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One Solution of Multi-term Fractional Differential Equations by Adomian Decomposition Method

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Abstract: *Inthis paper, we applied the Adomian decomposition method (ADM) to solving of nonlinear multi-term fractional differential equations of the Form*

$D_*^{\alpha} y(x) = \sum_{i=0}^{n} a_i(x) D_*^{\beta_i} y(x) + a_0(x) y(x) + N\left(x, y(x), D_*^{\beta_1} y(x), \dots, D_*^{\beta_n} y(x)\right) + g(x)$ *under the following initial conditions* $y^{(i)}(0) = c_i \quad (0 \le i \le m-1)$ *where N is nonlinear function* $x, y(x), D_*^{\beta_1} y(x), \dots, D_*^{\beta_n} y(x)$ *and* $\alpha > \beta_n > \cdots > \beta_1 > 0, (m-1 < \alpha \le m, and\ m \in N)$. *We show the ability of the method for solving Multi-term Fractional Differential Equations by some examples.*

Keywords: *Multi-term Fractional Differential Equations, Riemann-Liouville fractional integral, Caputo Derivative, Adomian polynomials.*

1. INTRODUCTION

The Adomian decomposition method (ADM) is a well-known for analytical approximate solutions of linear or nonlinear ordinary differential equations (ODEs), Fractional Differential Equations (FDEs) , integral equations ,integral differential equations, etc. They arevery useful for application oriented problems .Numerical method is to obtain approximate solutions of fractional differential equations.Forexamples Legendre Pseudo-Spectral Method,vibrational iteration method, etc. H.Jafari andV.Gejji [5,10] have solved a system of nonlinear fractional differential equations and a multi order fractional differential equation using Adomian decomposition for nonlinear is functions of $x, y_1, \dots, y_n$.In 2006,S.Momania and Zaid Odibatb[7] used the vibrational iteration method and the Adomian decomposition method are implemented to give approximate solutions for linear and nonlinear systems of differential equations of fractional order . O.A. Taiwo and O.S. Odetunde [8]applied approximation of multi-order fractional differential equations by an iterative decomposition method**.** M. M. Khader, talaat S. El danaf and A. S. Hendy [9],used efficient spectral collocation method for solving multi-term fractional differential equations based on the generalized laguerre polynomials **.**VedatSuatErturkShaher Momani B, Zaid Odibat [6] presented application of generalized differential transform method to multi-order fractional differential equations. In this paper we consider the Multi-term Fractional Differential Equations as form:

$$D_*^{\alpha} y(x) = \sum_{i=0}^{n} a_i(x) D_*^{\beta_i} y(x) + a_0(x) y(x) + N\left(x, y(x), D_*^{\beta_1} y(x), \dots, D_*^{\beta_n} y(x)\right) + g(x) \tag{1}$$

Under the following initial conditions $y^{(i)}(0) = c_i (0 \le i \le m-1)$ where $\alpha > \beta_n > \cdots > \beta_1 > 0, (m-1 < \alpha \le m,$ and $m \in N)$, N is non-linear function of $x, y(x), D_*^{\beta_1} y(x), \dots, D_*^{\beta_n} y(x)$and $g(x)$ and $a_i(x)$ are function of x.

In this paper, In Section 2, we survey the basic definitions and the properties of the fractional calculus. In Section 3, the analysis of Adomian decomposition method. In Section 4, we present the solving of the multi-term fractional differential equation by ADM. Some numerical examples are provided in Section 5. Also a conclusion is given in the last section.

2. Basic Definitions and the Properties of the Fractional Calculus

We give some basic definitions and properties of fractional calculus theory which are further used in this paper.

