International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 4, Issue 7, July 2016, PP 24-33
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
DOI: http://dx.doi.org/10.20431/2347-3142.0407004
www.arcjournals.org

Complementary Colour Transversal Vertex Covering Set

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Abstract: *In this paper we introduce new concepts namely Complementary Colour Transversal Vertex Covering Set (CCTVC Set) and Complementary Colour Transversal Vertex Covering Number (CCTVC Number) of a graph. If G is a graph then this number is denoted as α_{*c} (G). We have also observed that α_{*c} (G)= α_0(G) or α_{*c} (G) = α_0(G) + 1 for any graph G, Where α_0(G) is the vertex covering number of a graph G.We proved several theorems regarding the effect of removing a vertex from a graph on this number.*

Keywords: *Transversal, Colour Transversal, Vertex Covering Set,Vertex Covering Number,Complementary Colour Transversal Vertex Covering Set, Complementary Colouring, Complementary Chromatic Number, Complementary Colour Transversal Vertex Covering Number.*

AMS Subject Classification (2010): 05C15, 05C69.

1. Introduction

The concept of a vertex covering set is well known and has been studied by several authors. The identity $\alpha_0(G) + \beta_0(G) = | V(G) |$ ($\alpha_0(G)$ = The vertex covering number & $\beta_0(G)$ = The independence number) is well known. The concept of colour transversal dominating set was studied in detail in Ph.D. Thesis of Manoharan [9].We introduce the concepts of colour transversal vertex covering set and colour transversal vertex covering number of a graph in [3].

In this paper we consider the concepts of complementary colouring and complementary chromatic number of a graph.These concepts were introduced in [2]. Now we introduce the concepts of Complementary Colour Transversal Vertex Covering Set (CCTVC Set) and Complementary Colour Transversal Vertex Covering Number (CCTVC Number) of a graph. The operation of removing a vertex from a graph may increase, decrease or keep the number unchanged. We consider the effect of this operation on complementary colour transversal vertex covering number (CCTVC Number) of a graph.

We assume that our graphs are finite, simple and undirected. If G is a graph then V(G) will denote the vertex set of G and E(G) will denote the edge set of G.

2. Results and Discussion

Definition 2.1 (Complementary Colouring) [2]

Let G be a graph. The Colouring f of vertices of G is said to be a complementary colouring if whenever vertices u and v have different colours then they must be adjacent.

Definition 2.2 (Complementary Chromatic Number) [2]

Let G be a graph. The maximum numbers of colours which can be assigned to the vertices so that the resulting colouring is a complementary colouring is called the complementary chromatic number of G & it is denoted as χ_C (G). This complementary colouring is called complementary chromatic colouring.

Remark 2.3

- The complementary colouring of a graph need not be a proper colouring.
- If a graph G has having complementary colouring then it may happen that two vertices are adjacent and they have the same colour.

- If a graph has been given a complementary colouring then two non-adjacent vertices cannot have different colours. Thus, in any independent set all the vertices must have the same colours.
- It may be noted that in general a colour class corresponding to a complementary colouring need not be an independent set.

Example 2.4

Consider the graph with vertices v_1, v_2, v_3, v_4

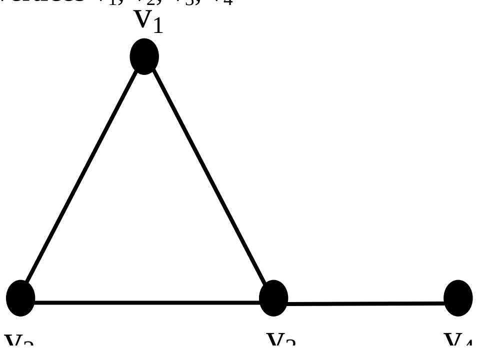

Fig.1

Consider complementary colouring in which v_1, v_2, v_3, v_4 receives colours as follows.

v_1 – colour 1, v_2 – colour 1, v_3 – colour 2, v_4 – colour 1

Here the colour classes corresponding to colour 1 is not an independent set.

