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Numerical Investigation of Heat Transfer in a Thin Liquid Film Flowing Down a Vertical Isothermal Surface

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Abstract: *Wavy falling films have been identified as an important aspect of absorption refrigeration systems. Much mass and heat transfer rates have been observed in such films. In this paper the energy equation for a laminar wavy film flowing over a vertical isothermal plate was solved numerically by the finite difference method making advantage of a previously available result for the hydrodynamic model from the same authors for Reynolds Numbers from 25 to 500. The results show that waves enhance the heat transfer by as much as 30% compared to the smooth film. This is due to the normal convective flux resulting from the transverse velocity component. The results for film heat heating is the same as for film cooling. This model is successful in predicting the periodic- and intermediate-wave regime characteristics.*

Keywords: *laminar, wavy, film, heat, thin film*

1. INTRODUCTION

Thin films flowing down vertical surfaces have been extensively studied because of their common occurrence in a variety of engineering applications. The transport properties typical of thin-film flows are especially suited to applications in industrial process equipment. Recently the absorption heat pumps and chillers have received considerable attention due to their low electricity consumption. The efficient heat- and mass-transfer characteristics of the film are primarily the result of the thinness of the film and are further enhanced by the presence of waves on the liquid-vapor interface. Gravity is the driving force which creates the film flow. The theoretical and numerical description of falling liquid films is still a real challenge. Existing models are either limited in their generality or need enormous computational resources.

Wavy motion in a falling liquid film has been investigated experimentally (Emmert and Pigford, [1]; Oliver and Atherinos, [2]; Yih and Seagrave, [3] and numerically by Yang, [4]; Patnaik and Blanco, [5]; Hantsch and Gross [6] and by a hybrid analytical-numerical method by Habib, [7].

Theoretical studies on the combined heat and mass transfer on smooth (waveless) film absorption have been reported by (Rotem and Neilson, [8]; Berhente and Ruckenstein, [9]; Shair, [10]; Nakoryakov et al., [11]; Anderberg and Vliet, [12]; Goff and Ramadance, [13]; Yang and Wood, [14]; Habib and wood, [15]. More recently experimental studies in heat transfer in two-dimensional falling film with uniform heat flux were performed by Haustein, [16]. All indicated that the film waves have a strong effect on the transfer rates. Theoretical studies of heat and mass transfer in a wavy film are rare in the literature. The difficulties are due to the coupled momentum, heat and mass transfer under conditions of the wavy motion. Yang et al. [17]; Patnaik [18]. Islama et al., [19] studied the heat and mass transfer in a wavy film absorption process.

Hirshburg and Florshuetz, [20] developed a wavy-film heat transfer model that included condensation. Faghri et al., [21] solved the energy equation assuming a simple sine wave profile. It is worth noting that Nusselt, [22] was the first to analyze laminar film condensation on an isothermal inclined surface by predicting the liquid film resistance. The objective of this work is to introduce a better heat transfer model using a more realistic wave profile developed previously by Habib et al., [7].

2. Analysis

To develop the mathematical model, consider a thin laminar wavy film flowing over a vertical isothermal plate as shown in Fig. 1. The following assumptions were used.

(1) The liquid properties are constant.

(2) Diffusion is neglected in the flow direction.

(3) Specified temperature at the wall and at the bulk of the surrounding vapor.

Under these assumptions the energy equation can be written as

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{1}$$

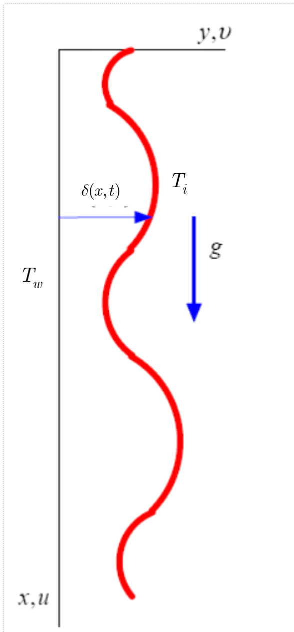

Fig1. *Coordinate System for the Wavy Film*

To solve the energy equation, a hydrodynamic solution is required a priori. Faghri [21] gives the following velocity profiles

$$u(x,y,t) = \frac{3}{2}V_o(x,t)\left[2y - \frac{y^2}{\delta_o(x,t)}\right]\left[\left(\frac{c}{V_o}-1\right)\phi - \left(\frac{c}{V_o}-1\right)\phi^2\right] \tag{2}$$

$$v = -3V_o\delta_o\frac{\partial\phi}{\partial x}\left\{-\left(\frac{c}{V_o}-1\right)(1-2\phi)(1+\phi) + \left[1+\left(\frac{c}{V_o}-1\right)\phi - \left(\frac{c}{V_o}-1\right)\phi^2\right]\left(\frac{y^2}{2}-\frac{y^3}{3}\right)\right\} \tag{3}$$