Definition 1: The Riemann-Liouville fractional integral operator of order $\alpha > 0$function$f(x)\epsilon C([a,b])$and $a < x < b$ is defined as:

$$J_a^\alpha f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1} f(t)dt \tag{2}$$

$$J_a^0 f(x) = f(x)$$

Definition 2: For$f(x)\epsilon C([a,b])$,the Caputo of fractional derivatives is defined by :

$$D_*^\alpha f(x) = \begin{cases} J^{m-\alpha} f^{(m)}(x) \ , m-1 < \alpha < m \\ \dfrac{d^m}{dt^m} f(t), \quad \alpha = m \end{cases} \tag{3}$$

Hence, we have some properties for derivatives and integral operator of fractional

1- $J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x) \qquad \alpha,\beta \geq 0$
2- $J^\alpha D_*^\alpha f(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(0^+)\frac{x^k}{\Gamma(k+1)} \qquad , m-1 < \alpha \leq m$
3- $J^\alpha D_*^\beta f(x) = J^{\beta-\alpha} f(x) - \sum_{k=0}^{m-1} f^{(k)}(0^+)\frac{x^{k-\alpha+\beta}}{\Gamma(k-\alpha+\beta+1)} \qquad , m-1 < \beta, \alpha \leq m$ and $\beta < \alpha$
4- $J^\alpha x^n = \frac{\Gamma(n+1)}{\Gamma(\alpha+n+1)} x^{\alpha+n} \qquad , x > 0, m-1 < \alpha \leq m, n > -1$
5- $J^\alpha \sum_{i=0}^n a_i f(x) = \sum_{i=0}^n a_i J^\alpha f(x)$, a_i is a constant
6- $D_*^\alpha C = 0$, C is a constant
7- $D_*^\alpha x^n = \begin{cases} 0 & , n < m-1 \\ \frac{\Gamma(n+1)}{\Gamma(n+1-\alpha)} x^{n-\alpha} & , n \geq m-1 \end{cases}$
8- $D_*^\alpha \sum_{i=0}^n a_i f(x) = \sum_{i=0}^n a_i D_*^\alpha f(x)$, a_i is a constant

3. Analysis of ADM

We consider the differential equation in the form:

$$Ly(x) + Ry(x) + Ny(x) = g\,(x) \tag{4}$$

With initial value $y^{(i)}(0) = \alpha_i, i = 0,1,2,3,\dots,n$

Where L is and invertible linear operator,R is the remainder of the linear operator,N is a nonlinear operator

Using the inverse operatorL^{-1} to both sides of Eq. (4), we obtain

$$y = \varphi + L^{-1}g(x) - L^{-1}Ry(x) - L^{-1}Ny(x) \tag{5}$$

Where φ arises from the given initial condition .The ADM suggests the solution $y(x)$ be decomposed by the infinite series of components

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \tag{6}$$

And the nonlinear operator $N(y)$can be decomposed by an infinite series of polynomials given by

$$N(y) = \sum_{n=0}^{\infty} A_n \tag{7}$$

And A_n are the so-called polynomials of $y_0, y_1, \ldots, y_n$defined by

$$A_n(y_0, y_1, \ldots, y_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}[F(\sum_{i=0}^{n} \lambda^i y_i)]_{\lambda=0} \quad , n = 0,1,2, \ldots \tag{8}$$

Substituting relations (6) and (8) in the differential equation (5) and solve it. We provide function of$y_0(x), y_1(x), \ldots$ and$y_n(x)$

4. Using ADM on Multi-Term Fractional Differential Equations (M-FDEs)

We can present the multi –term fractional equation (1) by using the ADM

$$Ly(x) = \sum_{i=0}^{n} a_i(x) D_*^{\beta_i} y(x) + a_0(x)y(x) + N\left(x, y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)\right) + g(x) \tag{9}$$

Where L is the Caputo of fractional derivatives D_*^{α} ,with invers L^{-1} is defined the Riemann-Liouville fractional integral operator ofJ^{α}.Using the inverse operator on (9) and we get:

$$J^{\alpha} D_*^{\alpha} y(x) = J^{\alpha} \sum_{i=0}^{n} a_i(x) D_*^{\beta_i} y(x) + J^{\alpha} a_0(x)y(x) + J^{\alpha} N\left(x, y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)\right) + J^{\alpha} g(x)$$

$$\Rightarrow y(x) - \sum_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{\Gamma(k+1)} = J^{\alpha} \sum_{i=0}^{n} a_i(x) D_*^{\beta_i} y(x) + J^{\alpha} a_0(x)y(x) + J^{\alpha} N\left(x, y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)\right) + J^{\alpha} g(x)$$

$$\Rightarrow y(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{\Gamma(k+1)} + J^{\alpha} g(x) + J^{\alpha} a_0(x)y(x) + J^{\alpha} \sum_{i=0}^{n} a_i(x) D_*^{\beta_i} y(t) + J^{\alpha} N\left(x, y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)\right) \tag{10}$$