Proposition 2.5 [2]

Let G be a graph. Then

- $\chi C(G) \leq \chi(G)$
- $\chi C(G) = \chi(G)$ iff G is a complete k – partite graph.

Proposition 2.6

Let G be a graph and suppose the colour classes of a complementary chromatic colouring of G are $C_1, C_2, \ldots, C_k$. Let T be a transversal of these colour classes then T is a dominating set.

PROOF

Let us assume that T intersect each C_i in a singleton set and therefore let $T \cap C_i = \{v_i\}$ for $i = 1,2, \ldots, k$. Let z be a vertex such that z does not belongs to T. Suppose $z \in C_i$ for some i. Then z is adjacent to v_j for every $j \neq i$.

Thus, T is a dominating set.

Corollary 2.7

Let G be a graph. Then $\gamma(G) \leq \chi_c(G)$

PROOF

From the above proposition $\gamma(G) \leq |T| = \chi_c(G)$

Proposition 2.8

Let G be a graph and $C_1, C_2, \ldots, C_k$ be the colour classes corresponding to some complementary chromatic colouring of G. Then for every colour class C_i with $|C_i| \geq 2$ & for every $v \in C_i$ $\exists$ some $u \in C_i \ni u$ is not adjacent to v.

PROOF

Suppose the statement does not hold.

Then for some colour class say C_1 with $|C_1| \geq 2$ there is a vertex v in C_1 such that v is adjacent to every vertex of C_1. Also v is adjacent to every vertex of every other colour class. Thus v is adjacent to every other vertex of G. Now, suppose we have used colours 1, 2, 3, …., k in complementary chromatic colouring of G. We may assign a new colour k + 1 to v and keep the colours of other vertices unchanged. Then we get a complementary colouring of G with k + 1 colours. This is a contradiction because complementary chromatic number of G = k.

Therefore the statement of the proposition must be true.

Proposition 2.9

Let G be a graph and suppose $C_1, C_2, \ldots, C_k$ are the colour classes corresponding to some complementary colouring of G. Let T be an independent subset of G. Then $T \subseteq C_i$ for some i.

PROOF

If T is a singleton set then obviously $T \subseteq C_i$ for some i.

Suppose T has at least two elements and suppose $T \cap C_i \neq \phi$ and $T \cap C_j \neq \phi$ for some $i \neq j$.

Let $v \in T \cap C_i$ and $u \in T \cap C_j$. Since $v \in C_i$ and $u \in C_j$ and $i \neq j$ v and u must be adjacent. This contradicts the fact that T is an independent set.

$\therefore$ T cannot intersect two distinct colour classes. Also $T \cap C_i$ is non-empty because the colour classes forms a partition of V (G). Thus $T \subseteq C_i$ for some i.

The following theorem is proved in [1]. We present a different proof for the sake of completeness.

THEOREM 2.10

Let G be a graph then the complementary chromatic colouring of G is unique. (in the sense that any two complementary chromatic colouring of G give rise to the same colour classes)

PROOF

Suppose there are two complementary chromatic colouring of G whose colour classes are $\{ C_1, C_2, \ldots\ldots, C_k\}$ and $\{ D_1, D_2, \ldots\ldots, D_k \}$. We will prove that for every i $C_i = D_j$ for some unique j.

For this first we prove that for every i there is some j $\ni C_i \subseteq D_j$.

Since $C_i \neq \phi$ & $D_1 \cup D_2 \cup \ldots\ldots \cup D_k = V(G)$, $C_i \cap D_j \neq \phi$ for some j

Claim

$C_i \subseteq D_j$

PROOF

Suppose $C_i \cap D_j \neq \phi$ for some j & for some j′ $C_i \cap D_{j'} \neq \phi$. For the sake of simplicity we assume that C_i intersects only these two sets D_j & $D_{j'}$.

