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Shifting Properties of Finite Sine Hyperbolic Transforms

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Abstract: *In this paper we have introduced the new concept of finite hyperbolic transforms. Transform of some standard functions are obtained and some properties are proved.*

Keywords: *Generalized Transform, Finite transform, Finite hyperbolic transform, Transform of some standard functions.*

1. Introduction

If the disturbance is $f(t)= e^{at^2}$, for $a> 0$, the usual Laplace transform cannot be used to find the solution of an initial value problem because Laplace transform of f does not exist. It is often true that the solution at times later than t would not affect the state at time t. This leads to define Finite Laplace transform.

The finite Laplace transform of a continuous or an almost piecewise continuous function f in $(0,T)$ is denoted by $L_T (f(t)) = F(p,T)$, and is defined as

$$L_T (f(t)) = F(p,T) = \int_0^T f(t)\, e^{-pt}\, dt \tag{1.1}$$

Where p is a real or complex number and T be a finite number which may be positive or negative.

Note: Above definition is defined for any bounded interval $(-T_1, T_2)$.

Finite Laplace transform motivate us to define Finite Sine Hyperbolic transform and RAM Finite Cosine Hyperbolic transform in $0 \le t \le T$ in order to extend the power and usefulness of usual Laplace transform in $0 \le t < \infty$. section 2 devotes some preliminaries containing some definitions and properties of finite sine hyperbolic transform In section 3.1 shifting properties of Finite Sine Hyperbolic Transform are obtained and In section 3.2 examples are given.

2. Preliminaries and Definitions

Definition 2.1 [1]: Let $p \in C$ and T be a finite number which may be positive or negative and f is a continuous or an almost piecewise continuous function defined over the interval $(0,T)$. Then RAM Finite Sine Hyperbolic transform of f is denoted by $R_{sh} (f(t)) = F_S(p,T)$, and defined by

$$R_{sh}(f(t)) = F_S(p,T)= \int_0^T \sinh(pt)\, f(t)\, dt$$

Where $\sinh(pt)$ is a Kernel of R_{sh}.

Here R_{sh} is called RAM Finite Sine Hyperbolic transformation operator.

Definition 2.2 [1]: Let $p \in C$ and T be a finite number which may be positive or negative and f is a continuous or an almost piecewise continuous function defined over the interval $(0,T)$. Then RAM Finite Cosine Hyperbolic transform of f is denoted by

$R_{ch}(f(t)) = F_C(p,T)$, and defined by

$$R_{ch}(f(t)) = F_C(p,T) = \int_0^T \cosh(pt)\, f(t)\, dt$$

where $\cosh(pt)$ is a Kernel of R_{ch}.

Here R_{ch} is called RAM Finite Cosine Hyperbolic transformation operator.

Note : sinht, cosh$t$ are bounded for any bounded interval $(-T_1,T_2)$.

Theorem 2.3 [1] If f is a piecewise continuous and absolutely integrable function on $(0,T)$, then $R_{sh}(f(t))$ exists.

Theorem 2.4 [1] If f is a piecewise continuous and absolutely integrable function on $(0, T)$, then $R_{ch}(f(t))$ exists.

Theorem 2.5 [1] If $f(t)$ is a piecewise continuous and bounded function on $(0, T)$, then $R_{sh}(f(t))$ exists.

Theorem 2.6 [1] If f is a piecewise continuous and bounded function on $(0, T)$, then $R_{ch}(f(t))$ exists.

2.1. RAM Finite Sine Hyperbolic Transform of Some Standard Functions [1]

1. $R_{sh}(1) = \dfrac{\cosh(pT)-1}{p}$

2. $R_{sh}(t) = \dfrac{T\cosh(pT)}{p} - \dfrac{\sinh(pT)}{p^2}$

3. $R_{sh}(t^2) = \dfrac{T^2\cosh(pT)}{p} - \dfrac{2T\sinh(pT)}{p^2} + \dfrac{(2\cosh(pT)-2)}{p^3}$

4. $R_{sh}(t^k) = \begin{cases} \dfrac{T^k\cosh(pT)}{p} - \dfrac{kT^{k-1}\sinh(pT)}{p^2} + ... + \dfrac{k!(-1)^k[\cosh(pT)-1]}{p^k}, \text{if } k \, is \, even \\ \dfrac{T^k\cosh(pT)}{p} - \dfrac{kT^{k-1}\sinh(pT)}{p^2} + ... + \dfrac{k!(-1)^k\sinh(pT)}{p^k}, \text{if } k \, is \, odd \end{cases}$

5. $R_{sh}(\sin(at)) = \left(\dfrac{-a}{p^2+a^2}\right)\sinh(pT)\cos(aT) + \left(\dfrac{p}{p^2+a^2}\right)\cosh(pT)\sin(aT)$.

