfrac{1}{1+\ldots\ldots} = [25,1,1,1,2,3,1,1,1,2]. \qquad \text{.....(2.7)}$$

$$1+\cfrac{1}{1+\cfrac{1}{2}}$$

$$A_8 = 38+\cfrac{1}{2+\ldots\ldots} = [38,2,3,1,10,1,1,2]. \qquad \text{.....(2.8)}$$

$$1+\cfrac{1}{1+\cfrac{1}{2}}$$

$$A_9 = 57 + \cfrac{1}{1+\ldots\ldots} = [57,1,1,1,67,1,1,2]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.9)$$

$$1+\cfrac{1}{1+\cfrac{1}{2}}$$

2.2. Quotients of 4 and 3 with powers of 2 to 10

Let B_1, B_2, B_3, ….., B_9 represent the quotients of the numbers 4 and 3 raised to the same powers starting from power 2 to power 10.

$$B_1 = 1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2}}} = [1,1,3,2]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.10)$$

$$B_2 = 2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3}}}} = [2,2,1,2,3]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.11)$$

$$B_3 = 3+\cfrac{1}{6+\cfrac{1}{4+\cfrac{1}{3}}} = [3,6,4,3]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.12)$$

$$B_4 = 4+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{17}}}} = [4,4,1,2,17]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.13)$$

$$B_5 = 5+\cfrac{1}{1+\ldots\ldots} = [5,1,1,1,1,1,1,1,5,6]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.14)$$

$$1+\cfrac{1}{5+\cfrac{1}{6}}$$

$$B_6 = 7+\cfrac{1}{2+\cfrac{1}{29+\cfrac{1}{18+\cfrac{1}{2}}}} = [7,2,29,18,2]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.15)$$

$$B_7 = 9+\cfrac{1}{1+\cfrac{1}{87+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{24}}}}}} = [9,1,87,1,1,1,24]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.16)$$

$$B_8 = 13 + \cfrac{1}{3 + \cfrac{1}{7 + \cfrac{1}{18 + \cfrac{1}{8 + \cfrac{1}{6}}}}} = [13,3,7,18,8,6]. \qquad \text{..................................(2.17)}$$

$$B_9 = 17 + \cfrac{1}{1 + \dots\dots} = [17,1,3,7,1,5,4,1,4,1,1,5]. \qquad \text{..................................(2.18)}$$

$$1 + \cfrac{1}{1 + \cfrac{1}{5}}$$

2.3. Quotients of 5 and 4 with powers of 2 to 10

Let C_1, C_2, C_3, ….., C_9 represent the quotients of the numbers 5 and 4 raised to the same powers starting from power 2 to power 10.

$$C_1 = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{2}}}} = [1,1,1,3,2]. \qquad \text{..................................(2.19)}$$

$$C_2 = 1 + \cfrac{1}{1 + \cfrac{1}{20 + \cfrac{1}{3}}} = [1,1,20,3]. \qquad \text{..................................(2.20)}$$

$$C_3 = 2 + \cfrac{1}{2 + \dots\dots} = [2,2,2,1,6,1,1,2]. \qquad \text{..................................(2.21)}$$

$$1 + \cfrac{1}{1 + \cfrac{1}{2}}$$

$$C_4 = 3 + \cfrac{1}{19 + \cfrac{1}{3 + \cfrac{1}{8 + \cfrac{1}{2}}}} = [3,19,3,8,2]. \qquad \text{..................................(2.22)}$$

$$C_5 = 3 + \cfrac{1}{1 + \dots\dots} = [3,1,3,1,3,6,3,3]. \qquad \text{..................................(2.23)}$$

$$6 + \cfrac{1}{3 + \cfrac{1}{3}}$$

$$C_6 = 4 + \cfrac{1}{1 + \dots\dots} = [4,1,3,3,6,1,1,2,1,1,1,9]. \qquad \text{..................................(2.24)}$$

$$1 + \cfrac{1}{1 + \cfrac{1}{9}}$$

$$C_7 = 5 + \cfrac{1}{1 + \cfrac{\ldots\ldots}{2 + \cfrac{1}{8 + \cfrac{1}{18}}}} = [5,1,24,3,2,2,8,18]. \quad \ldots\ldots(2.25)$$

$$C_8 = 7 + \cfrac{1}{2 + \cfrac{\ldots\ldots}{7 + \cfrac{1}{1 + \cfrac{1}{2}}}} = [7,2,4,1,1,3,1,3,10,3,7,1,2]. \quad \ldots\ldots(2.26)$$

$$C_9 = 9 + \cfrac{1}{3 + \cfrac{\ldots\ldots}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}} = [9,3,5,5,5,7,1,1,1,2,2,1,1,1,1,2]. \quad \ldots\ldots(2.27)$$