Let $C_{i'} = C_i \cap D_j$ & $C_{i''} = C_i \cap D_{j'}$

$\therefore C_{i'} \cup C_{i''} = C_i$

Now we assign a new colouring to vertices of G as follows.

For every $r \neq i$ the colours of vertices of the colour class C_r are unchanged.

If $x \in C_i \cap D_j$ then we assign colour i′ to x.

If $x \in C_i \cap D_{j'}$ then we assign colour i″ to x.

Then we have a new complementary chromatic colouring of G consisting of colours 1,2,3, ….., i - 1, i′, i″, i + 1, …… , k.

This colouring uses k + 1 colours & it is a complementary colouring. This contradicts the fact that the complementary chromatic number of G is k.

$\therefore C_i \cap D_j \neq \phi$ for unique j.

$\therefore C_i \subseteq D_j$ for some unique j.

If C_i is a proper subset of D_j for some i then $C_1 \cup C_2 \cup \ldots\ldots \cup C_k \neq V(G)$ because $D_1 \cup D_2 \cup \ldots\ldots \cup D_k = V(G)$.

Thus $C_i = D_j$ for some unique j.

$\therefore \{ C_1, C_2, \ldots\ldots, C_k\} = \{ D_1, D_2, \ldots\ldots, D_k \}$.

This proves that this colouring is unique.

Proposition 2.11

Let G be a graph and $v \in V(G)$. Let f be a complementary colouring of G then the restriction g of f on G – v is also a complementary colouring of G – v.

PROOF

Let x and y be two vertices of G – v such that $g(x) \neq g(y)$ then $f(x) \neq f(y)$.

Since f is a complementary colouring, it follows that x and y are adjacent vertices of G and therefore adjacent vertices of G – v.

THEOREM 2.12

Let G be a graph & $v \in V(G)$. Then the following statements are equivalent

(1) $\chi_C(G - v) < \chi_C(G)$

(2) v is adjacent to every other vertex of G.

(3) {v} is colour class in the complementary chromatic colouring of G.

PROOF

(1) $\Rightarrow$ (3)

Suppose {v} is not a colour class in the complementary chromatic colouring of G. Therefore there is a vertex different from v which has the same colour as v. Now, consider the restriction g of the complementary chromatic colouring f of G. There is a vertex u in G – v such that f(u) = f(v). Then g is a complementary chromatic colouring of G – v. Also g is a complementary colouring of G – v.

$\therefore \chi_C(G - v) \geq$ The number of colours used by g = The number of colours used by f = $\chi_C(G)$

$\therefore \chi_C(G - v) \geq \chi_C(G)$

This is a contradiction.

$\therefore$ {v} is colour class in the complementary chromatic colouring of G.

(3) $\Rightarrow$ (2)

For any complementary colouring of a graph G a vertex in any colour class is adjacent to every vertex in every other colour class. Since {v} is a colour class, {v} is adjacent to every vertex of every other colour class. Equivalently v is adjacent to every other vertex of G.

Therefore (2) is proved.

(2) $\Rightarrow$ (1)

Suppose v is adjacent to every other vertex of G.

Consider any complementary chromatic colouring of G – v which uses colours 1, 2, 3, …… ,k. Now, assign colour k + 1 to v. Then obviously we get a complementary colouring of vertices of G which uses k + 1 colours.

$\therefore \chi_C(G) \geq k + 1 > k = \chi_C(G - v)$

$\therefore \chi_C(G - v) < \chi_C(G)$

Corollary 2.13

Let G be a graph & $v \in V(G)$. If $\chi_c(G - v) = \chi_c(G)$ then {v} is not a colour class in the complementary chromatic colouring of G.

PROOF

Since $\chi_c(G - v) = \chi_c(G)$

$\chi_c(G - v) \not< \chi_c(G)$

So, {v} is not a colour class in the complementary chromatic colouring of G.