Where c is the wave speed and $V_o(x,t)$ is the cross-sectional mean velocity defined as

$$V_o(x,t) = \frac{1}{\delta_o}\int_0^{\delta_o} u(x,y,t)\,dy \tag{4}$$

And δ and ϕ respectively are the local film thickness and the dimensionless surface deflection as expressed by Habib [7] are

$$\delta(\xi) = \delta_o(1 + \phi(\xi)) \tag{5}$$

$$\phi(\xi) = \sum_{n=1}^{N} A_n \sin(2\pi n\xi) + B_n \cos(2\pi n\xi) \tag{6}$$

Some of the wave profiles are shown in Fig. 2, Habib [7]

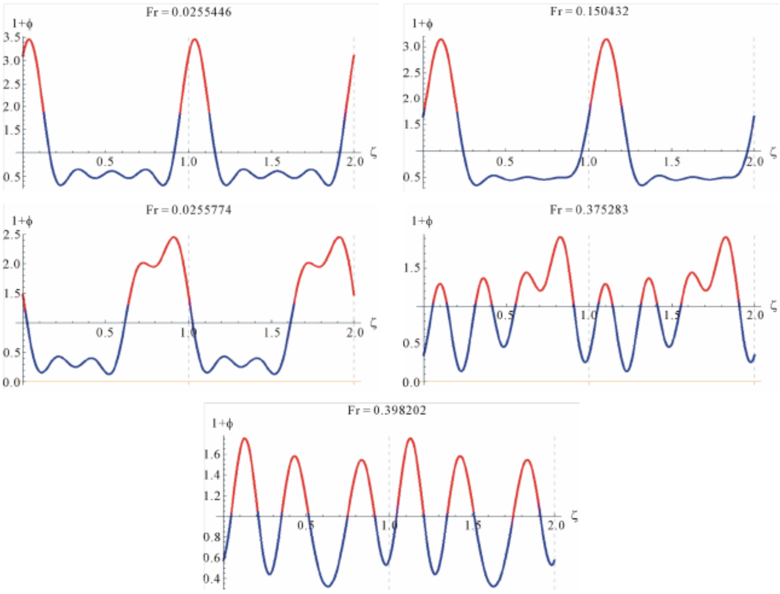

Fig2. *The wave profile for water (Reynolds number, Re = 25), (From Habib [7]).*

The main difficulty in seeking solution for Eq. [1] is that, in physical space at $y=\delta$, i. e., the boundary of computational domain is changing with space and time. To overcome this moving boundary difficulty, recall that the definition $\eta = y / \delta(x,t)$ converts the domain (x,y,t) with ripple interface, to a domain (x,η,t) with flat interface at the free surface, $\eta = 1$.

Substituting $\eta = y / \delta(x,t)$ into Eq. [2] and [3] respectively gives.

$$u = \frac{3}{2}V_o[2\eta - \eta^2]\left[1 + \left(\frac{c}{V_o} - 1\right)\phi - \left(\frac{c}{V_o} - 1\right)\phi^2\right] \tag{7}$$

$$\begin{aligned} \upsilon = &- 3V_o\delta_o\frac{\partial\phi}{\partial x}\left\{-\left(\frac{c}{V_o} - 1\right)(1 - 2\phi)(1 + \phi)\left(\frac{\eta^2}{2} - \frac{\eta^3}{6}\right) + \right. \\ &\left.\left[1 + \left(\frac{c}{V_o} - 1\right)\phi - \left(\frac{c}{V_o} - 1\right)\phi^2\right]\left[\left(\frac{\eta^2}{2} - \frac{\eta^3}{3}\right)\right]\right\} \end{aligned} \tag{8}$$

For closure, Eq. [1] requires an initial condition. To do this, recall that for periodic wave states, there exists a permanent wave transformation variable $\xi = x - ct$ which reduces Eq. [1] to

$$A_1\frac{\partial T}{\partial\xi} + A_2\frac{\partial T}{\partial\eta} = A_3\frac{\partial^2 T}{\partial\eta^2} + A_4\frac{\partial^2 T}{\partial\eta\partial\xi} \tag{9}$$

