International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 3, Issue 5, May 2015, PP 41-48
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Conformal Euler-Lagrange Mechanical Equations on Contact 5-Manifolds

Zeki Kasap

Faculty of Education/ Elementary Mathematics Education,
Pamukkale University,
Denizli,Turkey.
zekikasap@hotmail.com

Abstract: *The differential geometry of surfaces captures many of the key ideas and techniques characteristic of this field. Contact geometry is the odd-dimensional analogue of symplectic geometry in differential geometry. It is close to symplectic geometry and like the latter it originated in questions of classical and analytical mechanics. Contact geometry has, as does symplectic geometry, broad applications in mathematical physics, geometrical optics, classical mechanics, analytical mechanics, mechanical systems, thermodynamics, geometric quantization and applied mathematics such as control theory. A conformal map is a function that preserves angles locally. Conformal mapping is extremely important in complex analysis, as well as in many areas of physics and engineering. The Lagrangian mechanics was time ago expressed in the language of symplectic geometr. It is well known that one way of solving problems in classical mechanics is with the help of the Euler-Lagrange equations. In this study, conformal Euler-Lagrange mechanical equations as representing the motion of the object, we found on contact 5-manifolds. Also, implicit solutions of the differential equations found in this study are solved by Maple computation program and implicit graphs were drawn for the special value of closed function.*

Keywords: *Contact Manifold, Mechanical System, Dynamic Equation, Lagrangian Formalism.*

1. Introduction

Differential geometry is a mathematical discipline such that uses the techniques of differential calculus, integral calculus, linear algebra and multilinear algebra to study problems in geometry. A dynamical system concept is mathematical or differential geometry formalization for any fixed rule which describes the time dependence of a point's position in its defined in space. At any given time a dynamical system has a state given by a set of real numbers (a vector) that can be represented by a point in an appropriate state space (a geometrical manifold). Also, dynamical systems theory is an area of mathematics used to describe the behavior of complex dynamical systems, usually by employing differential equations or difference equations. Contact geometry is the study of a geometric structure on smooth manifolds given by a hyperplane distribution in the tangent bundle and specified by a one-form. Contact geometry is in many ways an odd-dimensional counterpart of symplectic geometry. Both contact and symplectic geometry are motivated by the mathematical formalism of classical mechanics, where one can consider either the even-dimensional phase space of a mechanical system or the odd-dimensional extended phase space that includes the time variable. A conformal mapping, also called a conformal map, conformal transformation, angle-preserving transformation, or biholomorphic map, is a transformation. Conformal maps preserve both angles and the shapes of infinitesimally small figures. In this study, conformal Euler-Lagrange differential equations modeling of the dynamic system will be obtained on contact 5-manifolds. Differential geometry of contact 5-manifolds is determined by the action of on contact 5-manifolds structure.

Hamilton proved some results on symplectic structures on 4-dimensional manifolds and contact structures on 5-dimensional manifolds [1]. *Srivastava* et al introduced the concept of (ε)-almost paracontact manifolds, and in particular, of (ε)-para Sasakian manifolds [2]. *Cho* et al shown that (X, J) is symplectic if and only if the contact structure ξ_1 on ∂X_1 is isomorphic to the standard contact structure on the 3-sphere S^3 and ∂X_1 is J-concave [3]. *Eliashberg* et al examined that Lagrangian and Hamiltonian intersections in contact geometry [4]. *Piercey* examined certain canonical constructions of contact manifolds as well as various interactions between symplectic geometry and contact geometry [5] *Takamura* reviewed the method due to Gel'f and and Fuks to show the finite dimensionality of the cohomology ring of the Lie algebra of formal contact vector fields [6]. *Bellettini* examined almost complex structures J that satisfy, for any vector v in the horizontal distribution, $d\alpha(v,Jv)=0$ such that a contact manifold is (M^5,α) [7]. *Janssens* and *Vanhecke* submitted an orthogonal decomposition of the vector space of some curvature tensors on a co-Hermitian real vector space [8]. *Chaubey* introduced some geometrical properties of almost contact metric manifolds equipped with semi-symmetric non-metric connection [9]. *Kodama* classified the local structure of complex contact manifolds of dimension three with Legendrian vector fields [10]. *Piercey* proved contact manifolds and identify simple examples and basic properties [11]. *Doubrov* and *Komrakov* announced the complete classification of all real Lie algebras of contact vector fields on the first jet space of one-dimensional submanifolds in the plane [12]. *Attarchi* and *Rezaii* viewed that a comprehensive study of contact and Sasakian structures on the indicatrix bundle of Finslerian warped product manifolds is reconstructed [13]. *Etnyre* explored that any almost contact structure on a 5-manifold is homotopic to a contact structure [14]. Kasap and Tekkoyun introduced Lagrangian and Hamiltonian formalism for mechanical systems using para/pseudo-Kähler manifolds [15].

2. Preliminaries

Definition 1. A pseudo J-holomorphic curve is a smooth map from a Riemannian surface into an almost complex manifold such that satisfies the Cauchy-Riemann equation J [16].

