Physical Properties of Polychromatic Quasi Crystals

M. Radha Madhavi	**D.Bharathi**
Sr. Associate Professor	Professor
Department of Mathematics	Department of Engg. Mathematics
Aarupadai Veedu Institute of Technology	A.U.College of Engineering
Vinayaka Missions University	Andhra University
Chennai, INDIA	Visakhapatnam, INDIA
mrmadhavi5@gmail.com	*bharathidora57@yahoo.co.in*

Abstract: *The polychromatic point groups with the help of crystallographic point groups containing one dimensional complex irreducible representations of generator groups were derived by Indenbom et al (1960). Rama Mohana Rao K (1987) had studied the magnetic properties pertaining to the ten crystallographic point groups that generate the 18 polychromatic point groups has already been covered during the study of the 90 magnetic classes. In this paper 18 polychromatic point groups with the help of quasi crystals with 5,8,10 & 12 symmetries are derived. The 54 polychromatic classes with the help of 11 non-crystallographic point groups containing one dimensional complex irreducible representations are given. The maximum number of non-vanishing independent constants required to describe a chosen physical property by the 54 polychromatic classes by using Bhagavantham's formula are determined and tabulated these constants to 10 known physical properties.*

Keywords: *Quasi Crystals, Polychromatic point group, Magnetic properties, Irreducible representation*

1. INTRODUCTION

The discovery of icosahedral quasi crystals in Al-Mn alloys by Schechtman et al (1984), other quasi crystals with ten, twelve, eight and five-fold symmetries has been obtained in succession. Levine and Steinhart (1984) introduced the term quasi crystals for these unusual systems exhibiting diffraction patterns with non-crystallographic symmetry. The decagonal symmetry found in NiCr, the octagonal symmetry in NiSi and the pentagonal symmetry found in AlCoNiTb. Feng and co-workers have discovered an incommensurate structure with cubic point group symmetry in V-Ni-Si alloys. Janssen gave a clear theoretical explanation for quasi periodic structures which may have either crystallographic or non-crystallographic point group symmetry. They can be obtained by projecting a higher dimensional periodic structure upon the physical space or by cutting a higher dimensional structure. Bradnd muller & Clauss (1988) Provided the irreducible tensors of rank 1-4 for all the irreducible representations of the pentagonal and icosahedral point groups. He also gave the Raman and hyper – Raman tensors for groups with five-fold symmetry.

Qusai crystals possess various point group symmetries. So many researchers worked on the physical properties of quasi crystals in both theoretical and experimental aspects. Some researchers derived the physical properties of quasi crystals using group theoretical methods. In chapter 2 we gave the number of independent constants required to describe a physical property of quasi crystals with 5,7,8,10,12 and icosahedral symmetries and also the number of independent constants required to describe a physical property of ferroic species of quasi crystals with 5,7,8,10,12 and icosahedral symmetries using Bhagavantham's formula.

Belov and Tarkhova (1956) gave the interpretation of anti-symmetry as two colour symmetry and also gave the concept of polychromatic symmetry. Consequently in 1960 Indenbom et al derived the 18 poly chromatic point groups by introducing the colour change operation R_n , n= 3,4 or 6 with $R_n^n = E$ with the help of 10 crystallographic point groups containing one dimensional complex irreducible representations and associated with the 18 pairs of one dimensional complex irreducible representations of the generator groups.

Naish (1963) described that the polychromatic structure, and hence the polychromatic point groups and space groups have a useful role to play in many physical applications such as the derivation and description of similarity symmetry point groups and space groups, in description of stem and layer symmetry groups in higher dimensional space and describing the magnetic symmetry and screw structure and where the traditional magnetic groups cannot adequately describe the situation.

Rama Mohana Rao K (1987) had studied the magnetic properties pertaining to the ten crystallographic point groups that generate the 18 polychromatic point groups has already been covered during the study of the 90 magnetic classes. He studied the 18 polychromatic point groups whose geometric configuration was provided by Indenbom et al and Bradley and Cracknel with a view to enumerating the number of constants required to describe a chosen magnetic property and occurring before an irreducible representation of the variant by G. In 1988 the same author obtained the maximum number of non-vanishing independent constants required to describe a chosen physical property by the 18 polychromatic classes and he tabulated the constants of six known physical properties. He also studied theoretically as the effect of spins in crystals in their magnetic state on the non-magnetic physical properties.