6. $R_{sh}(\cos(at)) = \left(\dfrac{a}{p^2+a^2}\right)\sinh(pT).\sin(aT) + \left(\dfrac{p}{p^2+a^2}\right)[\cosh(pT).\cos(aT)-1]$.

7. $R_{sh}(e^{at}) = \left(\dfrac{-a}{p^2-a^2}\right)\sinh(pT).e^{aT} + \left(\dfrac{p}{p^2-a^2}\right)[\cosh(pT).e^{aT}-1]$, provided $p^2 \neq a^2$.

8. $R_{sh}(e^{-at}) = \left(\dfrac{a}{p^2-a^2}\right)\sinh(pT).e^{-aT} + \left(\dfrac{-p}{p^2-a^2}\right)[1-\cosh(pT).e^{-aT}]$, provided $p^2 \neq a^2$.

2.2. RAM Finite Cosine Hyperbolic Transform of Some Standard Functions [1]

1. $R_{ch}(1) = \dfrac{\sinh(pT)}{p}$.

2. $R_{ch}(t) = \dfrac{T\sinh(pT)}{p} - \left(\dfrac{\cosh(pT)-1}{p^2}\right)$

3. $R_{ch}(t^2) = \frac{T^2.\sinh(pT)}{p} - \frac{2.T.\cosh(pT)}{P^2} + \frac{2.\sinh(pT)}{p^3}$.

4. $R_{ch}(t^k) = \begin{cases} \frac{T^k \sinh(pT)}{p} - \frac{kT^{k-1}\cosh(pT)}{p^2} + ... + \frac{k!(-1)^k \sinh(pT)}{p^k}, \text{if k is even,} \\ \frac{T^k \sinh(pT)}{p} - \frac{kT^{k-1}\cosh(pT)}{p^2} + ... + \frac{k!(-1)^k [\cosh(pT)-1]}{p^k}, \text{if k is odd.} \end{cases}$

5. $R_{ch}(\sin(at)) = \left(\frac{a}{p^2+a^2}\right)[1 - \cosh(pT)\cos(aT)] + \left(\frac{p}{p^2+a^2}\right)\sinh(pT)\sin(aT)$.

6. $R_{ch}(\cos(at)) = \left(\frac{a}{p^2+a^2}\right)\cosh(pT)\sin(aT) + \left(\frac{p}{p^2+a^2}\right)\sinh(pT)\cos(aT)$.

7. $R_{ch}(e^{at}) = \left(\frac{a}{p^2-a^2}\right)[\cosh(pT)e^{aT} - 1] + \left(\frac{p}{p^2-a^2}\right)\sinh(pT)e^{aT}$, provided $p^2 \neq a^2$.

8. $R_{ch}(e^{-at}) = \left(\frac{a}{p^2-a^2}\right)\cosh(pT)e^{-aT} + \left(\frac{-p}{p^2-a^2}\right)[1-\sinh(pT)e^{-aT}]$, provided $p^2 \neq a^2$.

2.3. Some Properties of RAM Finite Sine Hyperbolic Transform [1]

1. **Linearity:** $R_{sh}(f_1(t) + f_2(t)) = R_{sh}(f_1(t)) + R_{sh}(f_2(t))$.
2. **Scalar Multiplication:** If c be any constant, then $R_{sh}(cf(t)) = cR_{sh}(f(t))$.
3. **Scaling:** If $R_{sh}(f(t)) = F_S(p, T)$ then $R_{sh}(f(at)) = \frac{F_s\left(\frac{p}{a}, aT\right)}{a}$