Definition 2. Let M be a differentiable manifold of dimension (2n+1), and suppose J is a differentiable vector bundle isomorphism $J: TM\to TM$ such that $J_x: T_xM\to T_xM$ is a almost complex structure for T_xM, i.e. $J^2=J\circ J=-I$ where I is the identity (unit) operator on vector field V.

Definition 3. Symplectic geometry is a branch of differential geometry and differential topology that studies symplectic manifolds; differentiable manifolds equipped with a closed, nondegenerate 2-form. Symplectic geometry has its origins in the Lagrangian formulation of classical mechanics where the phase space of certain classical systems takes on the structure of a symplectic manifold.

Definition 4. Let V be a vector space. Let $\omega: V\times V\to\mathbb{R}$ be a skew-symmetric, bilinear 2-form, $\omega\in\Lambda^2V^*$. The form ω is nondegenerate if for every $v\in V$, $\omega(v,u)=0$, $\forall u\in V\Rightarrow v=0$. Note that since ω is skew-symmetric $\omega(v,v)= - \omega(v,v)$, hence $\omega(v,v)=0$.

Definition 5. A symplectic manifold is a smooth manifold (M) equipped with a closed nondegenerate differential 2-form (ω) called the symplectic form.

The study of symplectic manifolds is called symplectic geometry or symplectic topology. Symplectic manifolds arise naturally in abstract formulations of classical mechanics and analytical mechanics as the cotangent bundles of manifolds, e.g., in the Lagrangian formulation of classical mechanics, which provides one of the major motivations for the field.

Theorem 1. Every symplectic manifold (M,ω) of dimension 2n is locally symplectomorphic to an open subset of $(\mathbb{R}^{2n},\omega_0)$ and $\omega_0=\sum_{i=1}^{n}dx_i\wedge dy_i$.

Example 1. An almost complex symplectic manifold is standard Euclidean space $(\mathbb{R}^{2n},\omega_0)$ with its standard almost complex structure J_0 obtained from the usual identification with $\mathbb{C}^n$. Thus, one sets $z_j=x_{2j-1}+ix_{2j}$ for j=1,...,n and defines J_0 by $J_0(\partial_{2j-1})=\partial_{2j}$, $J_0(\partial_{2j})=-\partial_{2j-1}$, where $\partial_j=\partial/\partial x_j$ is the standard basis of $T_x\mathbb{R}^{2n}$ [16].

Definition 6. Let M be a manifold of odd dimension (2n+1). A contact structure is a maximally non-integrable hyperplane field $\xi=\ker\alpha\subset TM$, that is, the defining 1--form α is required to satisfy $\alpha\wedge(d\alpha)^n\neq0$. Such a 1-form α is called a contact form. The pair (M,ξ) is called a contact manifold.

Theorem 2. Let M be a 5-dimensional manifold endowed with a contact form and let J be an almost-complex structure defined on the horizontal distribution $H=\mathrm{Ker}\alpha$, such that $d\alpha(Jv,v)=0$ for any $v\in H$.

Proposition 1. Given a contact 5-manifold (M,α), there exist almost complex structures J such that $d\alpha(Jv,v)=0$ for all horizontal vectors v.

3. CONFORMAL GEOMETRY

The approach for studying conformal field theories is somewhat different from the usual approach for quantum and electromagnetic field theories.

Definition 7. A conformal map or transformations is a function which preserves angles.
Conformal geometry is the study of the set of angle-preserving (conformal) transformations on a space.

Theorem 3. A conformal manifold is a differentiable manifold equipped with an equivalence class of Riemann metric tensors, in which two metrics g_1 and g_2 are equivalent if and only if

$$g_2=\Psi^2 g_1, \tag{1}$$

where $\Psi>0$ is a smooth positive function. An equivalence class of such metrics is known as a conformal metric or conformal class [17].

Theorem 4. A conformal transformation is a change of coordinates $\sigma^\alpha \to \tilde{\sigma}(\alpha)$ such that the metric changes by

$$g_{\alpha\beta}(\sigma)\to\Omega^2(\sigma)\, g_{\alpha\beta}(\sigma). \tag{2}$$

A conformal field theory (CFT) is a field theory which is invariant under these transformations. Transformation of the form (2) has a different interpretation depending on whether we are considering a fixed background metric $g_{\alpha\beta}$, or a dynamical background metric. When the background is fixed, physical symmetry, taking the point σ^α to point $\tilde{\sigma}(\alpha)$.

4. COMPLEX STRUCTURES ON CONTACT 5-MANIFOLDS

Theorem 5. Assume that, on a contact 5-manifold (M,α), given a horizontal 2-form ω is given, that satisfies $\omega\wedge d\alpha=0$ and $\omega\wedge\omega\neq 0$.