In this paper the concept of 18 polychromatic point groups with the help of 10 crystallographic point groups containing one dimensional complex irreducible representations (Indenbom et al) was extended to the quasi crystals with 5,8,10 & 12 symmetries. The 54 polychromatic classes with the help of 11 non-crystallographic point groups containing one dimensional complex irreducible representations are given. The maximum number of non-vanishing independent constants required to describe a chosen physical property by the 54 polychromatic classes by using Bhagavantham's formula are determined and tabulated these constants to 10 known physical properties.

2. POLYCHROMATIC CLASSES OF QUASI CRYSTALS

The polychromatic point group C, is given by

$$C = \alpha_1 H + \alpha_2 R_p H + \alpha_3 R_p^2 H + \text{-------} + \alpha_p R_p^{p-1} H$$

Where H is a subgroup of some point group G and is of order (1/p) × the order of G and $\alpha_1, \alpha_2, \alpha_3 - --- , \alpha_p$ are the left coset representatives in the expansion of G.

$$G = \alpha_1 H + \alpha_2 H + \alpha_3 H + \text{-------} + \alpha_p H$$

If p = 3,4 or 6 we can derive p-coloured point groups. Bradley and cracknel gave the 58 black and white point groups are derived by studying the real one dimensional representation of the ordinary point groups and multiplying an element whose character is -1 by the colour changing operation. In the same way it is possible to use the complex one dimensional representation of the ordinary point groups to derive the 3-coloured,4-coloured and 6-coloured point groups(Indenbom et al). The complex numbers $\omega \left(= e^{\frac{2\pi i}{3}} \right), i \left(= e^{\frac{2\pi i}{4}} \right) and - \omega^* \left(= -e^{\frac{2\pi i}{6}} \right)$ can be associated with colour changing operations for 3-coloured,4-coloured and 6-coloured point groups respectively. The two members of a pair of complex conjugate representations of an ordinary point group both lead to the same polychromatic point group.

Similarly we defined 5-coloured,8-coloured 10-coloured and 12-coloured point groups by taking the complex one dimensional representation of the quasi crystals. The complex numbers

$\rho(= e^{\frac{2\pi i}{5}}), \rho(= e^{\frac{2\pi i}{8}}), \rho(= e^{\frac{2\pi i}{10}}), \rho(= e^{\frac{2\pi i}{12}})$, can associated with colour changing operations for 5-coloured,8-coloured 10-coloured and 12-coloured point groups respectively. All the 54 polychromatic classes of quasi crystals are given in table 1.

Table 1. *54 polychromatic variants of quasi crystals*

S.NO	C(poly chromatic point group)	G (point group)	S.NO	C(poly chromatic point group)	G (point group)	S.NO	C(poly chromatic point group)	G (point group)
1	$5_1^{(5)}$	5	19	$10/m^{(5)}$	10/m	37	$12_3^{(12)}$	12
2	$5_2^{(5)}$	5	20	$10/m'^{(5)}$	10/m	38	$12_1^{(6)}$	12

