

Oscillation Criteria for First Order Dynamic Equations with Maxima on Time Scales

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Abstract: In this work, we establish some new oscillation results for the first order dynamic equations with maxima

$$x^\Delta(t) + q(t)\max_{s \in [t-\delta, t]} x(s) = 0,$$

$$x^\Delta(t) + p_1(t)\max_{s \in [\tau_1(t), t]} x(s) - p_2(t)\max_{s \in [\tau_2(t), t]} x(s) = 0,$$

and

$$x^\Delta(t) + \sum_{i=1}^n p_i(t)\max_{s \in [\delta_i(t), \sigma(t)]} x(s) + \sum_{j=1}^m q_j(t)x(\eta_j(t)) = 0,$$

Our results not only complement and generalize some existing results, but also can be applied to some oscillation problems that were not covered before, we also give some examples to illustrate our main results

Keywords: Oscillation, dynamic equations, time scales, maxima.

1. INTRODUCTION

The theory of time scales was introduced by Hilger [1] in order to unify, extend and generalize ideas from discrete calculus, quantum calculus, and continuous calculus to arbitrary time scale calculus. A time scale T is a nonempty closed subset of the real numbers \mathbb{R} . When the time scale equals to the real numbers or integer numbers, the obtained results represent the classical theories of the differential and difference equations. Many other interesting time scales exist and give arise to many applications. The new theory of the so - called " dynamic equation" not only unify the theories of differential equations and difference equations, but also extends these classical cases to the so - called q - difference equations (when $T = q^{\mathbb{N}_0} := \{q^t : t \in \mathbb{N}_0 \text{ for } q > 1\}$ or $T = q^{\mathbb{Z}} = q^{\mathbb{Z}} \cup \{0\}$) which have important applications in quantum theory see [2]. Also, it can be applied on different types of time scales like $T = h\mathbb{Z}$ and $T = \mathbb{N}_0^2$. For an introduction to time scale calculus and dynamic equations, see Bohner and Peterson books [3, 4].

We note that equations with maxima occur in the problem of automatic regulation of various real system is considered in [5]. The maxima arise when the regulation law corresponds to the maximal deviation of the regulable quantity. The paper in which oscillatory properties of equations with maxima are considered in [6].

In this paper, we present time scale analogies of some result for continuous case $T = \mathbb{R}$ and the discrete case $T = \mathbb{Z}$. A nontrivial solution of the following equations is said to be oscillatory if it is neither eventually positive nor eventually negative. Otherwise, the solution is called nonoscillatory. A solution is called z -type if it is either nonpositive or nonnegative.

Next, let us recall some known oscillation results on this subject. The first order delay differential equation

$$x'(t) + q(t)\max_{s \in [t-\delta, t]} x(s) = 0, \quad (1)$$

equation (1) is nonoscillatory if $q(t)$ is of one sign as in [7] and if $q(t)$ is oscillatory, then equation (1) has at least one nonoscillatory solution. For multidelays H. A. Agwo considered in [8] the following equation

$$y^\Delta(t) + \sum_{i=1}^n p_i(t)y(\tau_i(t)) = 0, \tag{2}$$

Where

- (i) $p_i(t), i=1,2,\dots,n$, are nonnegative real valued rd-continuous function defined on T ,
- (ii) $T_i(\cdot):T \rightarrow T$ and $\tau_i(t) < t$, for all $t \in T, i=1,2,\dots,n$.

Then equation (2) is oscillatory if

$$\lim_{t_0 \rightarrow \infty} \sup_{t > t_0} \sup_{\lambda \in E} \left\{ \frac{\lambda \sum_{i=1}^n p_i(t)}{\sum_{i=1}^n p_i(t) \exp\left[-\int_{\tau_i(t)}^t \xi_{\mu(s)}(-\lambda \sum_{i=1}^n p_i(s)) \Delta s\right]} \right\} < 1,$$

Where $E = \{\lambda > 0 : 1 - \lambda \sum_{i=1}^n p_i(t)\mu(t) > 0\}$. This result will be used in the proof of our main result.

2. MAIN RESULTS

2.1 Theorem

If $q(t)$ is of one sign of equation

$$x^\Delta(t) + q(t) \max_{s \in [t-\delta, t]} x(s) = 0, \tag{3}$$

then, every solution of (3) is nonoscillatory.