Here, it should be understood ω is horizontal. Decompose $\omega=\omega_+ + \omega_-$, where ω_+ is the self-dual part and ω_- is the anti-self-dual part and $\omega\wedge d\alpha=\omega_+\wedge d\alpha+\omega_-\wedge d\alpha$. The notation $\|\ \|$ denotes here the standard norm for differential forms coming from the metric on the manifold and $\tilde{\omega}_+ = \sqrt{2}\,\omega_+/\|\omega_+\|$. We can choose an orthonormal basis for $P\in M$ of the form $\{e_1=X, e_2=IX, e_3=Y, e_4=IY\}$ and denote by $\{e^1, e^2, e^3, e^4\}$ the dual basis of orthonormal one-forms. Then $d\alpha$ has the form $e^1\wedge e^2+e^3\wedge e^4$. The forms $e^1\wedge e^2+e^3\wedge e^4$, $e^1\wedge e^3+e^4\wedge e^2$ and $e^1\wedge e^4+e^2\wedge e^3$ are an orthonormal basis for $\wedge_+^2$. The fact that ω_+ is orthogonal to $d\alpha$ implies that $\omega_+=a(e^1\wedge e^3+e^4\wedge e^2)+b(e^1\wedge e^4+e^2\wedge e^3)$ and $\|\omega_+\|^2=2(a^2+b^2)$, therefore $\tilde{\omega}_+=\cos\theta(e^1\wedge e^3+e^4\wedge e^2)+\sin\theta(e^1\wedge e^2+e^3\wedge e^4)$ for some θ depending on the chosen point,

$$\cos\theta=\frac{a}{\sqrt{a^2+b^2}},\ \sin\theta=\frac{b}{\sqrt{a^2+b^2}}. \tag{3}$$

Example 2. Then the explicit expression J are, any point $v\in p$, there exist local coordinates (x_i,y_j,θ); $i,j=1,2,3,4$ centered at p,

$$J(e_1)=\cos\theta e_3+\sin\theta e_4,\ J(e_2)=-\cos\theta e_4+\sin\theta e_3,$$

$$J(e_3)=-\cos\theta e_1-\sin\theta e_2,\ J(e_4)=\cos\theta e_2-\sin\theta e_1. \tag{4}$$

and an easy computation shows that $d\alpha(v,J(v))=0$ for any $v\in P$. The above structures (4) have been taken from [7].

Definition 8. In three dimensions, the vector from the origin to the point with cartesian coordinates (x,y,z) can be written as [18]:

$$r=xi+yj+zk=x\left(\frac{\partial}{\partial x}\right)+y\left(\frac{\partial}{\partial y}\right)+z\left(\frac{\partial}{\partial z}\right). \tag{5}$$

Proposition 2. Let $e_i=\partial/(\partial x_i)$ be a base on M. J denote conformal to the structure coefficient $\Omega=\Omega(x_1,x_2,x_3,x_4)$,using **Theorem 1** and **2**;

$$J(\partial/(\partial x_1))=\cos\theta\,\Omega^2(\partial/(\partial x_3))+\sin\theta\,\Omega^2(\partial/(\partial x_4)),$$

$$J(\partial/(\partial x_2))=-\cos\theta\,\Omega^2(\partial/(\partial x_4))+\sin\theta\,\Omega^2(\partial/(\partial x_3)),$$

$$J(\partial/(\partial x_3))=-\cos\theta\,\Omega^{-2}(\partial/(\partial x_1))-\sin\theta\,\Omega^{-2}(\partial/(\partial x_2)),$$

$$J(\partial/(\partial x_4))=\cos\theta\Omega^{-2}(\partial/(\partial x_2))-\sin\theta\Omega^{-2}(\partial/(\partial x_1)). \quad (6)$$

Proof : We examine the holomorphic property at (6).

$J^2(\partial/(\partial x_1))=\cos\theta\Omega^2 J(\partial/(\partial x_3))+\sin\theta\Omega^2 J(\partial/(\partial x_4))$

$=\cos\theta\Omega^2 J(-\cos\theta\Omega^{-2}(\partial/(\partial x_1))-\sin\theta\Omega^{-2}(\partial/(\partial x_2))+\sin\theta\Omega^2(\cos\theta\Omega^{-2}(\partial/(\partial x_2))-\sin\theta\Omega^{-2}(\partial/(\partial x_1))$

$$=-\cos\theta\cos\theta(\partial/(\partial x_1))-\cos\theta\sin\theta(\partial/(\partial x_2))+\sin\theta\cos\theta(\partial/(\partial x_2))-\sin\theta\sin\theta(\partial/(\partial x_1))=-(\partial/(\partial x_1)), \quad (7)$$

and similar manner it is shown that

$$J^2(\partial/(\partial x_i))=-\partial/(\partial x_i),\ i=1,2,3,4. \quad (8)$$

As seen above; holomorphic structures $(J^2\partial/(\partial x_i))=-\partial/\partial x_i$ or $J^2=-I$) are complex.

5. Lagrange Dynamics Equations

Theorem 6. The closed 2-form on a vector field and 1-form reduction function on the phase space defined of a mechanical system is equal to the differential of the energy function 1-form of the Lagrangian and the Hamiltonian mechanical systems [19, 20].