3	$\bar{5}_1^{(5)}$	$\bar{5}$	21	$10/m''^{(5)}$	10/m	39	$12_2^{(6)}$	12
4	$\bar{5}_2^{(5)}$	$\bar{5}$	22	$10/m'''^{(5)}$	10/m	40	$\overline{12}_1^{(12)}$	$\overline{12}$
5	$\bar{5}_{1'}^{(5)}$	$\bar{5}$	23	$8_1^{(8)}$	8	41	$\overline{12}_2^{(12)}$	$\overline{12}$
6	$\bar{5}_{2'}^{(5)}$	$\bar{5}$	24	$8_2^{(8)}$	8	42	$\overline{12}_3^{(12)}$	$\overline{12}$
7	$10_1^{(10)}$	10	25	$8_1^{(4)}$	8	43	$\overline{12}_1^{(6)}$	$\overline{12}$
8	$10_2^{(10)}$	10	26	$\bar{8}_1^{(8)}$	$\bar{8}$	44	$\overline{12}_2^{(6)}$	$\overline{12}$
9	$10_1^{(5)}$	10	27	$\bar{8}_2^{(8)}$	$\bar{8}$	45	$12/m^{(12)}$	12/m
10	$10_1^{(5)}$	10	28	$\bar{8}_1^{(4)}$	$\bar{8}$	46	$12/m'^{(12)}$	12/m
11	$\overline{10}_1^{(10)}$	$\overline{10}$	29	$8/m^{(8)}$	8/m	47	$12/m''^{(12)}$	12/m
12	$\overline{10}_2^{(10)}$	$\overline{10}$	30	$8/m'^{(8)}$	8/m	48	$12/m'''^{(12)}$	12/m
13	$\overline{10}_1^{(5)}$	$\overline{10}$	31	$8/m''^{(8)}$	8/m	49	$12/m^{1V\,(12)}$	12/m
14	$\overline{10}_2^{(5)}$	$\overline{10}$	32	$8/m^{(4)}$	8/m	50	$12/m^{(6)}$	12/m
15	$10/m^{(10)}$	10/m	33	$8/m'^{(4)}$	8/m	51	$12/m'^{(6)}$	12/m
16	$10/m'^{(10)}$	10/m	34	$8/m''^{(4)}$	8/m	52	$12/m''^{(6)}$	12/m
17	$10/m''^{(10)}$	10/m	35	$12_1^{(12)}$	12	53	$12/m'''^{(6)}$	12/m
18	$10/m'''^{(10)}$	10/m	36	$12_2^{(12)}$	12	54	$12/m^{1V\,(6)}$	12/m

3. PHYSICAL PROPERTIES OF QUASI CRYSTALS HAVING POLYCHROMATIC SYMMETRY

The physical property of substances depends on the relation between two quantities such as scalar-scalar, scalar-vector or symmetry tensor of rank two etc. In all the cases, each of the relation requires a certain number of independent constants connecting the components of the quantities involved for its complete description. The number of independent constants required to describe a physical property depends on the property chosen and on the crystallography symmetry of the material in solid state.

Bhagavantam [4] and other researchers gave the number of independent constants to describe the physical properties of the crystallographic point groups by using formula

$$n_i = \frac{1}{N}\sum_j h_j \chi_j'(R)\chi_i(R) \tag{1}$$

Where N is the total number of elements of the group G and h_j is the number of elements in the j th class. χ_j' (R) is the character of the appropriate transformation matrix relative to the operation R and χ_i (R) is the character relative to the operation R in the particular irreducible representation of a group G. The classifications of ten physical properties are listed in table 3 as given by Bhagavantam (1966).

For example, the thermal conductivity or electrical resistivity χ_ρ (R) is given by

χ_ρ (R) = $4\cos^2\phi \pm 2\cos\phi$

The maximum number of independent constants to describe the thermal conductivity is 6. Consider the point group 5, which contains two pairs of one dimensional complex representation.

Table2. *The character table for a point group 5*

5 (C_5)	E	C_5	C_5^2	C_5^3	C_5^4	n_i
A	1	1	1	1	1	2
E_1	1	ρ	ρ^2	ρ^{-2}	ρ^{-1}	1
	1	ρ^{-1}	ρ^{-2}	ρ^2	ρ	1
E_2	1	ρ^2	ρ^{-1}	ρ	ρ^{-2}	1
	1	ρ^{-2}	ρ	ρ^{-1}	ρ^2	1
χ_ρ (R)	6	1	1	1	1	

The polychromatic variants of the point group 5 are $5^{(5)}$ and $5'^{(5)}$, by using eq. (1) the independent thermal conductivity constants of $5^{(5)}$ and $5'^{(5)}$ is one. Similarly the number of independent constants to describe the physical properties of all 54 polychromatic variants are tabulated in table 4.