Where

$$\begin{aligned} A_1 = &- 2\frac{c\pi}{\lambda} - 3\frac{\pi\eta^2}{\lambda}\left[V_o + (c - V_o)\phi - (c - V_o)\phi^2\right] + \\ &6\frac{\pi\eta}{\lambda}\left[V_o + (c - V_o)\phi - (c - V_o)\phi^2\right] \end{aligned}$$

$$A_2 = \frac{\eta}{(1+\phi)}\frac{\partial \phi}{\partial t} - 2\frac{\eta\alpha}{(1+\phi)^2}\left(\frac{\partial \phi}{\partial x}\right)^2 + \frac{\eta\alpha}{(1+\phi)}\left(\frac{\partial^2 \phi}{\partial x^2}\right)$$

$$-\frac{\eta^2}{2}\Big[(1+\phi)^2 - 3c + 12V_o c\eta(1+\phi) - 6V_o\eta(1+\phi) +$$

$$\left\{9c - 6c\eta(1+\phi) + 2V_o\eta(1+\phi)\right\}\phi + 3(c-V_o)\left[3+\eta(1+\phi)\right]\phi^2 - 3(c-V_o)\phi^3]\Big]\frac{\partial \phi}{\partial x}$$

$$A_3 = \frac{\alpha}{(1+\phi)^2} + \frac{\eta^2\alpha}{(1+\phi)^2}\left(\frac{\partial \phi}{\partial x}\right)^2$$

$$A_4 = -\frac{4\alpha\pi\eta}{\lambda}\frac{\frac{\partial \phi}{\partial x}}{(1+\phi)} \tag{10}$$

In the evaporation problem of water, δ_o, the average film thickness over the wavelength will not vary substantially with distance x. The boundary conditions are

Specified temperature at the wall $\quad T(\xi,0)=1$

Specified temperature at the free surface $\quad T(\xi,0)=0$

Periodic boundary conditions in the streamwise direction $T(0,\eta)=T(2\pi,\eta)$

The transformed energy equation [9] is discretized by a finite difference method. The mesh size used is (60×60) with equal spacing in both directions. Convergence of the solution was achieved when the maximum absolute value did not exceed prescribed limit of 10^{-8}.

3. Results and Discussions

Figure 3-a shows the isotherms for smooth film (waveless) compared to the isotherms in wavy film (Fig 3-b) for film Reynold number equals 50.

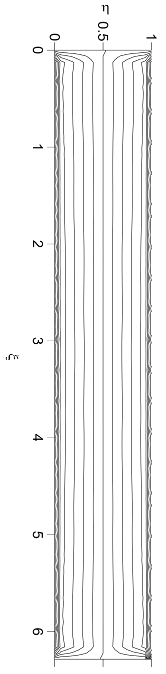

Fig3-a. *Isotherms for Waveless case, Re=50*

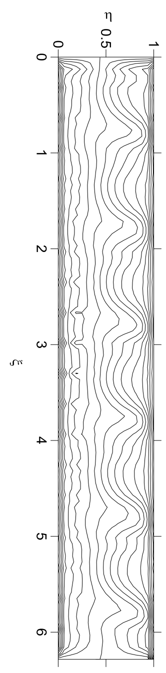

Fig3-b. *Isotherms for Wavy case, Re=50*

Fig3. *Isotherms for smooth and wavy film for Re=50*

As observed from Fig. 3-b the waves enhance the heat transfer as they penetrate the film. The normal convective flux resulting from the transverse velocity component is responsible for this enhancement. More isotherms are shown in Fig. 4 for Reynolds numbers form 25 to 500.