Proof:-

When $q(t)=0$, then $x^\Delta(t) = 0$ and consequently, $x(t) = x(t_0) = \alpha, \alpha \in R$. Also when $\delta = 0$, then equation (3) becomes $x^\Delta(t) + q(t)x(s) = 0$. This equation is not oscillatory. If $q(t) > 0$, suppose that $x(t)=0$ is an oscillatory solution of (3), then $x(t)$ is not z-type solution and otherwise, $x(t)=0$ eventually. Therefore, there exist t_1, t_2, t_3 such that $x(t_1) = x(t_2) = x(t_3) = 0$ and $x(t) < 0$.

For $t \in (t_1, t_2)_T, x(t) > 0$ for $t \in (t_2, t_3)_T$. Thus $x^\Delta(t_2) = -q(t_2) \max_{[t_2-\delta, t_2]} x(s) \leq 0$, which is a contradiction.

For $q(t) < 0$, the proof is similar, hence it's omitted. This completes the proof.

2.2 Theorem

Assume that we have

$$x^\Delta(t) + p_1(t) \max_{s \in [\tau_1(t), t]} x(s) - p_2(t) \max_{s \in [\tau_2(t), t]} x(s) = 0 \tag{4}$$

- (i) $\tau_2(t) > \tau_1(t), p_1, p_2 > 0$,

- (ii) there exists $t_1 \geq t_0$ such that

$$p_1(t) \geq p_2(\tau_2^{-1}(\tau_1(t))) \quad \text{for all } t \geq t_1 \geq t_0$$

- (iii) $\int_{t_0}^\infty p_2(s) \Delta s < \infty, \liminf_{t \rightarrow \infty} \int_{\tau_1(t)}^t p_1(s) \Delta s > c, c > 0$.

Then every solution of equation (4) is either oscillatory or tends to zero.

Proof:

Assume that equation (4) has a nonoscillatory solution $x(t)$ on $[t_0, \infty)_T$. Suppose that there exists $t \geq \tau_1^{-1}(t_1)$.

Now, we consider the following two cases:

I. $x(t)$ is bounded on $[t_1, \infty)_T$

Let

$$z = x - \int_{\tau_2^{-1}(\tau_1(t))}^t p_2(s) \max_{\xi \in [\tau_2(s), s]} x(\xi) \Delta s. \tag{5}$$

Then,

$$z^\Delta = x^\Delta - p_2(t) \max_{\xi \in [\tau_2(t), t]} x(\xi) + p_2(\tau_2^{-1}(\tau_1(t))) \max_{\xi \in [\tau_1(t), \tau_2^{-1}(\tau_1(t))]} x(\xi). \quad (6)$$

From (4) and (6) we have

$$z^\Delta(t) + p_1(t) \max_{\xi \in [\tau_1(t), t]} x(\xi) - p_2(\tau_2^{-1}(\tau_1(t))) \max_{\xi \in [\tau_1(t), \tau_2^{-1}(\tau_1(t))]} x(\xi) = 0, \quad (7)$$

since $\tau_1(t) < \tau_2(t)$, $x(t) > 0$, then we have

$$\max_{\xi \in [\tau_1(t), t]} x(\xi) > \max_{\xi \in [\tau_1(t), \tau_2^{-1}(\tau_1(t))]} x(\xi).$$

Due to (7) and condition (ii), we get $z^\Delta(t) \leq 0$ for all $t > \tau_1^{-1}(t_1)$ and consequently $z(t)$ is nonincreasing on $[t_1, \infty)_T$. Since $\int_{t_0}^{\infty} p_2(s) \Delta s < \infty$, and $x(t)$ bounded then

$$\int_{\tau_1^{-1}(t)}^{\infty} p_2(t) \max_{\xi \in [\tau_2(s), s]} x(\xi) \Delta s < \infty,$$

then,

$$\lim_{t \rightarrow \infty} \int_{\tau_2^{-1}(\tau_1(t))}^{\infty} p_2(t) \max_{\xi \in [\tau_2(t), t]} x(\xi) \Delta s = 0. \quad (8)$$

Hence $z(t)$ is bounded on $[\tau_1^{-1}(t), \infty)_T$, so $\lim_{t \rightarrow \infty} z(t)$ exists.