2.4. Quotients of 6 and 5 with powers of 2 to 10

Let D_1, D_2, D_3, ….., D_9 represent the quotients of the numbers 6 and 5 raised to the same powers starting from power 2 to power 10.

$$D_1 = 1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2}}}} = [1,2,3,1,2]. \quad \ldots\ldots(2.28)$$

$$D_2 = 1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{11}}}}} = [1,1,2,1,2,11]. \quad \ldots\ldots(2.29)$$

$$D_3 = 2 + \cfrac{1}{13 + \cfrac{\ldots\ldots}{2 + \cfrac{1}{1 + \cfrac{1}{2}}}} = [2,13,1,1,2,2,1,2]. \quad \ldots\ldots(2.30)$$

$$D_4 = 2 + \cfrac{1}{2 + \cfrac{\ldots\ldots}{9 + \cfrac{1}{2 + \cfrac{1}{3}}}} = [2,2,20,1,9,2,3]. \quad \ldots\ldots(2.31)$$

$$D_5 = 3 + \cfrac{1}{187 + \cfrac{1}{20 + \cfrac{1}{1 + \cfrac{1}{3}}}} = [3,187,20,1,3]. \quad \ldots\ldots(2.32)$$

$$D_6 = 3 + \cfrac{1}{1 + \cfrac{\ldots\ldots}{6 + \cfrac{1}{1 + \cfrac{1}{7}}}} = [3,1,1,2,1,1,44,1,16,1,7]. \quad \ldots(2.33)$$

$$D_7 = 4 + \cfrac{1}{3 + \cfrac{\ldots\ldots}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}} = [4,3,2,1,53,1,13,1,8,1,1,2]. \quad \ldots(2.34)$$

$$D_8 = 5 + \cfrac{1}{6 + \cfrac{\ldots\ldots}{18 + \cfrac{1}{2 + \cfrac{1}{3}}}} = [5,6,3,1,6,1,1,9,1,3,18,2,3]. \quad \ldots(2.35)$$

$$D_9 = 6 + \cfrac{1}{5 + \cfrac{\ldots\ldots}{3 + \cfrac{1}{2 + \cfrac{1}{3}}}} = [6,5,4,1,1,1,3,1,1,2,1,1,9,8,1,3,2,3]. \quad \ldots(2.36)$$

2.5. Quotients of 7 and 6 with powers of 2 to 10

Let E_1, E_2, E_3, ….., E_9 represent the quotients of the numbers 7 and 6 raised to the same powers starting from power 2 to power 10.

$$E_1 = 1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{3}}}} = [1,2,1,3,3]. \quad \ldots(2.37)$$

$$E_2 = 1 + \cfrac{1}{1 + \cfrac{\ldots\ldots}{2 + \cfrac{1}{1 + \cfrac{1}{12}}}} = [1,1,1,2,2,1,12]. \quad \ldots(2.38)$$

$$E_3 = 1 + \cfrac{1}{5 + \cfrac{\ldots\ldots}{1 + \cfrac{1}{1 + \cfrac{1}{13}}}} = [1,5,1,3,1,1,1,13]. \quad \ldots(2.39)$$

$$E_4 = 2 + \cfrac{1}{6 + \cfrac{\ldots\ldots}{5 + \cfrac{1}{1 + \cfrac{1}{36}}}} = [2,6,4,1,5,1,36]. \quad \ldots(2.40)$$

$$E_5 = 2 + \cfrac{1}{1 + \cfrac{......}{2 + \cfrac{1}{1 + \cfrac{1}{3}}}} = [2,1,1,11,184,2,1,3]. \qquad ......(2.41)$$