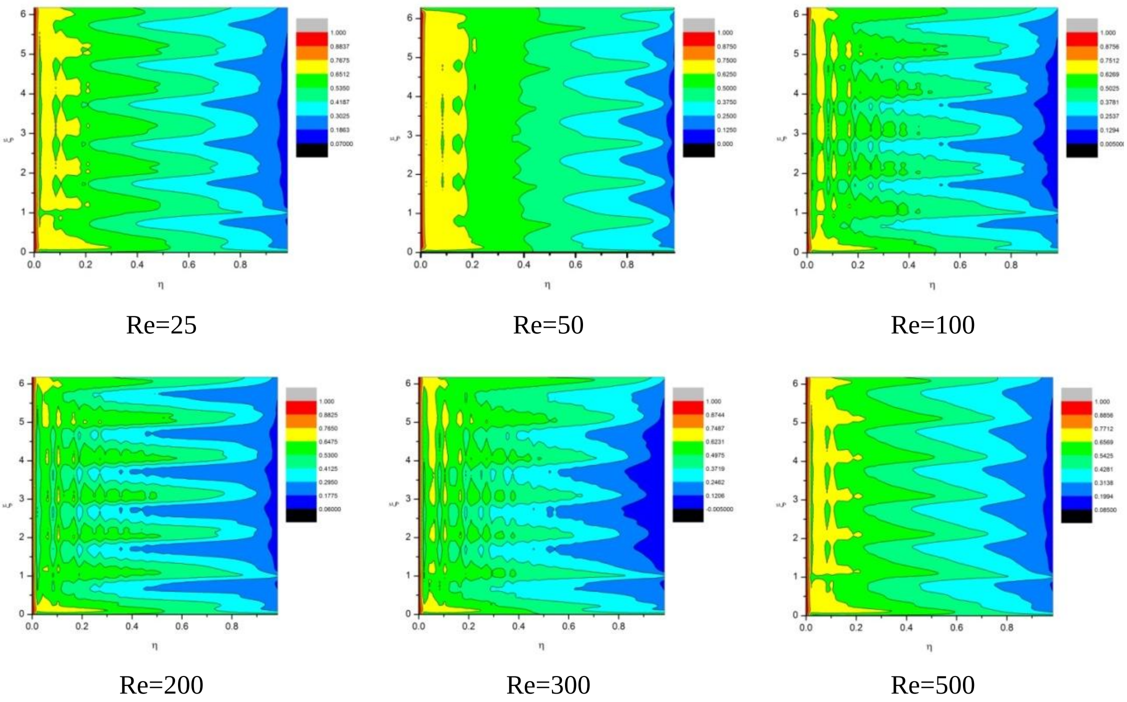

Fig4. *Isotherms at different Reynolds numbers*

To study the effect of film heating and cooling, two representative cases were studied for water at Reynold number = 25. In the first case the wall temperature was kept at a normalized isothermal temperature of 1 while the free surface was kept at a normalized temperature of 0. In the second case the situation was reversed, the free surface was kept at a normalized isothermal temperature of 1 while the wall temperature was kept at 0. Figure 5-a and Fig. 5-b show the temperature contour levels within the film for the first and second case, respectively.

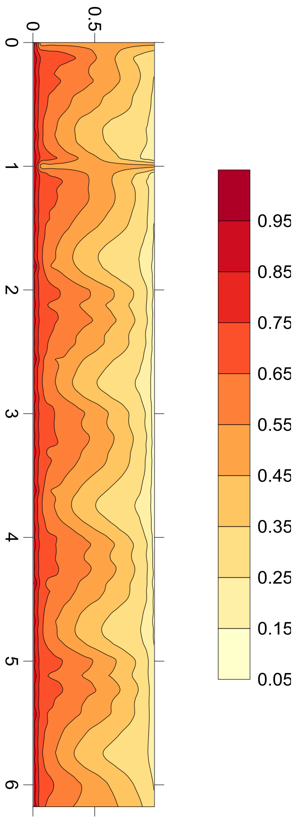

Fig5-a. *Isotherms for the Wavy Film for the first case, Re=25*

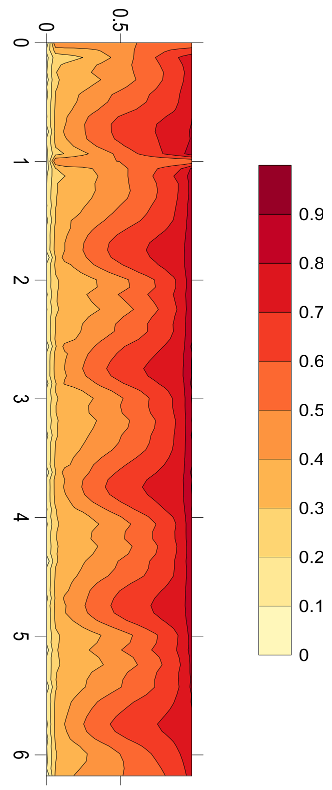

Fig5-b. *Isotherms for the Wavy Film for the second case, Re=25*

Fig5. *Isotherms for the Wavy Film for film heating and film cooling*

The local rate of heat transfer can be determined by knowledge of the temperature difference between the bulk vapor and the isothermal wall and the thermal resistance between theses points. The magnitudes of local heat flux at the wall and at the free surface are calculated respectively from