The decomposition method suggests that the solution $y(x)$ by the infinite series solution

$$y(x) = \sum_{j=0}^{\infty} y_j(x)$$

Substituting relations (7) in the differential equation (1)

$$\Rightarrow \sum_{j=0}^{\infty} y_j(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{\Gamma(k+1)} + J^{\alpha} g(x) + J^{\alpha} a_0(x) \sum_{j=0}^{\infty} y_j(x) + J^{\alpha} \sum_{i=0}^{n} a_i(x) D_*^{\beta_i} \sum_{j=0}^{\infty} y_j(x) + J^{\alpha} N\left(x, y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)\right) \tag{11}$$

And the nonlinear operator $N\left(x, y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)\right)$can be decomposed by an infinite series of polynomials given by

$$N\left(x, y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)\right) = \sum_{j=0}^{\infty} A_j\left(x, y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)\right) \tag{12}$$

And A_n are the Adomian polynomials of $x, y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)$defined by

$$A_n\left(y(x), D_*^{\beta_1}y(x), \ldots, D_*^{\beta_n}y(x)\right) = \frac{1}{n!}\frac{d^n}{d\lambda^n}[F(\sum_{i=0}^{n} \lambda^i y_i, \sum_{i=0}^{n} \lambda^i D_*^{\beta_1} y_i, \ldots, \sum_{i=0}^{n} \lambda^i D_*^{\beta_n} y_i)]_{\lambda=0} ,$$

$$n = 0,1,2, \ldots$$

We obtain the formulas of the first several Adomian polynomials for the variable simple analytic nonlinearity $N\left(x, y(x), D_*^{\beta_1} y(x), \ldots, D_*^{\beta_n} y(x)\right) = F(\sum_{i=0}^{n} \lambda^i y_i, \sum_{i=0}^{n} \lambda^i D_*^{\beta_1} y_i, \ldots, \sum_{i=0}^{n} \lambda^i D_*^{\beta_n} y_i)]$ from A_0 through A_2, inclusively, for convenient reference as:

$$A_0 = F\left(y_0, D_*^{\beta_1} y_0, \ldots, D_*^{\beta_n} y_0\right)$$

$$A_1 = \frac{d}{d\lambda} F\left(y_0 + \lambda y_1, D_*^{\beta_1} y_0 + \lambda D_*^{\beta_1} y_1, \ldots, D_*^{\beta_n} y_0 + \lambda D_*^{\beta_n} y_1\right)_{\lambda=0}$$

$$A_2 = \frac{1}{2}\frac{d^2}{d\lambda^2} F\left(y_0 + \lambda y_1 + \lambda^2 y_2, D_*^{\beta_1} y_0 + \lambda D_*^{\beta_1} y_1 + \lambda^2 D_*^{\beta_1} y_2, \ldots, D_*^{\beta_n} y_0 + \lambda D_*^{\beta_n} y_1 + \lambda^2 D_*^{\beta_n} y_2\right)$$

Substituting relations (3), (4) and (7) in the differential equation (9)

$$\Rightarrow \sum_{j=0}^{\infty} y_j(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+) \frac{x^k}{\Gamma(k+1)} + J^\alpha g(t) + \sum_{j=0}^{\infty} J^\alpha (a_0(x) y_j(x)) + \sum_{j=0}^{\infty} J^\alpha \sum_{i=0}^{n} a_i(x) D_*^{\beta_i} y_j(x) + J^\alpha \sum_{j=0}^{\infty} A_j$$