$$E_6 = 2 + \cfrac{1}{1 + \cfrac{......}{8 + \cfrac{1}{1 + \cfrac{1}{12}}}} = [2,1,16,4,1,2,1,5,1,8,1,12]. \qquad ......(2.42)$$

$$E_7 = 3 + \cfrac{1}{2 + \cfrac{......}{1 + \cfrac{1}{5 + \cfrac{1}{4}}}} = [3,2,3,5,3,6,1,2,26,1,5,4]. \qquad ......(2.43)$$

$$E_8 = 4 + \cfrac{1}{235 + \cfrac{......}{1 + \cfrac{1}{1 + \cfrac{1}{4}}}} = [4,235,2,1,284,1,4,1,1,4]. \qquad ......(2.44)$$

$$E_9 = 4 + \cfrac{1}{1 + \cfrac{......}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}} = [4,1,2,22,12,1,1,2,6,2,2,83,1,1,2]. \qquad ......(2.45)$$

2.6. Quotients of 8 and 7 with powers of 2 to 10

Let F_1, F_2, F_3, ….., F_9 represent the quotients of the numbers 8 and 7 raised to the same powers starting from power 2 to power 10.

$$F_1 = 1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3}}}} = [1,3,3,1,3]. \qquad ......(2.46)$$

$$F_2 = 1 + \cfrac{1}{2 + \cfrac{1}{33 + \cfrac{1}{1 + \cfrac{1}{4}}}} = [1,2,33,1,4]. \qquad ......(2.47)$$

$$F_3 = 1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{6 + \cfrac{1}{26}}}}} = [1,1,2,3,6,26]. \qquad ......(2.48)$$

$$F_4 = 1 + \cfrac{1}{1 + \cfrac{......}{2 + \cfrac{1}{18 + \cfrac{1}{3}}}} = [1,1,18,1,6,2,18,3]. \qquad ....(2.49)$$

$$F_5 = 2 + \cfrac{1}{4 + \cfrac{......}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}} = [2,4,2,1,1,1,1,2,65,2,2,2]. \qquad ....(2.50)$$

$$F_6 = 2 + \cfrac{1}{1 + \cfrac{......}{1 + \cfrac{1}{1 + \cfrac{1}{6}}}} = [2,1,1,4,1,7,11,4,1,7,1,1,1,6]. \qquad ....(2.51)$$

$$F_7 = 2 + \cfrac{1}{1 + \cfrac{......}{1 + \cfrac{1}{3 + \cfrac{1}{2}}}} = [2,1,10,6,1,4,1,4,2,6,1,16,1,3,2]. \qquad ....(2.52)$$

$$F_8 = 3 + \cfrac{1}{3 + \cfrac{......}{20 + \cfrac{1}{31 + \cfrac{1}{2}}}} = [3,3,14,1,9,4,1,1,7,20,31,2]. \qquad ....(2.53)$$

$$F_9 = 3 + \cfrac{1}{1 + \cfrac{......}{2 + \cfrac{1}{4 + \cfrac{1}{2}}}} = [3,1,4,33,2,3,1,2,6,3,2,2,4,6,2,4,2]. \qquad ....(2.54)$$

2.7. Quotients of 9 and 8 with powers of 2 to 10

Let G_1, G_2, G_3, ….., G_9 represent the quotients of the numbers 9 and 8 raised to the same powers starting from power 2 to power 10.

$$G_1 = 1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4}}}} = [1,3,1,3,4]. \qquad ....(2.55)$$

$$G_2 = 1 + \cfrac{1}{2 + \cfrac{......}{1 + \cfrac{1}{2 + \cfrac{1}{3}}}} = [1,2,2,1,3,1,1,2,3]. \qquad ....(2.56)$$

$$G_3 = 1+\cfrac{1}{2+\cdots\cfrac{}{8+\cfrac{1}{1+\cfrac{1}{29}}}} = [1,2,1,3,1,8,1,29]. \quad \ldots\ldots(2.57)$$

$$G_4 = 1+\cfrac{1}{1+\cdots\cfrac{}{12+\cfrac{1}{3+\cfrac{1}{4}}}} = [1,1,4,19,2,12,3,4]. \quad \ldots\ldots(2.58)$$