$$q_o = -\frac{k}{\delta}\frac{\partial T}{\partial \eta}\bigg|_o \quad h_o = \frac{q_o}{T_o - T_i} = \frac{q_o}{1} \tag{11}$$

$$q_i = \frac{k}{\delta}\frac{\partial T}{\partial \eta}\bigg|_i \left[1 + \left(\frac{\partial \delta}{\partial x}\right)^2\right]^{-1/2} \tag{12}$$

Figure 6-a, b show a comparison of the local Nusselt number along the wall and the free surface respectively compared to the local Nusselt Number for a waveless or smooth film at Re=25 for the first case.

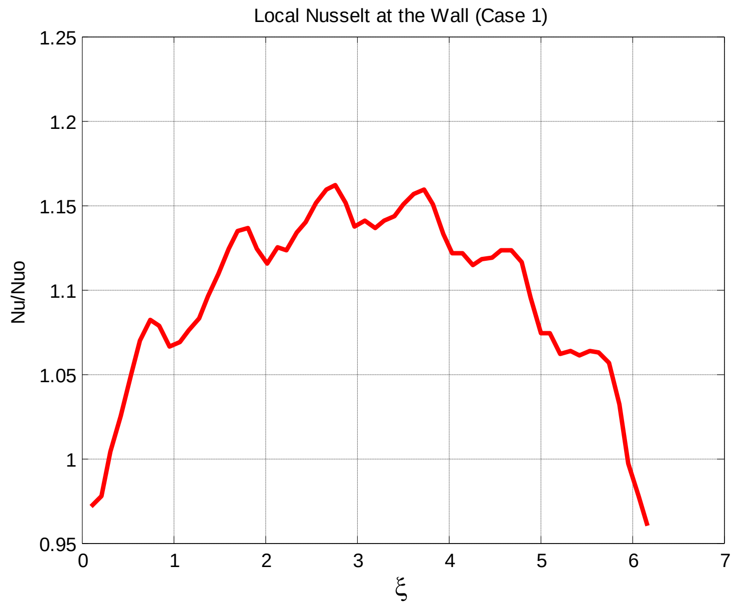

Fig6-a. *Comparison of the Local Nusselt Number at the wall for the Wavy Film for the first case for* Re=25.

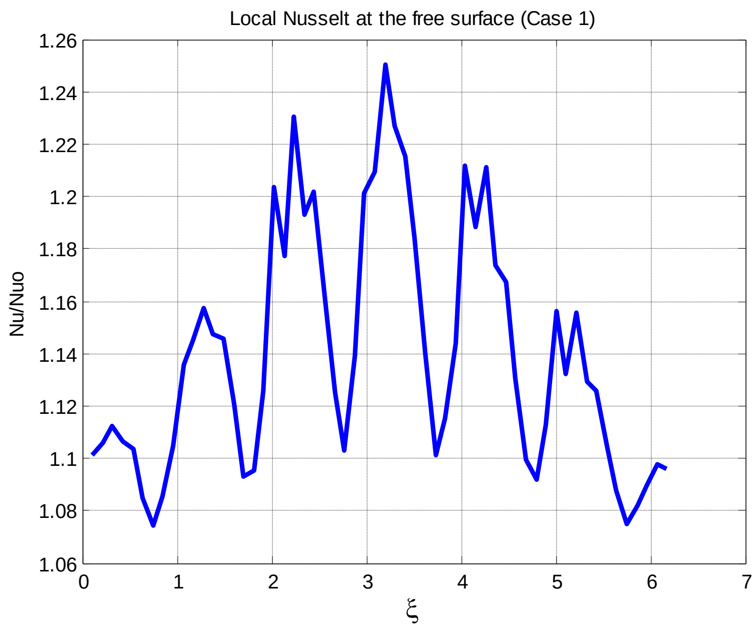

Fig6-b. *Comparison of the Local Nusselt Number at the free surface for the Wavy Film for the first case for Re=25.*

Figure 7-a, b show a comparison of the local Nusselt number along the wall and the free surface respectively compared to the local Nusselt Number for a waveless or smooth film at Re=25 for the second case.