Definition 9 [21, 22]. Let M be an n-dimensional manifold and TM its tangent bundle with canonical projection τ_M: TM→M. TM is called the phase space of velocities of the base manifold M. Let L: TM→R be a differentiable function on TM called the Lagrangian function. Here, L=T-V such that T is the kinetic energy and V is the potential energy of a mechanical system. In the problem of a mass on the end of a spring, $T=m\dot{x}^2/2$ and $V=kx^2/2$, so we have $L=m\dot{x}^2/2-(kx^2)/2$. We consider the closed 2-form and base space (J) on TM given by $\Phi_L=-d(\mathbf{d}_J L)=-d(J(\mathbf{d}))$. Consider the equation

$$i_\xi\Phi_L=dE_L. \quad (9)$$

Where i_ξ is reduction function and $i_\xi\Phi_L=\Phi_L(\xi)$ is defined in the form. Then ξ is a vector field, we shall see that (9) under a certain condition on ξ is the intrinsically expression of the Euler-Lagrange equations of motion. This equation (9) is named as Lagrange dynamical equation.

Definition 10. We shall see that for motion in a potential, $E_L=VL-L$ is an energy function and $V=J\xi$ a Liouville vector field. Here dE_L denotes the differential of E. The triple (TM,Φ_L,ξ) is known as Lagrangian system on the tangent bundle TM. If it is continued the operations on (9) for any coordinate system then infinite dimension Lagrange's equation is obtained the form below. The equations of motion in Lagrangian mechanics are the Lagrange equations of the second kind, also known as the Euler-Lagrange equations;

$$\partial/(\partial t)((\partial L)/(\partial x))=(\partial L)/(\partial x). \quad (10)$$

Definition 11. We have $(\partial L)/(\partial x)=mx$ and $(\partial L)/(\partial x)=-kx$, so eq. (10) gives $m\ddot{x}=-kx$ which is exactly the result obtained by using F=ma at Newton's second law for the mechanical problem. The Euler-Lagrange equation, eq. (10), gives $m\ddot{x}=-dV/dx$. In a three-dimensional setup written in terms of cartesian coordinates, the potential takes the form V(x,y,z), so the Lagrangian is $L=m(\dot{x}^2+\dot{y}^2+\dot{z}^2)/2-V(x,y,z)$. So, the three Euler-Lagrange equations may be combined into the vector statement $m\ddot{x}=-\nabla V$.

6. Euler-Lagrangian Mechanical Equations

We can be obtained, using **Theorem 3,4,6**, Euler-Lagrange eq9uations for classical and analytical mechanics on contact 5-manifold and its shown that by (TM,g,J). Let (x_i,y_j,θ); i,j=1,2,3,4 be coordinate functions.

Proposition 4. Let ξ be the vector field characterized by

$$\xi=\sum_{i=1}^{4}\left(X^i\otimes\frac{\partial}{\partial x_i}\right),\quad X^i=\dot{x}_i. \quad (11)$$

on (TM,g,J). Then the vector field defined by

$$J(\xi)=J\left(\sum_{i=1}^{4}(X^i\frac{\partial}{\partial x_i})\right)=\sum_{i=1}^{4}X^iJ\left(\frac{\partial}{\partial x_i}\right)$$

$$=X^1\left(\cos\theta\Omega^2\left(\frac{\partial}{\partial x_3}\right)+\sin\theta\Omega^2\left(\frac{\partial}{\partial x_4}\right)\right)+X^2\left(-\cos\theta\Omega^2\left(\frac{\partial}{\partial x_4}\right)+\sin\theta\Omega^2\left(\frac{\partial}{\partial x_3}\right)\right)$$

$$+X^3\left(-\cos\theta\Omega^{-2}\left(\frac{\partial}{\partial x_1}\right)-\sin\theta\Omega^{-2}\left(\frac{\partial}{\partial x_2}\right)\right)+X^4\left(\cos\theta\Omega^{-2}\left(\frac{\partial}{\partial x_2}\right)-\sin\theta\Omega^{-2}\left(\frac{\partial}{\partial x_1}\right)\right), \quad (12)$$

is thought to be Liouville vector field on contact 5-manifold (TM,g,J). $\Phi_L=-d(\mathbf{d}_J L)=-d(J(\mathbf{d}))$ is the closed 2-form given by (9) such that $d=\sum_{i=1}^{4}\left(\frac{\partial}{\partial x_i}dx_i\right)$, $d_J:F(M)\to\wedge^1 M$ and $d_J=i_J d-di_J$,

$$d_J=J(d)=\sum_{i=1}^{4}J\left(\frac{\partial}{\partial x_i}\right)dx_i \quad (13)$$