3. MAIN RESULTS

3.1. Shifting Properties of RAM Finite Sine Hyperbolic Transform

Theorem 3.1.1 If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(\cosh(at).f(t)) + R_{ch}(\sinh(at).f(t)) = F_S((p + a), T)$$

Proof: Let $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(\cosh(at).f(t)) + R_{ch}(\sinh(at).f(t))$$

$$= \int_0^T f(t)(\cosh(at).\sinh(pt) + \sinh(at).\cosh(pt))\,dt$$

$$= \int_0^T f(t)\sinh((p + a)t)\,dt$$

$$= F_S((p + a), T)$$

Theorem 3.1.2 If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$R_{sh}((\cosh)at).f(t)) - R_{ch}(\sinh(at).f(t)) = F_S((p - a), T)$

Proof: Let $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$R_{sh}(\cosh(at).f(t)) - R_{ch}(\sinh(at).f(t))$

$$= \int_0^T f(t)(\cosh(at).\sinh(pt) - \sinh(at).\cosh(pt))\,dt$$

$= \int_0^T f(t) \sinh((p-a)t)\,dt$

$= F_S((p-a), T)$

Theorem 3.1.3 If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(\cosh(at).f(t)) = \frac{F_S((p+a),T)+F_S((p-a),T}{2}$$

Proof: By using shifting properties (3.1.1) and (3.1.2) we have,

$R_{ch}(\sinh(at).f(t)) - R_{sh}(\cosh(at).f(t)) = -F_S((p-a), T)$ and

$R_{sh}(\cosh(at).f(t)) + R_{ch}(\sinh(at).f(t)) = F_S((p+a), T)$

$$\Rightarrow R_{sh}(\cosh(at).f(t)) = \frac{F_S((p+a),T)+F_S((p-a),T}{2}$$

Theorem 3.1.4 If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(\sinh(at).f(t)) = \frac{F_C((p+a),T)-F_C((p-a),T)}{2}$$

Proof: By using Shifting properties (3.1.1) and (3.1.2) we have

$R_{sh}(\sinh(at).f(t)) + R_{ch}(\cosh(at).f(t)) = F_C((p+a), T)$ and

$R_{ch}(\cosh(at).f(t)) - R_{sh}(\sinh(at).f(t)) = F_C((p-a), T)$

$$\Rightarrow R_{sh}(\cosh(at).f(t)) = \frac{F_C((p+a),T)-F_C((p-a),T)}{2}$$

Theorem 3.1.5 If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(e^{-at}.f(t)) = \frac{F_S((p+a),T)+F_S((p-a),T)+F_C((p-a),T)-F_C((p+a),T)}{2}$$

Proof: By using Shifting properties (3.1.3) and (3.1.4) we have

$$R_{sh}(\cosh(at).f(t)) = \frac{F_S((p+a),T)+F_S((p-a),T)}{2} \text{ and}$$

$$R_{sh}(\sinh(at).f(t)) = \frac{F_C((p-a),T)-F_C((p+a),T)}{2}$$

$$\Rightarrow R_{sh}(e^{-at}.f(t)) = \frac{F_S((p+a),T)+F_S((p-a),T)+F_C((p-a),T)-F_C((p+a),T)}{2}$$

Theorem 3.1.6 If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(e^{at}.f(t)) = \frac{F_S((p+a),T)+F_S((p-a),T)-F_C((p-a),T)+F_C((p+a),T)}{2}$$

Proof: By using Shifting properties (3.1.3) and (3.1.4) we have

$$R_{sh}(\cosh(at).f(t)) = \frac{F_S((p+a),T)+F_S((p-a),T)}{2} \text{ and}$$

$$R_{sh}(\sinh(at).f(t)) = \frac{F_C((p+a),T)-F_C((p-a),T)}{2}$$

$$\Rightarrow R_{sh}(e^{-at}.f(t)) = \frac{F_S((p+a),T)+F_S((p-a),T)-F_C((p-a),T)+F_C((p+a),T)}{2}$$

Theorem 3.1.7 If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(\cos(at).\, f(t)) = \frac{F_S((p+ia),T)+F_S((p-ia),T)}{2}$$