Denote

$$l = \lim_{t \rightarrow \infty} z(t), \quad (9)$$

from (5), (8) and (9) we have

$$\lim_{t \rightarrow \infty} z(t) = \lim_{t \rightarrow \infty} x(t) = l, \quad (10)$$

where $l > 0$ or $l = 0$

If $l > 0$, integrating (4) from $\tau_1(t)$ to t , we get

$$x(t) - x(\tau_1(t)) + \int_{\tau_1(t)}^t p_1(s) \max_{\xi \in [\tau_1(s), s]} x(\xi) \Delta s - \int_{\tau_1(t)}^t p_2(s) \max_{\xi \in [\tau_2(s), s]} x(\xi) \Delta s = 0, \quad t \geq \tau_1^{-2}(t_1). \quad (11)$$

From (10), there is a $t_2 \geq \tau_1^{-2}(t_1)$ such that

$$\frac{l}{2} < x(t) < 2l \quad \text{for all } t \geq t_2. \quad (12)$$

Hence, from (11), (12), we have

$$x(t) - x(\tau_1(t)) + \frac{l}{2} \int_{\tau_1(t)}^t p_1(s) \Delta s - 2l \int_{\tau_1(t)}^t p_2(s) \Delta s < 0 \text{ for all } t \geq t_2. \quad (13)$$

Taking the lower limit in (13) as $t \rightarrow \infty$, we get

$$\int_{t-\tau_1(t)}^t p_1(s) \Delta s < 0.$$

Which is a contradiction.

II. If $x(t)$ is unbounded on $[t_1, \infty)$ from (4), we have

$$x^\Delta(t) \leq p_2(t) \max_{s \in [\tau_2(t), t]} x(s), \quad t \geq \tau_1(t_1). \quad (14)$$

Integrating (14) from $\tau_1(t)$ to t , we get

$$x(t) \leq x(\tau_1(t)) + \int_{\tau_1(t)}^t p_2(s) \max_{\xi \in [\tau_2(s), s]} x(\xi) \Delta s.$$

Since $x(t)$ is positive and unbounded, we must have,

$$x^\Delta(t) > 0 \text{ for } t \geq \tau_1^{-1}(t_1),$$

and so,

$$x(t) \leq x(\tau_1(t)) + \int_{\tau_1(t)}^t p_2(s) \Delta s.$$

By the Gronwall inequality

$$x(t) \leq x(\tau_1(t)) e_{p_2}(t, \tau_1(t)) < \infty,$$

which is a contradiction.

Assume that equation (4) has eventually negative solution $x(t)$ on $[t_0, \infty)_T$ then, there exists $t_3 \geq t_0$ such that $x(t) < 0$ on $[\tau_1^{-1}(t_4), \infty)_T$.

Let $y = -x$, then equation (2.2) takes the form

$$y^\Delta(t) + p_1(t) \min_{s \in [\tau_1(t), t]} y(s) - p_2(t) \min_{s \in [\tau_2(t), t]} y(s) = 0. \tag{15}$$

Assume that

$$w(t) = y(t) + \int_t^\infty p_2(s) \min_{\xi \in [\tau_2(s), s]} y(\xi) \Delta s, \tag{16}$$

then

$$w^\Delta(t) = y^\Delta(t) - p_2(t) \min_{s \in [\tau_1(t), t]} y(s), \tag{17}$$

from (15), (17) we get

$$w^\Delta(t) + p_1(t) \min_{s \in [\tau_1(t), t]} y(s) = 0. \tag{18}$$

Since $p_1(t), y(t) > 0$, then $w^\Delta(t) \leq 0$.

Now, we consider the following two cases:

I. y is bounded on $[\tau_1^{-1}(t_3), \infty)_T$.

Since $\int p_2(s) \Delta s < \infty$, y is bounded, then w is bounded, then w is bounded $[\tau_1^{-1}(t_3), \infty)_T$.

Since w is bounded and $w^\Delta \leq 0$, then w is bounded. Hence.

$$\lim_{t \rightarrow \infty} w(t) = l. \tag{19}$$

From (16), we have $\lim_{t \rightarrow \infty} w(t)$ exists. $= \lim_{t \rightarrow \infty} y(t), y(t) > 0$. Hence $l \geq 0$.

If $l > 0$, then there exists $t_4 \geq \tau_1^{-1}(t_3)$ such that

$$\frac{l}{2} < y(t) < 2l \text{ for all } t \geq t_4. \tag{20}$$

Integrating (18) from $\tau_1(t)$ to t we get

$$w(t) - w(\tau_1(t)) + \frac{l}{2} \int_{\tau_1(t)}^t p_1(s) \Delta s < 0. \tag{21}$$

Taking the lower limit as $t \rightarrow \infty$, we get

$$\int_{\tau_1(t)}^t p_1(s) \Delta s < 0,$$

which is a contradiction.