Definition 2.14 (Complementary Colour Transversal Vertex Covering Set)

Let G be a graph. A subset S of V(G) is said to be a complementary colour transversal vertex covering set of G if

1. S is a transversal for the complementary chromatic colouring of G and
2. S is a vertex covering set of G

This set is also called CCTVC set of G.

Example 2.15

For the graph mentioned in example – 2.4, S = $\{v_1, v_3\}$ is a CCTVC set.

Definition 2.16 (Complementary Colour Transversal Vertex Covering Number)

Let G be a graph and $S \subseteq V(G)$. If S is a complementary colour transversal vertex covering set of G whose cardinality is minimum among all complementary colour transversal vertex covering set of G then S is said to be a minimum complementary colour transversal vertex covering set of G.

The cardinality of such a set is called complementary colour transversal vertex covering number (or CCTVC Number) of G. It is denoted as $\alpha_{*}c$ (G).

THEOREM 2.17

Let G be a graph. Then for G only one of the following two possibilities holds.

(1) $\alpha_{*}c(G) = \alpha_0(G)$

(2) $\alpha_{*}c(G) = \alpha_0(G) + 1$

PROOF

Let G be a graph. Consider any complementary chromatic colouring of G and suppose $C_1, C_2, \ldots, C_k$ are the colour classes corresponding to this colouring. Let S be a maximum independent subset of G so that $|S| = \beta_0(G)$. Now S is a subset of C_i for some unique i. Suppose S is a proper subset of C_i then,

(1) V(G) – S is a minimum vertex covering set of G.

(2) V(G) – S is a colour transversal for this complementary colouring of G

Therefore, V(G) – S is a minimum vertex covering set as well as a complementary colour transversal vertex covering set.

Since $\alpha_{*}(G) \geq \alpha_0(G)$ it follows that $\alpha_{*}c(G) = \alpha_0(G)$ in this case.

Suppose S is a subset of C_i and S = $C_i$then V(G) – S is a vertex covering set but it is not a transversal for this colouring. Let x be any vertex of S then the set $(V(G) - S) \cup \{x\}$ is a CCTVC set of G.

Let $T = (V(G) - S) \cup \{x\}$

$\therefore \alpha_{*}c(G) = |T| = |V(G) - S| + 1 = \alpha_0(G) + 1$

Thus for any graph G only one of the following two possibilities holds

(1) $\alpha_{*}c(G) = \alpha_0(G)$

(2) $\alpha_{*}c(G) = \alpha_0(G) + 1$

THEOREM 2.18

If G is a complete graph then for any vertex v of G

(1) $\chi_C (G - v) < \chi_C (G)$

(2) $\alpha_{*}c(G - v) < \alpha_{*}c(G)$

PROOF

Result (1) follows from the Theorem – 2.17

(2) Suppose | V (G) | = n. Since G is a complete graph χ_C (G) = n and $\alpha_{*}c(G)$ = n for any v $\in$V(G) , G – v is also a complete graph.

$\therefore \alpha_{*}c (G - v) = n - 1 < n = \alpha_{*}c(G)$

$\therefore \alpha_{*}c (G - v) < \alpha_{*}c(G)$

THEOREM 2.19

Let G be a graph with $\beta_0(G) \geq 2$. Let v $\in$V(G) $\ni$ $\chi_C (G - v) < \chi_C (G)$ then $\alpha_{*}c(G - v) < \alpha_{*}c(G)$

PROOF

Since $\beta_0(G) \geq 2$, G is not a complete graph. First suppose that $\alpha_{*}c (G) = \alpha_0 (G)$.

Let S be a minimum vertex covering set of G. Now, V (G) – S is a maximum independent set of G.

$\therefore$ v $\notin$ V(G) – S ($\because$ v is adjacent to every other vertex of G & $\beta_0(G) \geq 2$) and therefore v $\in$ S.

Now, S_1 = S – {v} is a vertex covering set of G – v also S_1 is a colour transversal for the complementary chromatic colouring of G – v which is induced from the complementary chromatic colouring of G.