Also, the vertical differentiation $\mathbf{d}_J$ is given by d is the usual exterior derivation. Then, there is the following result. Here, we can be account Euler-Lagrange equations for classical and analytical mechanics on contact 5-manifold (TM,g,J). We get the equations given by

$$d_J=\left(\cos\theta\Omega^2\left(\frac{\partial}{\partial x_3}\right)+\sin\theta\Omega^2\left(\frac{\partial}{\partial x_4}\right)\right)dx_1+\left(-\cos\theta\Omega^2\left(\frac{\partial}{\partial x_4}\right)+\sin\theta\Omega^2\left(\frac{\partial}{\partial x_3}\right)\right)dx_2$$

$$+\left(-\cos\theta\Omega^{-2}\left(\frac{\partial}{\partial x_1}\right)-\sin\theta\Omega^{-2}\left(\frac{\partial}{\partial x_2}\right)\right)dx_3+\left(\cos\theta\Omega^{-2}\left(\frac{\partial}{\partial x_2}\right)-\sin\theta\Omega^{-2}\left(\frac{\partial}{\partial x_1}\right)\right)dx_4. \quad (14)$$

Let, we account Φ_L

$\Phi_L=-d(\mathbf{d}_J L)$

$$=\sum_{i=1}^{4}\cos\theta\Omega^2\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)+\cos\theta 2\Omega\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_3}\right)+\sin\theta\Omega^2\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)$$

$$+\sin\theta 2\Omega\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_4}\right)dx_1\wedge dx_i$$

$$+\sum_{i=1}^{4}\left(-\cos\theta\Omega^2\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-\cos\theta 2\Omega\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_4}\right)+\sin\theta\Omega^2\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)\right.$$

$$\left.+\sin\theta 2\Omega\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_3}\right)\right)dx_2\wedge dx_i$$

$$+\sum_{i=1}^{4}\left(-\cos\theta\Omega^{-2}\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)+\cos\theta 2\Omega^{-3}\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_1}\right)-\sin\theta\Omega^{-2}\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)\right.$$

$$\left.+\sin\theta 2\Omega^{-3}\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_2}\right)\right)dx_3\wedge dx_i$$

$$+\sum_{i=1}^{4}\left(\cos\theta\Omega^{-2}\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)-\cos\theta 2\Omega^{-3}\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_2}\right)-\sin\theta\Omega^{-2}\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)\right.$$

$$\left.+\sin\theta 2\Omega^{-3}\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_1}\right)\right)dx_4\wedge dx_i, \quad (15)$$

and then we find using

(a) $(f\wedge g)(v)=f(v)g-g(v)f$, (b) $(dx_i\wedge dx_j)\left(\frac{\partial}{\partial x_k}\right)=dx_i\left(\frac{\partial}{\partial x_k}\right)dx_j-dx_j\left(\frac{\partial}{\partial x_k}\right)dx_i=\left(\frac{\partial x_i}{\partial x_k}\right)dx_j-\left(\frac{\partial x_j}{\partial x_k}\right)dx_i$, (c) $\frac{\partial x_i}{\partial x_k}=\delta_{ik}$,

So,

$$i_\xi\Phi_L=\Phi_L(\xi)=\Phi_L\left(\sum_{i=1}^{4}X^i\frac{\partial}{\partial x_i}\right)$$

$$=-\sum_{i=1}^{4}X^i\left[\left(\cos\theta\Omega^2\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)+\cos\theta 2\Omega\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_3}\right)+\sin\theta\Omega^2\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)\right.\right.$$

$$\left.+\sin\theta 2\Omega\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_4}\right)\right)dx_1+\left(-\cos\theta\Omega^2\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-\cos\theta 2\Omega\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_4}\right)\right.$$

$$\left.+\sin\theta\Omega^2\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)+\sin\theta 2\Omega\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_3}\right)\right)dx_2+\left(-\cos\theta\Omega^{-2}\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)\right.$$

$$\left.+\cos\theta 2\Omega^{-3}\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_1}\right)-\sin\theta\Omega^{-2}\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)+\sin\theta 2\Omega^{-3}\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_2}\right)\right)dx_3$$

$$+\left(\cos\theta\Omega^{-2}\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)-\cos\theta 2\Omega^{-3}\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_2}\right)-\sin\theta\Omega^{-2}\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)\right.$$

$$\left.\left.+\sin\theta 2\Omega^{-3}\left(\frac{\partial\Omega}{\partial x_i}\right)\left(\frac{\partial L}{\partial x_1}\right)\right)dx_4\right]. \quad (16)$$