II. $y(t)$ is unbounded on $[\tau_1^{-1}(t_3), \infty)_T$.

From (15) we have

$$y^\Delta(t) \leq p_2(t) \min_{s \in [\tau_2(t), t]} y(s). \tag{22}$$

Integrating (22) from $\tau_1^{-1}(t_4)$ to t , we get,

$$y(t) - y(\tau_1^{-1}(t_4)) \leq \int_{\tau_1^{-1}(t_4)}^t p_2(s) \min_{\xi \in [\tau_2(s), s]} y(\xi) \Delta s.$$

Let $\theta = \tau_2(s)$, then the above inequality can be written in the form

$$y(t) - y(\tau_1^{-1}(t_4)) \leq \int_{\tau_2(\tau_1^{-1}(t_4))}^{\tau_2(t)} p_2(\tau_2^{-1}(\theta)) \min_{\xi \in [\theta, \tau_2^{-1}(\theta)]} y(\xi) \Delta\theta. \tag{23}$$

Since $y > 0$, y is unbounded, then $y^\Delta > 0$, and consequently y is non-decreasing. So from (23), we have

$$\begin{aligned} y(t) &\leq y(\tau_1^{-1}(t_4)) + \int_{\tau_2(\tau_1^{-1}(t_4))}^{\tau_2(t)} p_2(\tau_2^{-1}(\theta)) y(\theta) \Delta\theta \\ &\leq y(\tau_1^{-1}(t_4)) + \int_{\tau_2(\tau_1^{-1}(t_4))}^t p_2(\tau_2^{-1}(\theta)) y(\theta) \Delta\theta, \end{aligned}$$

which implies that

$$y(t) \leq y(\tau_1^{-1}(t_4)) e_{p_2}(t, \tau_2(\tau_1^{-1}(t_4))) < \infty,$$

which is a contradiction. the proof is complete.

2.2.1 Remark

If we choose $\tau_1(t) = t - \delta_1$ and $\tau_2(t) = t - \delta_2$ in the above theorem, then equation (4) takes the form

$$x^\Delta(t) + p_1(t) \max_{s \in [t-\delta_1, t]} x(s) - p_2(t) \max_{s \in [t-\delta_2, t]} x(s) = 0 \tag{24}$$

where

(i) $\delta_1 > \delta_2, p_1, p_2 > 0$,

(ii) there exists $t_1 \geq t_0$ such that

$$p_1(t) \geq p_2(t - (\delta_1 - \delta_2)) \text{ for all } t \geq t_1 \geq t_0. \tag{25}$$

(iii) $\int_{t_0}^\infty p_2(s) \Delta s < \infty$ and $\liminf_{t \rightarrow \infty} \int_{t-\delta_1}^t p_1(s) \Delta s > c, c > 0$.

Then every solution of equation (24) is either oscillatory or tends to zero.

2.3 Theorem

Assume that

$$x^\Delta(t) + \sum_{i=1}^n p_i(t) \max_{s \in [\delta_i(t), \sigma(t)]} x(s) + \sum_{j=1}^m q_j(t) x(\eta_j(t)) = 0, \tag{26}$$

where

$\delta_i, \eta_j < t, i = 1, \dots, n, j = 1, \dots, m$, such that

$$\begin{aligned} \text{I. } &\lim_{t_0 \rightarrow \infty} \sup_{t > t_0} \sup_{\lambda \in E_1} \left\{ \frac{\lambda \sum_{k=1}^{n+m} h_k(t)}{\sum_{k=1}^{n+m} h_k(t) \exp\left[-\int_{\tau_k(t)}^t \xi_\mu(s) (-\lambda \sum_{k=1}^{n+m} h_k(s)) \Delta s\right]} \right\} < 1, \\ \text{II. } &\lim_{t_0 \rightarrow \infty} \sup_{t > t_0} \sup_{\lambda \in E_2} \left\{ \frac{\lambda \sum_{j=1}^m \frac{q_j(t)}{e_p(\eta_j(t), t)}}{\sum_{j=1}^m \frac{q_j(t)}{e_p(\eta_j(t), t)} \exp\left[-\int_{\eta_j(t)}^t \xi_\mu(s) (-\lambda \sum_{j=1}^m \frac{q_j(t)}{e_p(\eta_j(t), t)}) \Delta s\right]} \right\} < 1, \end{aligned}$$

where

$$\begin{aligned} E_1 &= \{ \lambda > 0 : 1 - \lambda \sum_{k=1}^{n+m} h_k(t) \mu(t) > 0 \}, E_2 = \{ \lambda > 0 : 1 - \lambda \sum_{j=1}^m \frac{q_j(t)}{e_p(\eta_j(t), t)} \mu(t) > 0 \}, \\ h_k(t) &= \begin{cases} p_k(t) & k = 1, 2, \dots, n \\ q_{k-n}(t) & k = 1, 2, \dots, m + n \end{cases} \end{aligned}$$

$$\tau_k(t) = \begin{cases} \delta_k(t) & k = 1, 2, \dots, n \\ \eta_{k-n}(t) & k = 1, 2, \dots, m+n, \end{cases}$$

and $P = \sum_{i=1}^n p_i(t)$

then equation (26) is oscillatory.

