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On Spectral Theory of Sequences Vector Spaces in a Space without Scalar Product: Case of a Banach's Space G

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Abstract: *With the question of knowing if it is possible to study sequences in a nonseparable space or not provided with a scalar product, we propose a possible technique for an unspecified Banach's space G, with or without unit element, separable or not.*

Keywords: *Sequence, Convergent sequence, Vector space, Banach's space, Hilbert's space, Isomorphism.*

1. Introduction

1.1 Useful Vector Spaces of Sequences in G

Let G be an arbitrary space of Banach, G^n the vector space of sequences in G, i.e. $G^n = \langle x = (x_1, x_2, \dots, x_i, \dots)$ such that $x_i \in G$ for $i = 1,2,3, \dots \rangle, G_C^n = \{x \in G^n : \exists x_\alpha \in$ and $\lim_{i\to\infty} \|x_i - x\alpha$=0the vectorial subspace of convergent sequences in G and, finally, the vectorial subspace of G_C^ndenoted $G^2$and defined as and by the following conditions:

$G^2 = \{x = (x_1, x_2, \dots, x_i, \dots) : x \in G_C^n\, et \sum_{i=1}^{\infty} \|x_i\|^2 < \infty\}$; it allows two vectorial subspaces denoted $G_p^n\, et G^P$.

1.2 Hilbert's spaces sequence

Now, Let $(H_i)_{i\geq 1}$ be a sequence of unspecified spaces of Hilbert and H a set of sequences defined as follows: $H = \{h = (h_1, h_2, \dots, h_i, \dots) : h_i \in H_i$ for $i = 1,2,3, \dots$ and $\sum_{i=1}^{\infty} \|h_i\|^2 < \infty\}$; it is known that H is a vector space such as, provided with the square form denoted and defined by $\alpha : H^2 \longrightarrow \mathbb{R} : \alpha(h, g) = \sum_{i=1}^{\infty} (h_i \uparrow g_i)$for all $h, g \in H$, it is a Hilbert's space called hilbertian sum of the sequence $(H_i)_{i\geq 1}$[1].

1.3 Towards the answer

Let us return to vector spaces G^2 and H, and consider the following maps: on the one hand $\gamma : G^2 \longrightarrow H : x \longrightarrow \gamma(x) = h \in H$for all $x \in G^2$and on the other hand $\omega : (G^2)^2 \longrightarrow H^2 : (x, y) \longrightarrow \omega(x, y) = (g, h)$for all $x, y \in G^2$and $h, g \in H$; it is clear that γ and ω are linear one-to-one maps, therefore isomorphisms(i); now, Let $(G^2)^2 : \varphi \longrightarrow\longrightarrow\longrightarrow \mathbb{R}$ be a map ; it is easy and favorable to note that starting from three applications $\omega, \alpha et \varphi$, one immediately obtains the diagram presented in section 2.

2. ANSWER

2.1 Observing and exploiting the diagram

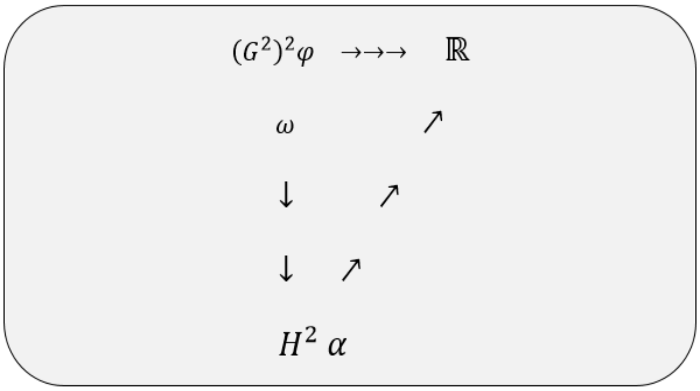

Figure 1. *Commutative diagram*

It is clear that $\varphi = \alpha o \omega$ such that, for all $x, y \in G^2$, one obtains following equalities :

$\varphi(x,y) = (\alpha o \omega)(x,y) = \alpha\{\omega((x,y))\} = \alpha(g,h) = \sum_{i=1}^{\infty}(g_i \uparrow h_i)$

or simply $\varphi(x,y) = \sum_{i=1}^{\infty}(g_i \uparrow h_i)$

2.2 Result

As H and $G^2$are isomorphic vector spaces $(i)$and H is a Hilbert's space whose scalar product is $\alpha(h,g) = \sum_{i=1}^{\infty}(h_i \uparrow g_i)$for all $h, g \in H$, it is easy to conclude that $G^2$is a space of Hilbert whose scalar product is $\varphi(x,y) = \sum_{i=1}^{\infty}(x_i \uparrow y_i)$for all $x, y \in G^2$; thus, one can easily establish the quadruple G^n, G_p^n, G^2, G^psimilar to the traditional quadruple $\mathbb{K}^n, \mathbb{K}_p^n, \ell^2, \ell_p$attributed to Riesz.

2.3 An illustration

In particular, Let $\mathcal{B}(H)$ be a Banach's space; by representing a sequence in $\mathcal{B}(H)$by $A = (A_1, A_2, A_3, \dots, A_i, ..)$ such that $A_i \in \mathcal{B}(H)$ for all $i = 1,2,3,$ ...and one can consider the vector space of the convergent sequences in $\mathcal{B}(H)$, denoted and defined as follows:

$\mathcal{B}^2 = \{A = (A_1, A_2, A_3, \dots, A_i, ..)$ such that $\sum_{i=1}^{\infty}\|A_i\|^2 < \infty$ }; it is provided with two vectorial subspaces denoted $\mathcal{B}_p^n$ and $\mathcal{B}^p$; it is easy to note that the vector space $\mathcal{B}^2$is isomorphic to the vector space G^2which is a Hilbert's space whose scalar product is denoted $\varphi(x,y) = \sum_{i=1}^{\infty}(x_i \uparrow y)$; it results from it that $\mathcal{B}^2$is a space of Hilbert whose scalar product is written as $\delta(A,D) = \sum_{i=1}^{\infty}(A_i \uparrow$ $\mathcal{D}i$for all $A,\mathcal{D}\in \mathcal{B}H$; thus, one can easily establish the quadruple $\mathcal{B}n, \mathcal{B}pn$,$\mathcal{B}2$, $\mathcal{B}p$similar to the traditional quadruple $\mathbb{K}^n, \mathbb{K}_p^n, \ell^2, \ell_p$attributed to Riesz.

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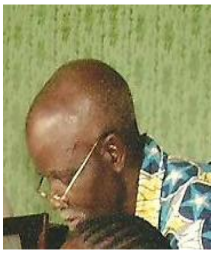

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