Also, the energy function of system is

$$E_L=J(\xi)-L=X^1\left(\cos\theta\Omega^2\left(\frac{\partial L}{\partial x_3}\right)+\sin\theta\Omega^2\left(\frac{\partial L}{\partial x_4}\right)\right)+X^2\left(-\cos\theta\Omega^2\left(\frac{\partial L}{\partial x_4}\right)\right.$$

$$\left.+\sin\theta\Omega^2\left(\frac{\partial L}{\partial x_3}\right)\right)+X^3\left(-\cos\theta\Omega^{-2}\left(\frac{\partial L}{\partial x_1}\right)-\sin\theta\Omega^{-2}\left(\frac{\partial L}{\partial x_2}\right)\right)$$

$$+X^4(\cos\theta\Omega^{-2}((\partial L)/(\partial x_2))-\sin\theta\Omega^{-2}((\partial L)/(\partial x_1)))-L, \quad (17)$$

and the differential of E_L is

$$dE_L=\sum_{i=1}^{4}([X^1(\cos\theta\Omega^2((\partial^2L)/(\partial x_3\partial x_i))+\cos\theta 2\Omega((\partial\Omega)/(\partial x_i))((\partial L)/(\partial x_3))+\sin\theta\Omega^2((\partial^2L)/(\partial x_4\partial x_i))$$
$$+\sin\theta 2\Omega((\partial\Omega)/(\partial x_i))((\partial L)/(\partial x_4)))dx_i]+[X^2(-\cos\theta\Omega^2((\partial^2L)/(\partial x_4\partial x_i))-\cos\theta 2\Omega((\partial\Omega)/(\partial x_i))((\partial L)/(\partial x_4))$$
$$+\sin\theta\Omega^2((\partial^2L)/(\partial x_3\partial x_i))+\sin\theta 2\Omega((\partial\Omega)/(\partial x_i))((\partial L)/(\partial x_3)))dx_i]+[X^3(-\cos\theta\Omega^{-2}((\partial^2L)/(\partial x_1\partial x_i))$$
$$+\cos\theta 2\Omega^{-3}((\partial\Omega)/(\partial x_i))((\partial L)/(\partial x_1))-\sin\theta\Omega^{-2}((\partial^2L)/(\partial x_2\partial x_i))+\sin\theta 2\Omega^{-3}((\partial\Omega)/(\partial x_i))((\partial L)/(\partial x_2)))dx_i]$$
$$+[X^4(\cos\theta\Omega^{-2}((\partial^2L)/(\partial x_2\partial x_i))-\cos\theta 2\Omega^{-3}((\partial\Omega)/(\partial x_i))((\partial L)/(\partial x_2))-\sin\theta\Omega^{-2}((\partial^2L)/(\partial x_1\partial x_i))$$
$$+\sin\theta 2\Omega^{-3}((\partial\Omega)/(\partial x_i))((\partial L)/(\partial x_1)))dx_i-((\partial L)/(\partial x_i))dx_i]). \quad (18)$$

Using $i_\xi\Phi_L=dE_L$ (9), we get first equations as follows:

$$-X^1(\cos\theta\Omega^2((\partial^2L)/(\partial x_3\partial x_1))+\cos\theta 2\Omega((\partial\Omega)/(\partial x_1))((\partial L)/(\partial x_3))+\sin\theta\Omega^2((\partial^2L)/(\partial x_4\partial x_1))$$
$$+\sin\theta 2\Omega((\partial\Omega)/(\partial x_1))((\partial L)/(\partial x_4)))dx_1-X^2(\cos\theta\Omega^2((\partial^2L)/(\partial x_4\partial x_1))+\cos\theta 2\Omega((\partial\Omega)/(\partial x_1))((\partial L)/(\partial x_4))$$
$$+\sin\theta\Omega^2((\partial^2L)/(\partial x_3\partial x_1))+\sin\theta 2\Omega((\partial\Omega)/(\partial x_1))((\partial L)/(\partial x_3)))dx_1-X^3(\cos\theta\Omega^{-2}((\partial^2L)/(\partial x_1\partial x_1))$$
$$-\cos\theta 2\Omega^{-3}((\partial\Omega)/(\partial x_1))((\partial L)/(\partial x_1))+\sin\theta\Omega^{-2}((\partial^2L)/(\partial x_2\partial x_1))-\sin\theta 2\Omega^{-3}((\partial\Omega)/(\partial x_1))((\partial L)/(\partial x_2)))dx_1$$
$$-X^4(\cos\theta\Omega^{-2}((\partial^2L)/(\partial x_2\partial x_1))-\cos\theta 2\Omega^{-3}((\partial\Omega)/(\partial x_1))((\partial L)/(\partial x_2))+\sin\theta\Omega^{-2}((\partial^2L)/(\partial x_1\partial x_1))$$
$$-\sin\theta 2\Omega^{-3}((\partial\Omega)/(\partial x_1))((\partial L)/(\partial x_1)))dx_1=-((\partial L)/(\partial x_1))dx_1, \quad (19)$$

or

$$-\cos\theta[X^1(\partial/(\partial x_1))+X^2(\partial/(\partial x_2))+X^3(\partial/(\partial x_3))+X^4(\partial/(\partial x_4))](\Omega^2((\partial L)/(\partial x_3)))$$
$$-\sin\theta[X^1(\partial/(\partial x_1))+X^2(\partial/(\partial x_2))+X^3(\partial/(\partial x_3))+X^4(\partial/(\partial x_4))](\Omega^2((\partial L)/(\partial x_4)))+((\partial L)/(\partial x_1))=0, \quad (20)$$

and

$$-\cos\theta\xi(\Omega^2((\partial L)/(\partial x_3)))-\sin\theta\xi(\Omega^2((\partial L)/(\partial x_4)))+((\partial L)/(\partial x_1))=0. \quad (21)$$