Proof:-

Suppose that $x(t)$ is an eventually positive solution of equation (26) then there exist $T \in [t_0, \infty)_T$ sufficiently large such that $x(t) > 0, x(\delta_i(t)) > 0$ for all $i = 1, 2, \dots, n, x(\eta_j(t)) > 0$, for all $j = 1, 2, \dots, m$, for all $t \geq T$. Then $x^\Delta(t) < 0$ i.e., x is non-increasing and consequently from (26) we have

$$x^\Delta(t) + \sum_{i=1}^n p_i(t)x(\delta_i(t)) + \sum_{j=1}^m q_j(t)x(\eta_j(t)) = 0,$$

the above equation can be written in the form

$$x^\Delta(t) + \sum_{k=1}^{n+m} h_k(t)x(\tau_k(t)) = 0, \tag{27}$$

where

$$h_k(t) = \begin{cases} p_k(t) & k = 1, 2, \dots, n \\ q_{k-n}(t) & k = 1, 2, \dots, m+n, \end{cases}$$

and

$$\tau_k(t) = \begin{cases} \delta_k(t) & k = 1, 2, \dots, n \\ \eta_{k-n}(t) & k = 1, 2, \dots, m+n. \end{cases}$$

But according to [1] all solutions of equation (27) are oscillatory if

$$\lim_{t_0 \rightarrow \infty} \sup_{t > t_0} \sup_{\lambda \in E_1} \left\{ \frac{\lambda \sum_{k=1}^{n+m} h_k(t)}{\sum_{k=1}^{n+m} h_k(t) \exp\left(-\int_{\tau_k(t)}^t \xi_\mu(s) (-\lambda \sum_{k=1}^{n+m} h_k(s)) \Delta s\right)} \right\} < 1,$$

which is a contradiction.

Suppose that $x(t)$ is an eventually negative solution of equation (26) then there exists $T \in [t_0, \infty)_T$ sufficiently large such that $x(t) < 0, x(\delta_i(t)) < 0$ for all $i = 1, 2, \dots, n, x(\eta_j(t)) < 0$ for all $j = 1, 2, \dots, m$. Take $y = -x$, then equation (26) takes the form

$$y^\Delta(t) + \sum_{i=1}^n p_i(t)y(\sigma(t)) + \sum_{j=1}^m q_j(t)y(\eta_j(t)) = 0. \tag{28}$$

Let $P = \sum_{i=1}^n p_i(t)$. Multiplying equation (28) by $e_P(t, t_0)$ then, we have

$$y^\Delta(t)e_P(t, t_0) + Py(\sigma(t))e_P(t, t_0) + \sum_{j=1}^m q_j(t)y(\eta_j(t))e_P(t, t_0) = 0,$$

i.e.,

$$(y^\Delta(t)e_P(t, t_0))^\Delta + \left(\sum_{j=1}^m q_j(t)y(\eta_j(t)) \right) e_P(t, t_0) = 0,$$

let $z(t) = y(t)e_P(t, t_0)$

$$z^\Delta(t) + e_P(t, t_0) \sum_{j=1}^m \frac{q_j(t)z(\eta_j(t))}{e_P(\eta_j(t), t_0)} = 0,$$

i.e.,

$$z^\Delta(t) + \sum_{j=1}^m \frac{q_j(t)z(\eta_j(t))}{e_P(\eta_j(t), t)} = 0,$$

but all solutions are oscillatory if

$$\lim_{t_0 \rightarrow \infty} \sup_{t > t_0} \sup_{\lambda \in E_2} \left\{ \frac{\lambda \sum_{j=1}^m \frac{q_j(t)}{e_p(\eta_j(t), t)}}{\sum_{j=1}^m \frac{q_j(t)}{e_p(\eta_j(t), t)} \exp\left(-\int_{\eta_j(t)}^t \xi_\mu(s) \left(-\lambda \sum_{j=1}^m \frac{q_j(t)}{e_p(\eta_j(t), t)}\right) \Delta s\right)} \right\} < 1,$$

which is a contradiction, the proof is complete.