$\therefore$ S_1 is a CCTVC set of G – v.

$\therefore \alpha_{*}c(G - v) \leq | S_1 | < | S | = \alpha_{*}c(G)$

Suppose $\alpha_{*}c (G) = \alpha_0 (G) + 1$

Let S be a minimum CCTVC set of G then v $\in$ S because {v} is a colour class in the unique complementary chromatic colouring of G.

Now, let S_1 = S – {v} then S_1 is a CCTVC set of G – v.

$\therefore \alpha_{*}c (G - v) \leq | S_1 | = \alpha_0 (G) < \alpha_{*}c(G)$

$\therefore \alpha_{*}c (G - v) < \alpha_{*}c(G)$

THEOREM 2.20

Let G be a graph & v $\in$V (G). If $\chi_C (G - v) = \chi_C (G)$ then $\alpha_{*}c(G - v) \leq \alpha_{*}c(G)$

PROOF

Since $\chi_C (G - v) = \chi_C (G)$, {v} is not a colour class in the complementary chromatic colouring of G. Let S be a minimum CCTVC set of G.

Case 1: v $\notin$ S

Then S is a vertex covering set of G – v & since it is a colour transversal of G it contains a vertex u different from v such that u has the same colour as v.

Thus S is a CCTVC set in G – v.

Case 2: v $\in$ S

Suppose S contains a vertex u different from v which has the same colour as v. Then S – {v} is a vertex covering set of G – v and it is also a colour transversal for the complementary chromatic colouring of G – v.

Suppose $v \in S$ & there is no other vertex which has the same colour as v & which is in S.

In this case let u be a vertex different from v such that u has the same colour as v.

Let $S_1 = (S - \{v\}) \cup \{u\}$

Then S_1 is a CCTVC set.

From both the cases above it follows that $\alpha_* c(G - v) \leq \alpha_* c(G)$

Now, we consider the possibility when $\chi_C (G - v) > \chi_C (G)$.

Let $\{ C_1, C_2, \ldots\ldots ,C_j\}$ be the set of all colour classes of G ($j \geq 1$) and let $\{ D_1, D_2, \ldots\ldots ,D_k \}$be the set of all colour classes of G – v.

THEOREM 2.21

$\chi_C (G - v) > \chi_C (G)$ iff

(1) There are at least two colour classes of G – v which are all subsets of the colour class C which contains v & there union = C – {v} and v is non-adjacent with some vertex in every such colour class.

(2) Other colour classes of G – v are just the colour classes of G different from C.

PROOF

(1) Suppose $\chi_C (G - v) > \chi_C (G)$ then $k > j$

Now, each colour class D_i intersect some colour class C_r of G. Suppose $D_i \cap C_r \neq \phi$ & $D_i \cap C_{r'} \neq \phi$. Now, let $D_i \cap C_r = D_{i'}$ & $D_i \cap C_{r'} = D_{i''}$

Then we can assign two distinct colours of vertices of $D_{i'}$& $D_{i''}$ in place of the single colour of $D_{i.}$ This will increase the number of colour used in complementary colouring of G – v. Which is a contradiction.

$\therefore$ $D_i \cap C_r \neq \phi$ for some unique r.

$\therefore$ $D_i \subseteq C_r$ for some unique r.

Also, there are colour classes of G – v which intersect the colour class C containing v. Therefore, there are colour classes which are subsets of C. Suppose there are only m colour classes of G –v which are containing in C and $m < k - j + 1$.

Now, we provide a new colouring of G as follows.

Assign the same colour as that of C to all the vertices which belong to the m colour classes mentioning above. Do not change the colours of the remaining $j - m$ colour classes which are disjoint from C. Thus we get a complementary chromatic colouring of G consisting of $k - m + 1$colours, which is greater than j. This is a contradiction as j is the highest number of colours which is assign to vertices of G so that resulting colouring is complementary colouring.