Table 3. *Classification of physical properties*

S.NO	Physical property	Character χ'_i (R)	χ_i(R)	Max.No.of constants
1	Pyro electricity: production of charges by hydrostatic pressure	$2\cos\phi \pm 1$	1	3
2	Electrical resistivity, thermal conductivity	$4\cos^2\phi \pm 2\cos\phi$	1	6
3	Hall Effect	$4\cos^2\phi \pm 4\cos\phi + 1$	1	9
4	Piezoelectricity	$(4\cos^2\phi \pm 2\cos\phi)(2\cos\phi \pm 1)$	1	18
5	Photoelasticity	$16\cos^4\phi \pm 16\cos^3\phi + 4\cos^2\phi$	1	36
6	Elasticity(Symmetric & symmetric tensor)	$16\cos^4\phi \pm 8\cos^3\phi - 4\cos^2\phi + 1$	1	21
7	Magnetothermoelectric power, piezo-Hall effect	$16\cos^4\phi \pm 24\cos^3\phi + 12\cos^2\phi \pm 2\cos\phi$	1	54
8	Piezoelectric Coefficients (Higher order)	$32\cos^5\phi \pm 32\cos^4\phi \mp 4\cos^2\phi + 2\cos\phi \pm 1$	1	63
9	Elastic Coefficients (Higher order)	$64\cos^6\phi \pm 32\cos^5\phi - 48\cos^4\phi \mp 8\cos^3\phi + 16\cos^2\phi$	1	56
10	Photo elastic Coefficients (Higher order)	$64\cos^6\phi \pm 64\cos^5\phi \mp 8\cos^3\phi + 4\cos^2\phi \pm 2\cos\phi$	1	126

Table 4. *Number of independent constants to describe the physical properties by the 54 polychromatic classes of quasi crystals*

S.NO	Polychromatic class	Number of independent constants to describe the physical properties in table 7.2									
		1	2	3	4	5	6	7	8	9	10
1	$5_1^{(5)}$	1	1	2	4	7	4	11	13	11	25
2	$5_2^{(5)}$	0	1	1	3	7	4	10	12	11	25
3	$\overline{5}_1^{(5)}$	0	1	2	0	7	4	11	0	11	25
4	$\overline{5}_2^{(5)}$	0	1	1	0	7	4	10	0	11	25
5	$\overline{5}_{1'}^{(5)}$	1	0	0	4	0	0	0	13	0	0
6	$\overline{5}_{2'}^{(5)}$	0	0	0	3	0	0	0	12	0	0
7	$10_1^{(10)}$	1	1	2	4	6	3	10	11	7	18
8	$10_2^{(10)}$	0	1	1	2	5	3	7	7	7	16
9	$10_1^{(5)}$	0	0	0	1	2	1	3	5	4	9
10	$10_1^{(5)}$	0	0	0	0	1	1	1	2	4	7
11	$\overline{10}_1^{(10)}$	1	0	0	4	1	1	1	11	4	7
12	$\overline{10}_2^{(10)}$	0	1	1	1	5	3	7	5	7	16
13	$\overline{10}_1^{(5)}$	0	1	2	0	6	3	10	2	7	18
14	$\overline{10}_2^{(5)}$	0	0	0	0	2	1	3	7	4	9
15	$10/m^{(10)}$	0	1	2	0	6	3	10	2	7	18
16	$10/m'^{(10)}$	0	1	1	0	5	3	7	0	7	17
17	$10/m''^{(10)}$	0	0	0	0	2	1	3	0	4	9
18	$10/m'''^{(10)}$	0	0	0	0	1	1	1	0	4	8
19	$10/m^{(5)}$	1	0	0	4	0	0	0	11	0	0
20	$10/m'^{(5)}$	0	0	0	2	0	0	0	7	0	0
21	$10/m''^{(5)}$	1	0	0	1	0	0	0	5	0	0
22	$10/m'''^{(5)}$	0	0	0	0	0	0	0	2	0	0
23	$8_1^{(8)}$	1	1	2	4	6	3	10	10	6	16