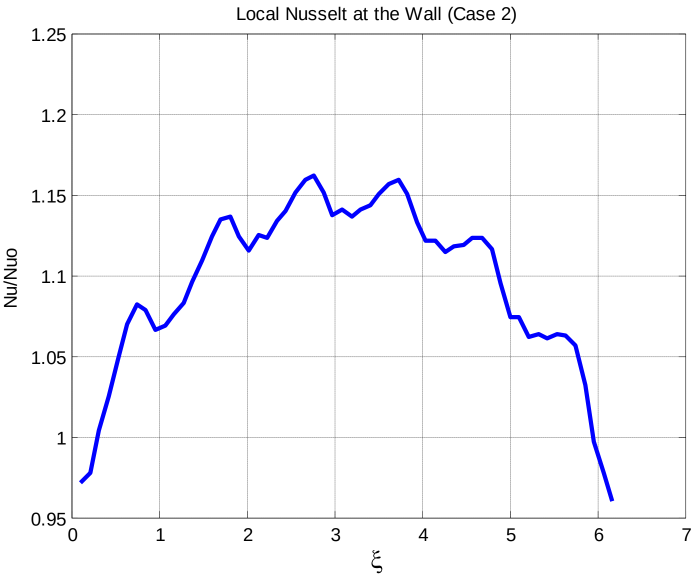

Fig7-a. *Comparison of the Local Nusselt Number at the wall for the Wavy Film for the second case for Re=25.*

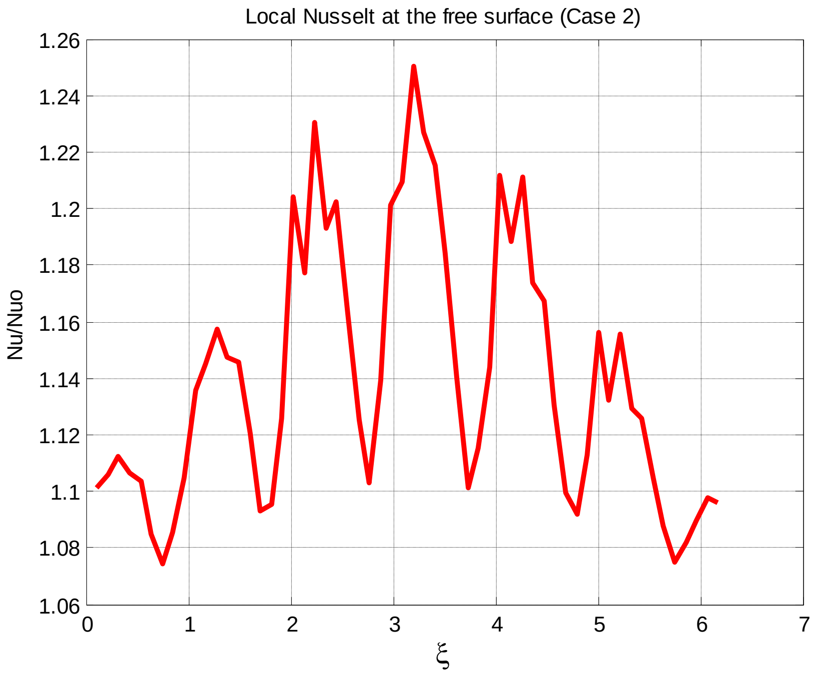

Fig7-b. *Comparison of the Local Nusselt Number at the free surface for the Wavy Film for the second case for Re=25.*

It is worth noting that the results for the local Nusselt numbers for the wall and the free surface for the first case are similar to the results for the free surface and the wall in the second case. The difference is the direction of heat transfer and, subsequently the direction of the net flux of the molecules at the liquid-vapor interface.

To calculate the average Nusselt number, as the wave passes a particular location on the flow surface, the local heat transfer rate will vary according to the local film thickness. The time average value of this local heat transfer rate is equal to the average heat transfer rate over a wavelength of the film. For small amplitude and large wavelengths the average flux for the surface $\eta = 1$ is calculated from

$$\frac{\overline{q}_i}{k} = \frac{1}{\lambda}\int_0^{\lambda} \frac{1}{\delta}\frac{\partial T}{\partial \eta}\bigg|_i \left[1 + \left(\frac{\partial \delta}{\partial x}\right)^2\right]^{-1/2} d\xi \qquad (13)$$