Suppose there are m colour classes of G –v which are containing in C and $m > k - j + 1$.

Then $k - m < j - 1$

Thus it must be true that the remaining k –m colour classes of G – v are contained in j – 1 colour classes of G. Which is impossible because $k - m < j - 1$.

Thus, $m < k - j + 1$ & $m > k - j + 1$ are impossibilities. Therefore, $m = k - j + 1$

Since C is a colour class in G containing v & union of the above mentioned colour classes = C – {v}, v must be non-adjacent to some vertex in the union & therefore v must be non-adjacent to some vertex in some colour class.

Claim

Now, we prove that v is non-adjacent with some vertex in every colour class of G – v which is contained in C.

PROOF OF THE CLAIM

Suppose there is a colour class of G –v say D such that D ⊂ C & v is adjacent with every vertex of D. Then we can assign a colour to the vertices of D which is different from v & it is also different from the colours of other colour classes of G.

Thus we get a complementary colouring of G which consists of j + 1 colours. This contradicts the fact that complementary chromatic number of G = j. Therefore, there is no colour class of G – v which is contained in D & v is adjacent with every vertex of that colour class.

(2) Now, consider the remaining k – (k – j + 1) = j -1 colour classes of G – v . Since there are j -1 colour classes of G different from C, each colour class is contained in a unique colour class of G. Since the union of both the colour classes = V(G), this j -1 colour classes of G – v are exactly the colour classes different from C.

Conversely suppose (1) and (2) holds then it follows that

The number of colour classes of G – v > The number of colour classes of G

$\therefore \chi_C(G - v) > \chi_C(G)$

THEOREM 2.22

Let G be a graph & v ∈V(G). Suppose $\chi_C(G - v) > \chi_C(G)$ than any of the following three possibilities can hold

(1) $\alpha_*c(G - v) > \alpha_*c(G)$

(2) $\alpha_*c(G - v) = \alpha_*c(G)$

(3) $\alpha_*c(G - v) < \alpha_*c(G)$

PROOF

First suppose that $\underline{\alpha_0(G - v) < \alpha_0(G)}$ then there is a minimum vertex covering set S of G such that v ∈ S. Let M = V(G) – S then M is a maximum independent subset of G & v ∉ M.

Now, $M \subseteq C_i$ for some i

Case (1) M is a proper subset of C_i

Then V(G) – M = S is a minimum vertex covering set & it is also a colour transversal of G.

∴ S is a CCTVC set of G & $| S | = n - \beta_0(G) = \alpha_0(G)$

Since M does not contain v, M is also a maximum independent subset of G – v. Therefore M is a subset of D_r for some unique r.

If M is a proper subset of D_r then Let $G_1 = G – v$

$\therefore S_1 = V(G_1) - M$ is a minimum vertex covering set of G – v & it is also a colour transversal of G – v.

∴ S_1 is a CCTVC set of G – v & $| S_1 | = n – 1 - \beta_0(G)$

$\therefore \alpha_*c(G - v) < \alpha_*c(G)$

Now, suppose $M = D_r$. Let $x \in D_r$

Consider the set $S_1 = (V(G_1) – M) \cup \{x\}$ then S_1 is a vertex covering set & it is also a colour transversal of G – v.

Also $| S_1 | = \alpha_0(G) + 1$

∴ S_1 is a CCTVC set of G – v

$\therefore \alpha_*c(G - v) = | S_1 | = n – \beta_0(G) = \alpha_*c(G)$

Case (2) Suppose $M = C_i$ for some i. Let x ∈ M.

Now, consider the set T = (V(G) – M) $\cup$ {x} then T is a vertex covering set of G & it is also a colour transversal for complementary chromatic colouring of G. Thus T is a minimum CCTVC set.