$$G_5 = 2+\cfrac{1}{36+\cdots\cfrac{}{2+\cfrac{1}{1+\cfrac{1}{2}}}} = [2,36,1,1,1,5,3,11,1,2,1,2]. \quad \ldots\ldots(2.59)$$

$$G_6 = 2+\cfrac{1}{3+\cdots\cfrac{}{1+\cfrac{1}{21+\cfrac{1}{3}}}} = [2,3,1,1,3,2,69,2,1,1,1,21,3]. \quad \ldots\ldots(2.60)$$

$$G_7 = 2+\cfrac{1}{1+\cdots\cfrac{}{1+\cfrac{1}{1+\cfrac{1}{3}}}} = [2,1,1,3,3,3,38,3,6,1,2,3,3,1,1,3]. \quad \ldots\ldots(2.61)$$

$$G_8 = 2+\cfrac{1}{1+\cdots\cfrac{}{9+\cfrac{1}{3+\cfrac{1}{4}}}} = [2,1,7,1,4,3,2,1,1,1,1,7,1,3,1,1,5,1,9,3,4]. \quad \ldots\ldots(2.62)$$

$$G_9 = 3+\cfrac{1}{4+\cdots\cfrac{}{2+\cfrac{1}{8+\cfrac{1}{6}}}} = [3,4,23,12,1,1,8,2,20,1,10,2,8,6]. \quad \ldots\ldots(2.63)$$

2.8. Quotients of 10 and 9 with powers of 2 to 10

Let H_1, H_2, H_3, ….., H_9 represent the quotients of the numbers 10 and 9 raised to the same powers starting from power 2 to power 10.

$$H_1 = 1+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{4}}}} = [1,4,3,1,4]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(2.64)}$$

$$H_2 = 1+\cfrac{1}{2+\ldots\ldots} = [1,2,1,2,4,2,2,1,2] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(2.65)}$$

$$2+\cfrac{1}{1+\cfrac{1}{2}}$$

$$H_3 = 1+\cfrac{1}{1+\ldots\ldots} = [1,1,1,9,1,5,1,1,1,1,9] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(2.66)}$$

$$1+\cfrac{1}{1+\cfrac{1}{9}}$$

$$H_4 = 1+\cfrac{1}{1+\ldots\ldots} = [1,1,2,1,4,9,1,4,3,1,1,2] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(2.67)}$$

$$1+\cfrac{1}{1+\cfrac{1}{2}}$$

$$H_5 = 1+\cfrac{1}{1+\ldots\ldots} = [1,1,7,2,4,1,1,1,4,3,1,3,13,2]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(2.68)}$$

$$3+\cfrac{1}{13+\cfrac{1}{2}}$$

$$H_6 = 2+\cfrac{1}{11+\ldots\ldots} = [2,11,52,2,1,1,1,3,2,8,1,5,2]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(2.69)}$$

$$1+\cfrac{1}{5+\cfrac{1}{2}}$$

$$H_7 = 2+\cfrac{1}{3+\ldots\ldots} = [2,3,10,2,11,1,1,2,4,1,7,2,2,2,1,2,5]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(2.70)}$$

$$1+\cfrac{1}{2+\cfrac{1}{5}}$$

$$H_8 = 2+\cfrac{1}{1+\ldots\ldots} = [2,1,1,2,1,1,2,1,1,1,2,1,2,1,1,2,1,12,1,4,7,2,18,2]. \qquad \ldots\ldots\ldots\ldots\ldots\text{(2.71)}$$

$$2+\cfrac{1}{18+\cfrac{1}{2}}$$

$$H_9 = 2 + \cfrac{1}{1 + \ldots\ldots \cfrac{}{12 + \cfrac{1}{3 + \cfrac{1}{16}}}} = [2,1,6,1,1,2,1,6,1,3,1,1,1,4,1,3,1,2,1,3,12,3,16]. \quad \ldots\ldots\ldots\ldots\ldots(2.72)$$

3. CONCLUSION

The continued fraction expansion algorithm has many specific features. However, the continued fraction expansion of any number or quotients of numbers is the efficient process of finding its best rational approximations. In fact, many continued fractions can be obtained in a similar way for different quotients with different powers.

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