We define:

$$y_0(x) = \sum_{k=0}^{m-1} y^{(k)}(0^+) \frac{x^k}{\Gamma(k+1)} + J^\alpha g(x) \tag{13}$$

$$y_{k+1}(x) = J^\alpha (a_0(x) y_k(x)) + J^\alpha \sum_{i=0}^{n} a_i(x) D_*^{\beta_i} y_k + J^\alpha A_k \quad , k > 0 \tag{14}$$

5. Numerical Examples

Example 1: Consider the following fractional nonlinear equation

$$D_*^{\frac{3}{2}} y(x) + y(x) D_*^{\frac{1}{2}} y(x) = g(x) \tag{15}$$

Where

$$g(x) = \frac{\Gamma(4)}{\Gamma\left(\frac{7}{2}\right)} x^{\frac{11}{2}} + \frac{\Gamma(4)}{\Gamma\left(\frac{5}{2}\right)} x^{\frac{3}{2}}$$

With initial value $y(0) = y'(0) = 0$

The exact solution of is $y(x) = x^3$

Apply ADM for equation (15), we have $\alpha = \frac{3}{2} (1 < \alpha < 2), \beta = \frac{1}{2}$ and $N = y(x) D_*^{\frac{1}{2}} y(x)$, Substituting in equation (13), we obtain:

$$y_0(x) = J^{\frac{3}{2}} \sum_{k=0}^{1} y^{(k)}(0^+) \frac{x^k}{\Gamma(k+1)} + J^{\frac{3}{2}}(g(x))$$

$$y_{k+1}(x) = -J^{\frac{3}{2}} A_k \quad , k > 0$$

For $N = y(x) D_*^{\frac{1}{2}} y(x)$ in nonlinear equation (15), we have

$$A_0 = F\left(y_0, D_*^{\frac{1}{2}} y_0\right) = y_0(x) D_*^{\frac{1}{2}} y_0(x)$$

$$A_1 = \frac{d}{d\lambda} F\left(y_0 + \lambda y_1, D_*^{\frac{1}{2}} y_0 + \lambda D_*^{\frac{1}{2}} y_1\right)_{\lambda=0} = \frac{d}{d\lambda}\left[(y_0 + \lambda y_1)\left(D_*^{\frac{1}{2}} y_0 + \lambda D_*^{\frac{1}{2}} y_1\right)\right] = y_1 D_*^{\frac{1}{2}} y_0 + y_0 D_*^{\frac{1}{2}} y_1$$

We can obtain $A_2, A_3, \ldots$ with relation (12)

The equation (15) where non-linear term $y(x)D_*^{\frac{1}{2}}y(x)$ is introduced. Thisproblem is solving by Adomian decomposition method and shows a behavior of the numerical solutionfor step size h = 0.01.[see Table:1]

Table: 1

X	*Exact y(x)*	*ADM*	*error*
0	*0*	*0*	*0*
0.01	*1.000000000000000e-06*	*1.000000001019345e-06*	*-1.019345071472480e-15*
0.02	*8.000000000000001e-06*	*8.000000130476152e-06*	*-1.304761505137525e-13*
0.03	*2.700000000000000e-05*	*2.700000222930459e-05*	*-2.229304590863242e-12*
0.04	*6.400000000000001e-05*	*6.400001670087061e-05*	*-1.670087059911420e-11*