$\therefore \alpha_* c(G) = |T| = n - \beta_0(G) + 1$

Suppose M is a proper subset of some colour class D_r of the complementary chromatic colouring of G – v. Then $T_1 = V(G_1) - M$ is is a minimum vertex covering set of G – v & it is also a colour transversal for this colouring of G – v.

$\therefore \alpha_* c(G - v) = |T_1| = n - 1 - \beta_0(G)$

$\therefore \alpha_* c(G - v) < \alpha_* c(G)$

On the other hand if M = D_r forsome r then let y $\in D_i$

Then $T_2 = (V(G_1) - M) \cup \{y\}$ is a vertex covering set & it is also a colour transversal of G – v.

$\therefore \alpha_* c(G - v) = n - 1 - \beta_0(G) + 1 = n - \beta_0(G)$

$\therefore \alpha_* c(G - v) < \alpha_* c(G)$

Now suppose $\underline{\alpha_0(G - v) = \alpha_0(G)}$

In this case v $\notin$ S for any minimum vertex covering set S of G.

$\therefore$ v$\in$ M for every maximum independent subset M of G. Now, M is a subset of C_i for some colour class C_i. Since v $\in$ M $\Rightarrow$ v $\in C_i$

Suppose M is a proper subset of C_i then as proved above $\alpha_* c(G) = n - \beta_0(G)$

1. Suppose M – {v} is a proper subset of D_r for some colour class D_r of G – v.

Then $\alpha_* c(G - v) = n - 1 - (\beta_0(G) - 1) = n - \beta_0(G)$

$\therefore \alpha_* c(G - v) = \alpha_* c(G)$

2. Suppose M – {v} = D_r for some colour class D_r of G – v.

Then $\alpha_* c(G - v) = n - 1 - (\beta_0(G) - 2) = n - \beta_0(G) + 1$

$\therefore \alpha_* c(G - v) > \alpha_* c(G)$

Suppose M = C_i for some colour class C_i of G.

Then $\alpha_* c(G) = n - \beta_0(G) + 1$

Suppose M – {v} is a proper subset of D_r for some colour class D_r of G – v. As proved in above theorem D_r & D_s are subsets of C_i for at least two distinct values r & s then M – {v} will be a proper subset of $D_r \cup D_s$ and therefore M – {v} will be proper subset of C – {v}.

$\therefore$ M is proper subset of C_i which is contradiction.

$\therefore$ M is proper subset of D_r is not possible for any r.

Hence, $\alpha_* c(G - v) = n - 1 - (\beta_0(G) - 2) = n - \beta_0(G) + 1$

$\therefore \alpha_* c(G - v) = \alpha_* c(G)$

Example 2.23

Consider the graph G in example 2.4

Here, $\chi_C(G) = 2$ & $\chi_C(G - v_4) = 3$

$\therefore \chi_C(G - v) > \chi_C(G)$

Also observe that

$\alpha_* c(G) = 2$ & $\alpha_* c(G - v_4) = 3$

Hence, $\alpha_* c(G - v) > \alpha_* c(G)$

Example 2.24

Consider the path graph with four vertices $G = P_4$

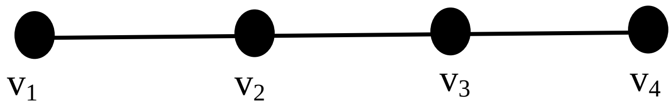

Fig. 2

Here, $\chi_C(G) = 1$ & $\chi_C(G - v_4) = 2$

$\therefore \chi_C(G - v) > \chi_C(G)$

Also observe that

$\alpha_* c(G) = 2 = \alpha_* c(G - v_4)$

Hence, $\alpha_* c(G - v) = \alpha_* c(G)$

3. Concluding Remark

There are enough number of examples of graph G for which $\chi_C(G - v) > \chi_C(G)$ and $\alpha_* c(G - v) \geq \alpha_* c(G)$. However, we do not know a graph G for which $\chi_C(G - v) > \chi_C(G)$ and $\alpha_* c(G - v) < \alpha_* c(G)$.

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