2.3.1 Remark

we note that if T=R equation (26) becomes

$$x'(t) + \sum_{i=1}^n p_i(t) \max_{s \in [\delta_i(t), t]} x(s) + \sum_{j=1}^m q_j(t) x(\eta_j(t)) = 0. \tag{29}$$

Then, if

$$\liminf_{t \rightarrow \infty} \frac{\sum_{i=1}^n p_i(t) \int_{\delta_i(t)}^t (\sum_{i=1}^n p_i(s) ds) + \sum_{j=1}^m q_j(t) \int_{\eta_j(t)}^t (\sum_{j=1}^m q_j(s) ds)}{\sum_{i=1}^n p_i(t) + \sum_{j=1}^m q_j(t)} > \frac{1}{e},$$

and,

$$\liminf_{t \rightarrow \infty} \frac{\sum_{j=1}^m q_j(t) e^{\int_{\eta_j(t)}^t (\sum_{i=1}^n p_i(s) ds)} \int_{\eta_j(t)}^t (\sum_{j=1}^m q_j(s) e^{\int_{\eta_j(s)}^s (\sum_{i=1}^n p_i(\xi) d\xi)} ds)}{\sum_{j=1}^m q_j(t) e^{\int_{\eta_j(t)}^t (\sum_{i=1}^n p_i(s) ds)}} > \frac{1}{e}, \tag{30}$$

then all solutions of equation (29) are oscillatory. Condition (30) is a new criteria for oscillation of equation (27).

3. EXAMPLES

In this section we give some examples to illustrate our main results.

3.1 Example

Take T=R, consider the equation

$$x'(t) - \frac{1}{t(t-1)} \max_{s \in [t-2, t]} x(s) = 0. \tag{31}$$

Here, $p(t) = \frac{1}{t(t-1)}$, $t > 3$ is of one sign then by Theorem (2.1) equation (31) is nonoscillatory. In fact $x(t) = \frac{t-1}{t}$ is a nonoscillatory solution for this equation.

3.2 Example

Take T=R, consider the equation

$$x'(t) + (e^{-t} + 1) e^{-\frac{3}{2}} \max_{s \in [t-\frac{3}{2}, t]} x(s) - e^{-(t+\frac{1}{2})} \max_{s \in [t-\frac{1}{2}, t]} x(s) = 0. \tag{32}$$

Here, $p_1(t) = (e^{-t} + 1) e^{-\frac{3}{2}}$, $p_2(t) = e^{-(t+\frac{1}{2})}$, $\delta_1 = \frac{3}{2}$ and $\delta_2 = \frac{1}{2}$. It is clear that $\delta_1 > \delta_2$, $p_1(t) > p_2(t-1)$, $t \geq 1$. Then according to the Remark (2.2.1) then every solution of this equation is oscillatory or tends to zero. In fact $x = e^{-t}$ is a solution which is tends to zero.

3.3 Example

Take T=R, consider the equation

$$x'(t) + \frac{1}{3} \max_{s \in [t-3, t]} x(s) + \frac{2}{3} \max_{s \in [t-2, t]} x(s) + \frac{2}{e} x(t-1) = 0. \tag{33}$$

Here, $p_1 = \frac{1}{3}$, $p_2 = \frac{2}{3}$, $\delta_1 = 3$ and $\delta_2 = 2$,

$$\liminf_{t \rightarrow \infty} \frac{\frac{1}{3} \int_{t-3}^t \left(\frac{1}{3} + \frac{2}{3}\right) ds + \frac{2}{3} \int_{t-2}^t \left(\frac{1}{3} + \frac{2}{3}\right) ds + \frac{2}{e} \int_{t-1}^t \frac{2}{e} ds}{\frac{1}{3} + \frac{2}{3} + \frac{2}{e}} > \frac{1}{e},$$

and

$$\liminf_{t \rightarrow \infty} \frac{\int_{t-1}^t \frac{2}{e} e^{\int_{t-1}^s (\frac{1}{3} + \frac{2}{3}) ds} \int_{t-1}^s \frac{2}{e} e^{\int_{t-1}^{\xi} (\frac{1}{3} + \frac{2}{3}) d\xi} ds}{\frac{2}{e} e^{\int_{t-1}^t (\frac{1}{3} + \frac{2}{3}) ds}} > \frac{1}{e},$$

then equation (33) is oscillatory.

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