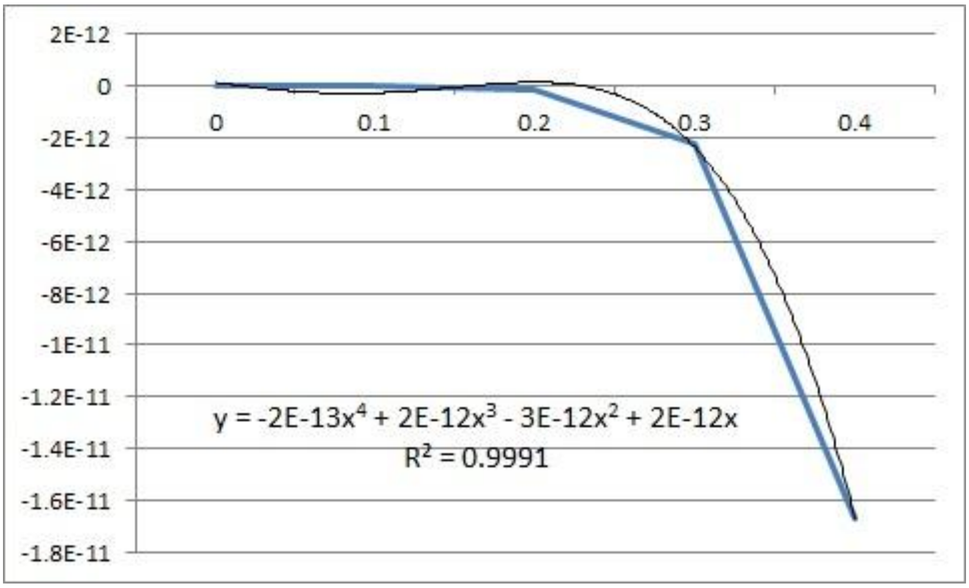


Figure5.1. *Show Error (where non-linear term* $y(x)D_*^{\frac{1}{2}}y(x)$*)*

Example2: consider the following initialvalue equations:

$$D_*^{2.5}y(x) + D_*^{1.5}y(x)D_*^{0.5}y(x) = g(x) \quad (16)$$

Where $g(x) = \frac{\Gamma(4)}{\Gamma(1.5)}x^{0.5} + (\frac{\Gamma(4)}{\Gamma(1.5)}\frac{\Gamma(4)}{\Gamma(3.5)})x^4$

With initial value $y(0) = 1, y'(0) = y''(0) = 0$

The exact solution of is $y(x) = x^3 + 1$

Using ADM on equation (16), where $2 < \alpha < 3$

$$y_0(x) = \sum_{k=0}^{2} y^{(k)}(0^+)\frac{x^k}{\Gamma(k+1)} + J^{2.5}(\frac{\Gamma(4)}{\Gamma(1.5)}x^{0.5} + (\frac{\Gamma(4)}{\Gamma(1.5)}\frac{\Gamma(4)}{\Gamma(3.5)})x^4)$$

$$y_0(x) = x + J^{2.5}(\frac{\Gamma(4)}{\Gamma(1.5)}x^{0.5} + (\frac{\Gamma(4)}{\Gamma(1.5)}\frac{\Gamma(4)}{\Gamma(3.5)})x^4)$$

The nonlinear operator $N\left(y(x), D_*^{1.5}y(x)\right) = D_*^{1.5}y(x)D_*^{0.5}y(x)$,we have

$$A_0 = A_0 = F(D_*^{1.5}y_0, D_*^{0.5}y_0) = D_*^{1.5}y_0, D_*^{0.5}y_0$$

$A_1 = \frac{d}{d\lambda}F(D_*^{0.5}y_0 + \lambda D_*^{0.5}y_1, D_*^{1.5}y_0 + \lambda D_*^{1.5}y_1)_{\lambda=0}$

$=\frac{d}{d\lambda}[(D_*^{0.5}y_0 + \lambda D_*^{0.5}y_1)(D_*^{1.5}y_0 + \lambda D_*^{1.5}y_1)] = D_*^{0.5}y_1 D_*^{1.5}y_0 + D_*^{0.5}y_0 D_*^{1.5}y_1$