24	$8_2^{(8)}$	0	1	1	2	5	3	7	7	8	17
25	$8_1^{(4)}$	0	0	0	1	2	1	3	7	6	13
26	$\bar{8}_1^{(8)}$	1	0	0	1	2	1	3	7	6	13
27	$\bar{8}_2^{(8)}$	0	1	1	2	5	3	7	7	8	17
28	$\bar{8}_1^{(4)}$	0	1	2	4	6	3	10	10	6	16
29	$8/m^{(8)}$	0	1	2	0	6	3	10	0	6	16
30	$8/m'^{(8)}$	0	1	1	0	5	3	7	0	8	17
31	$8/m''^{(8)}$	0	0	0	0	2	1	3	0	6	13
32	$8/m^{(4)}$	1	0	0	4	0	0	0	10	0	0
33	$8/m'^{(4)}$	0	0	0	2	0	0	0	7	0	0
34	$8/m''^{(4)}$	0	0	0	1	0	0	0	7	0	0
35	$12_1^{(12)}$	1	1	2	4	6	3	10	11	7	18
36	$12_2^{(12)}$	0	1	1	2	5	3	7	7	7	16
37	$12_3^{(12)}$	0	0	0	1	2	1	3	5	4	9
38	$12_1^{(6)}$	0	0	0	0	1	1	1	2	3	6
39	$12_2^{(6)}$	0	0	0	0	0	0	0	1	1	2
40	$\overline{12}_1^{(12)}$	1	0	0	4	0	0	0	11	1	2
41	$\overline{12}_2^{(12)}$	0	1	1	0	5	3	7	2	7	16
42	$\overline{12}_3^{(12)}$	0	0	0	1	2	1	3	5	4	9
43	$\overline{12}_1^{(6)}$	0	0	0	2	1	1	1	7	3	6
44	$\overline{12}_2^{(6)}$	0	1	2	0	6	3	10	1	7	18
45	$12/m^{(12)}$	0	1	2	0	6	3	10	0	7	18
46	$12/m'^{(12)}$	0	1	1	0	5	3	7	0	7	16
47	$12/m''^{(12)}$	0	0	0	0	2	1	3	0	4	9
48	$12/m'''^{(12)}$	0	0	0	0	1	1	1	0	3	6
49	$12/m^{1V(12)}$	0	0	1	0	0	0	0	0	1	2
50	$12/m^{(6)}$	1	0	0	4	0	0	0	11	0	0
51	$12/m'^{(6)}$	0	0	0	2	0	0	0	7	0	0
52	$12/m''^{(6)}$	0	0	0	1	0	0	0	5	0	0
53	$12/m'''^{(6)}$	0	0	0	0	0	0	0	2	0	0
54	$12/m^{1V(6)}$	0	0	0	0	0	0	0	1	0	0

4. CONCLUSION

In this paper the theoretical study of non vanishing independent constants of physical properties of quasi crystals with polychromatic symmetry are discussed. The 54 polychromatic classes with help of 11 non-crystallographic point groups are tabulated and Using Bhagavantam's formula, the number of independent constants needed to describe a physical property of quasi crystals with polychromatic symmetry are calculated. The experimental investigations suggests to a physicist to identify the quasi crystals possessing the symmetry of the polychromatic groups.

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AUTHOR'S BIOGRAPHY

Radha Madhavi, was born in Tenali, India, in 1979. She received the M.Sc degree in Applied Mathematics from the Sri Padmavathi Mahila University, Tirupathi, India, in 2000, and the M.Phil. and Ph.D. degrees in Applied Mathematics from the Andhra University, Andhra Pradesh, India, in 2002 and 2013, respectively. In 2002, she joined the Department of mathematics, M.V.G.R.College of Engineering as a Assistant professor, and in 2007 became a Sr. Assistant professor. In 2009 She Joined as Sr. Assistant Professor in Velammal Institute of Technology, Chennai. In 2015 She joned in AVIT, Vinayaka missons University, Chennai as Sr. Associate professor to till date. Her Current research interests are Applications of Group theory and Graph Theory.