And for the wall it is

$$\frac{\overline{q}_o}{k} = \frac{1}{\lambda}\int_0^{\lambda} \frac{1}{\delta}\frac{\partial T}{\partial \eta}\bigg|_o d\xi \qquad (14)$$

Consider the characteristics of an idealized film at a particular location on the flow surface by assuming the dominance of cross-film conduction. The ratio of the local Nusselt number considering wave effects to that of the Nusselt smooth-film theory is simply

$$\frac{Nu}{Nu_N} = \frac{\delta_N}{\lambda}\int_0^{\lambda} \frac{d\xi}{\delta(\xi)} \qquad (15)$$

The local film thickness is given by Eq. 5 consequently, the integration in (15) was performed numerically.

Figure 8 shows a comparison of the Nusselt for wavy film with Nusselt smooth film at different values of Reynold number. The results show that waves enhance the heat transfer by as much as 30% compared to the smooth film. The periodic solution essentially yields the smooth-film result just below a Reynolds number of 20. From the previous discussions, it is apparent that the heat transfer model incorporating the results of the hydrodynamic model is quite successful in predicting local results. However, the application of this heat transfer model must be restricted to situations in which the hydrodynamic model satisfactorily predicts the hydrodynamic characteristics of the film. This model is successful in predicting the periodic- and intermediate-wave regime characteristics. As the Reynolds number is increased, the periodic wave region shortens until, at high Reynolds numbers, it is virtually nonexistent. The smooth-film entry length increases with larger Reynolds numbers. It is expected that the model predictions will not be successful for higher Reynolds numbers because of the influence of wave speed and profile.

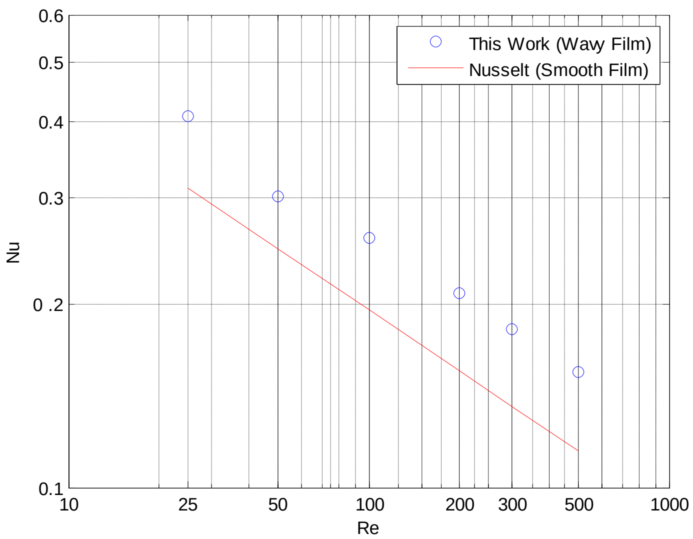

Fig8. *Comparison of the Nusselt numbers for wavy film with Nusselt smooth film, (Nusselt, [22]), at different values of Reynold number.*

4. Conclusion

In this paper a theoretical model for heat transfer is investigated for Reynolds Numbers from 25 to 500. The results show that waves enhance the heat transfer by as much as 30% compared to the smooth film. This is due to the normal convective flux resulting from the transverse velocity. The results for film heating is the same as for film cooling. The difference is in the direction of heat flux only. This model is successful in predicting the periodic- and intermediate-wave regime characteristics. The application of this heat transfer model must be restricted to situations in which the hydrodynamic model satisfactorily predicts the hydrodynamic characteristics of the film. The numerical algorithm developed for the model accelerated the solution. In conclusion the theoretical and numerical description of falling liquid will remain a real challenge

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Nomenclature

c	= wave speed (m/s)
Nu	= Film Nusselt Number
Re	= Reynolds number = $4\Gamma_o / \nu$
t	= time(s)
T	= Temperature, $^\circ C$
u	= x-direction velocity (m/s)
v	= y-direction velocity (m/s)
V_o	= x-direction mean velocity over film thickness
x	= coordinate parallel to the wall
y	= coordinate normal to the wall
α	= thermal diffusivity
δ	= local film thickness (m)
δ_o	= mean film thickness over a wavelength (m)
η	= $y / \delta(x,t)$
λ	= wavelength (m)
ν	= kinematic viscosity (m^2/s)
ξ	= $(x - ct) / \lambda$
ϕ	= dimensionless free surface deflection

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