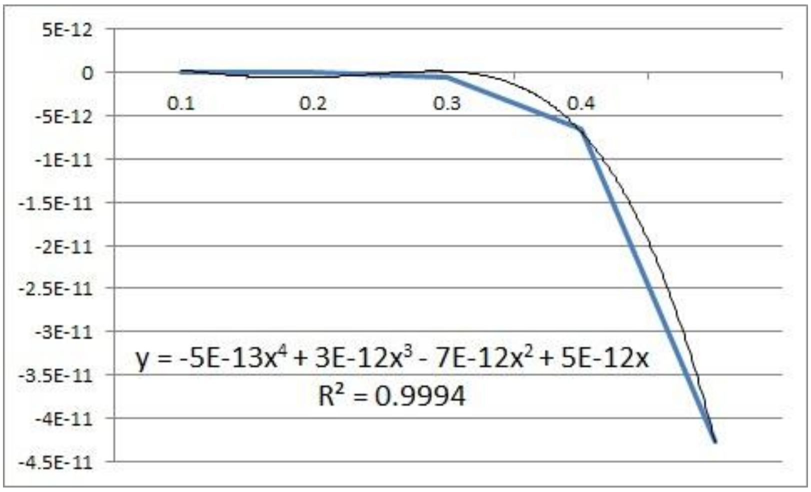
….

We define

$$y_0(x) = 1 + x^3 + (\frac{\Gamma(4)}{\Gamma(1.5)}\frac{\Gamma(4)}{\Gamma(3.5)}\frac{\Gamma(5)}{\Gamma(7.5)})x^{6.5} \quad , y_{k+1}(x) = -J^{2.5}A_k \quad , k \geq 0$$

Table2. *Show the analytic approximate solution for equation (16) where nonlinear is define* $D_*^{1.5}y(x)D_*^{0.5}y(x)$*and* $h = 0.1$

x	*Exact y(x)*	*ADM*	*Error*
0	*1*	*1*	*0*
0.1	*1.000001000000000*	*1.000001000000005*	*-5.329070518200751e-15*
0.2	*1.000008000000000*	*1.000008000000473*	*-4.729550084903167e-13*
0.3	*1.000027000000000*	*1.000027000006598*	*-6.598277479952230e-12*
0.4	*1.000064000000000*	*1.000064000042808*	*-4.280797938349679e-11*



Example 3. Consider the nonlinear equation

$$D_*^2 x(t) + D_*^{\alpha_2} x(t) + \left(D_*^{\alpha_1} x(t)\right)^2 + \left(x(t)\right)^3 = f(t) \quad (17)$$

Where

$$f(t) = 2t + \frac{2}{\Gamma(4-\alpha_2)} t^{3-\alpha_2} + \left(\frac{2}{\Gamma(4-\alpha_1)} t^{3-\alpha_1}\right)^2 + \left(\frac{1}{3}t^3\right)^3$$

With initial value $x(0) = 1, x'(0) = 0$

The exact solution of is $x(t) = \frac{1}{3}t^3$

Using ADM on equation (17), for$\alpha_1 = 0.555$ and$\alpha_2 = 1.455$.

$$x(t) = tx'(0) + x(0) + \int_0^t \int_0^t f(t)dtdt - \int_0^t \int_0^t D_*^{\alpha_2} x(t)dtdt - \int_0^t \int_0^t (\left(D_*^{\alpha_1} x(t)\right)^2 + \left(x(t)\right)^3)dtdt$$

The nonlinear operator $N\left(x(t), D_*^{\alpha_1} x(t)\right) = \left(D_*^{\alpha_1} x(t)\right)^2 + \left(x(t)\right)^3$, we have

$$A_0 = A_0 = F\left(x_0(t), D_*^{\alpha_1} x_0(t)\right) = \left(D_*^{\alpha_1} x_0(t)\right)^2 + \left(x_0(t)\right)^3$$

$A_1 = \frac{d}{d\lambda} F\left(x_0(t) + \lambda x_1(t), D_*^{\alpha_1} x_0(t) + \lambda D_*^{\alpha_1} x_1(t)\right)_{\lambda=0} = \frac{d}{d\lambda}\left[(\left(D_*^{\alpha_1} x_0(t) + \lambda D_*^{\alpha_1} x_1(t)\right)^2 + \right.$
x0t+λx1t3)=2D∗α1x0tD∗α1x1t+3x1tx0t2

….