If we take of the curve α, for all equations, as an integral curve of ξ such that it is $\xi(\alpha)=(\partial/(\partial t))(\alpha)$. We, as similar operations performed at (19), find the following equations:

$$\text{dif1. } -\cos\theta(\partial/(\partial t))(\Omega^2((\partial L)/(\partial x_3)))-\sin\theta(\partial/(\partial t))(\Omega^2((\partial L)/(\partial x_4)))+((\partial L)/(\partial x_1))=0,$$
$$\text{dif2. } \cos\theta(\partial/(\partial t))(\Omega^2((\partial L)/(\partial x_4)))-\sin\theta(\partial/(\partial t))(\Omega^2((\partial L)/(\partial x_3)))+((\partial L)/(\partial x_2))=0,$$
$$\text{dif3. } \cos\theta(\partial/(\partial t))(\Omega^{-2}((\partial L)/(\partial x_1)))+\sin\theta(\partial/(\partial t))(\Omega^{-2}((\partial L)/(\partial x_2)))+((\partial L)/(\partial x_3))=0,$$
$$\text{dif4.} -\cos\theta(\partial/(\partial t))(\Omega^{-2}((\partial L)/(\partial x_2)))+\sin\theta(\partial/(\partial t))(\Omega^{-2}((\partial L)/(\partial x_1)))+((\partial L)/(\partial x_4))=0, \quad (22)$$

such that the differential equations (22) are named conformal Euler-Lagrange mechanical equations on contact 5-manifold such that this is shown in the form of (TM,g,J). Additionally, therefore the triple (TM,Φ_L,ξ) is called a conformal Euler-Lagrangian mechanical system on (TM,g,J).

6. Equations Solving with Computer

The location of each object in space represented by three dimensions in physical space. Three-dimensional space is a geometric three-parameter model of the physical universe in which all known matter exists. These three dimensions can be labeled by a combination of three chosen from the terms length, width, height, depth, mass, density and breadth. Any three directions can be chosen, provided that they do not all lie in the same plane. So, each vector represents the speed and direction of the movement of air at that point. We can solve these equations system (22) using Maple computation software. The number of dimensions of the equation (23) will be reduced to three and behind the graphics will be drawn. First, implicit function at (23) will be selected as a special. After, the figure of the equation (23) has been drawn for the route of the movement of objects in the electromagnetic field.

The solution (22) system found according to the specific value of θ and graph will be drawn. (22) are partial differential equations on contact 5-manifolds.

(1) $L(x_1,x_2,x_3,x_4,t)=(((x_1*c_2+x_2*c_4)*t^3+(-c_1*x3+x_4*c_3)*t-x_4*c_4+x3*c_2)*\cos(t)$

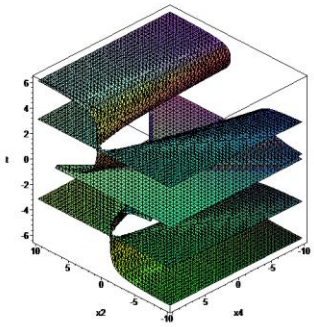
$+((x_1*c_1+c_3*x_2)*t^3+(-x_4*c_4+x_3*c_2)*t+c_1*x_3-x_4*c_3)*\sin(t)+F_1(t)*t^2)/t^2$, for $\theta=0$ and $\Omega=t$.

(2) $L(x_1,x_2,x_3,x_4,t)=F_1(t)+\exp(-i*t)*F_2((x_2*\Omega^2+x_4*i)/\Omega^2,(x_1*\Omega^2-i*x3)/\Omega^2)$

$+\exp(t*i)*F_3((x_2*\Omega^2-i*x_4)/\Omega^2,(x_1*\Omega^2+x_3*i)/\Omega^2)$, $i^2=-1$, for $\theta=\pi$ and $\Omega=t$. (23)

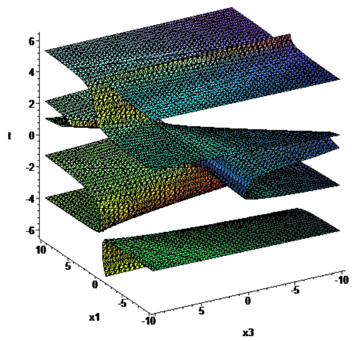
Example 3. Implicit solutions (23) obtained with a special selection of the closed function as follows:

(1) $L(x_1,x_2,x_3,x_4,t)=(\cos(t)*t*x_4+(t^3*x_2-x_4)*\sin(t)+t^3)/t^2$,



(24)

(2) $L(x_1,x_2,x_3,x_4,t)=((x_1*t^3+x_3*t-x_3)*\cos(t)+(x_1*t^3-x_3*t-x_3)*\sin(t)+t^3)/t^2$.