We define

$$y_0=\frac{t^3}{3}+\frac{2t^{3.545}}{\Gamma(2.545)(2.545)(3.545)}+\frac{4t^{6.89}}{\Gamma(3.445)\Gamma(3.445)(5.89)(6.89)}+\frac{1}{270}\frac{t^{11}}{11}$$

$$y_{k+1}(x)=-\int_0^t\int_0^t A_k dtdt \quad ,k\geq 0$$

We found thefollowing results (see Table 3).and Error [1] show in paper [1]

Table: 3

t	Error (ADM)	Error [1]
0.1	5.6877e-06	0.009878278000
0.01	2.0721e-08	0.000250220300
0.001	4.3455e-12	0.000006325524

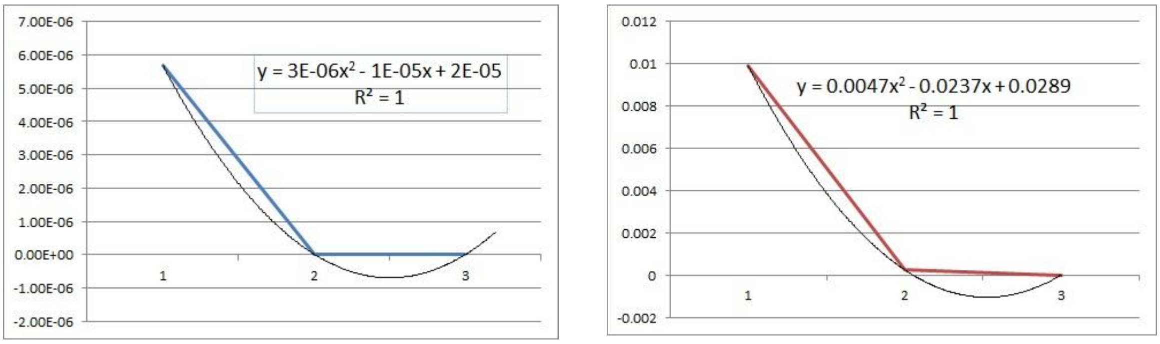


Figure 5.4. *Show Errors E [1] and E (ADM)*

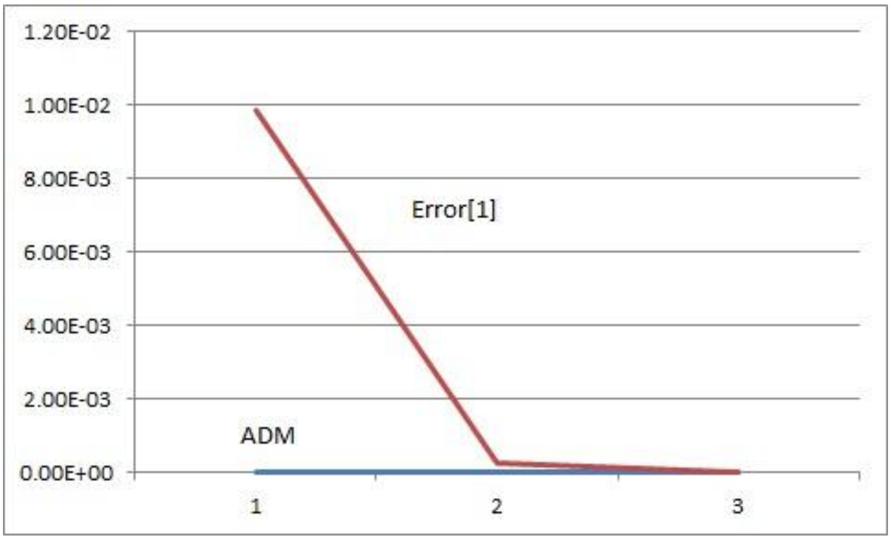


Figure5.4. *Show Error (compare of E (ADM) and E[1]*)

6. CONCLUSION

In this paper, the Adomian Decomposition method has been successfully applied to find the approximate solution of some multi – fractional differential equations for nonlinear function is of $x, y(x), D_*^{\beta_1}y(x), \dots, D_*^{\beta_n}y(x)$.In Example 3, Comparison of ADM's errors with other tools , in paper [1] shows that ADM provides better approximation. Numerical examples show that the Adomian series solution converges faster.

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