(25)

7. CONCLUSION

A classical field theory explains the study of how one or more physical fields interact with matter which is used quantum and classical mechanics of physics branches. Also, the classical theory of electromagnetism deals with electric and magnetic fields and their interaction with each other and with charges and currents. An electromagnetic field is a physical field produced by electrically charged objects. How the movement of objects in electrical, magnetically and gravitational fields force is very important. For example, on a weather map, the surface wind velocity is defined by assigning a vector to each point on a map. So, said that each vector represents the speed and direction of the movement of air at this point. In this study, the conformal Euler-Lagrange mechanical equations system (22) derived on a generalized on contact 5-manifold may be suggested to deal with problems in electrical, magnetically and gravitational fields for the path of movement (24), (25) of defined space moving objects [23, 24].

REFERENCES

[1]. Bellettini M., On symplectic 4-manifolds and contact 5-manifolds, Dissertation an der Fakultat fur Mathematik, Informatik und Statistik der Ludwig-Maximilians-Universitat Munchen, (2008).

[2]. Srivastava S.Kr., Narain D. and Srivastava K., Properties of ε-paracontact manifold, VSRD-TNTJ, Vol. 2 (11), 559-569, (2011).

[3]. Cho M.S. and Cho Y.S., Note on contact structure and symplectic structure, Bull. Korean Math. Soc. 37, No. 1, 181-189, (2000).
[4]. Eliashberg Y., Hofery H. and Salamonz D., Lagrangian intersections in contact geometry, Stanford University Ruhr-Universitat Bochumy Mathematics Institutez University of Warwick Coventry CV4 7AL Great Britain, (1994).
[5]. Piercey V.I., Contact geometry, University of Arizona, Math 537B, 1-16, (2008).
[6]. Takamura M., The relative cohomology of formal contact vector fields with respect to formal poisson vector fields, J. Math. Soc. Japan, Vol. 60, No. 1, 117-125, (2008).
[7]. Bellettini C., Almost complex structures and calibrated integral cycles in contact 5-manifolds, Advances in Calculus of Variations, Vol. 6, Issue 3, 339-374, (2013).
[8]. Janssens D. and Vanhecke L., Almost contact structures and curvature tensors, Kodai Math. J., 4, 1-27, (1981).
[9]. Chaubey S.K., Almost contact metric manifolds admitting semi-symmetric non-metric connection, Bulletin of Mathematical Analysis and Applications, 3 (2), 252-260, (2011).
[10]. Kodama H., Complex contact three manifolds with Legendrian vector fields, Proc. Japan Acad. Ser. A. Math. Sci. Vol. 78, No 4, 51-54, (2002).
[11]. Piercey V.I., Contact geometry, University of Arizona, (2008).
[12]. Doubrov B.M. and Komrakov B.P., Contact Lie algebras of vector fields on the plane, geometry & topology, Vol. 3, 1-20, (1999).
[13]. Attarchi H. and Rezaii M.M., Contact structure on the indicatrix bundle of finslerian warped product manifolds, http://arxiv.org/abs/1106.2663v1, (2011).
[14]. Etnyre J.B., Contact Structures on 5-Manifolds, http://arxiv.org/abs/1210.5208, (2013).
[15]. Kasap Z. and Tekkoyun M., Mechanical systems on almost para/pseudo-Kahler Weyl manifolds, IJGMMP, Vol. 10, No. 5, 1350008-1350015. (2013).
[16]. Dusa McD. and Salamon D., J-holomorphic curves and quantum cohomology, (1995).
[17]. http://en.wikipedia.org/wiki/Conformal_geometry, (2015).
[18]. Griffiths D.J., Introduction to electrodynamics. Prentice Hall. ISBN 0-13-805326-X, (1999).
[19]. Klein J., Escapes variationnels et mécanique, Ann. Inst. Fourier, Grenoble, 12, (1962).
[20]. Ibord A., The geometry of dynamics, Extracta Mathematicable, 11 (1), 80-105, (1996).
[21]. Leon M.de and Rodrigues P.R., Methods of differential geometry in analytical mechanics, North-Hol. Math. St., 152, Elsevier Sc. Pub. Com. Inc., Amsterdam, 263-397, (1989).
[22]. Abraham R., Marsden J.E. and Ratiu T., Manifolds tensor analysis and applications, Springer, 483-542, (2001).
[23] Weyl H., Space-time-matter, Dover Publ. 1922.Translated from the 4th German edition by H. Brose. London: Methuen. Reprint New York, Dover (1952).
[24]. Thidé B., Electromagnetic field theory, (2012).

AUTHOR'S BIOGRAPHY

Dr. Zeki Kasap: He is working as a Faculty of Education in Pamukkale University, Kinikli Campus, Denizli/TURKEY. His research arears of interest are Analytical Mechanics, Dynamical Systems, Equations of Motion of Objects in Space, Differential Geometry, Mathematical Physics and Mathematics Education. His papers more than 12 were published in various reputed international journals. He presented six papers in International Conferences.