Proof: By using Shifting properties (3.1.3) and (3.1.4) we have

$$R_{sh}(\cosh(at).\, f(t)) = \frac{F_S((p+a),T)+F_S((p-a),T)}{2} \text{ and}$$

$$\Rightarrow R_{sh}(\sinh(iat).\, f(t)) = \frac{F_C((p+ia),T)+F_S((p-ia),T)}{2}$$

$$\Rightarrow R_{sh}(\cos(at).\, f(t)) = \frac{F_S((p+ia),T)+F_S((p-ia),T)}{2}$$

Theorem 3.1.8 If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(\sin(at).\, f(t)) = \frac{F_C((p+ia),T)-F_C((p-ia),T)}{2i}$$

Proof: By using Shifting properties (3.1.4) we have

$$R_{sh}(\sinh(at).\, f(t)) = \frac{F_C((p+a),T)-F_C((p-a),T)}{2}$$

$$\Rightarrow R_{sh}(\sinh(iat).\, f(t)) = \frac{F_C((p+ia),T)-F_C((p-ia),T)}{2}$$

$$\Rightarrow R_{sh}(\sinh(at).\, f(t)) = \frac{F_C((p+ia),T)-F_C((p-ia),T)}{2i}$$

Theorem 3.1.9 Suppose $f(t) = 0$ for $t < 0$. If $R_{sh}(f(t)) = F_s(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, Then

$R_{sh}(f(t-a)) = \sinh(pa)\, F_C(p,(T-a)) + \cosh(pa)\, F_S(p,(T-a))$

Proof: Let $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(f(t-a)) = \int_0^T f(t-a)\sinh(pt)\,dt$$

$$= \int_{-a}^{T-a} f(x)\sinh(p(a+x))\,dt$$

$$= \int_0^{T-a} f(x)\sinh(pa)\cosh(px) + \cosh(pa)\sin(px)\,dx$$

$$= \sinh(pa)\int_0^{T-a} f(x)\cosh(px)\,dt + \cosh(pa)\int_0^{T-a} f(x)\sinh(px)dx$$

$$= \sinh(pa)\, F_C(p,(T-a)) + \cosh(pa)\, F_S(p,(T-a))$$

3.2. Examples

3.2.1. Find $R_{sh}(t\sinh(t))$

Solution: We know that

$$R_{sh}(t) = \frac{T\cos(pT)}{p} - \frac{\sinh(pT)}{p^2}$$

And R_{sh} (sinh(t). $f(t)$) $= \dfrac{F_C((p+a),T) - F_C((p-a),T)}{2}$

$$\Rightarrow R_{sh}(t\sinh(t)) = \frac{\frac{t\cosh((p+1)T)}{(p+1)} - \frac{\sinh(p+1)T)}{(p+1)^2} - \frac{T\cosh(p-1)T)}{(p-1)} + \frac{\sinh(p-1)T)}{(p-1)^2}}{2}$$

3.2.2. Find R_{sh} (t cosh(t))

Solution: We know that

$$R_{ch}(t) = \frac{T\cosh(pT)}{p} - \frac{\sinh(PT)}{P^2}$$

And R_{sh} (cosh(at).$f(t)$) $= \dfrac{F_S((p+a),T) + F_S((p-a),T)}{2}$

$$\Rightarrow R_{sh}(t\cosh(t)) = \frac{\frac{T\cosh(p+1)T)}{(p+1)} - \frac{\sinh((p+1)T)}{(p+1)^2} + \frac{T\cosh((p-1)T)}{(p-1)} - \frac{\sinh(p-1)T)}{(p-1)^2}}{2}$$

3.2.3. Find R_{sh} (te^t)

Solution: We know that

$$R_{sh}(t) = \frac{T\cosh(pT)}{p} - \frac{\sinh(pT)}{p^2}$$

And $R_{sh}(e^{at}.f(t)) = \dfrac{F_S((p+a),T) + F_S((p-a),T) - F_C((p-a),T) + F_C((p+a),T)}{2}$

$\Rightarrow R_{sh}(te^t)$

$$= \frac{\frac{T\cosh((p+1)T)}{(p+1)} - \frac{\sinh((p+1)T)}{(p+1)^2} + \frac{T\cosh((p-1)T)}{(p-1)} - \frac{T\cosh((p-1)T)}{(p-1)} + \frac{T\cosh((p-1)T)}{(p-1)} - \frac{T\cosh((p-1)T)}{(p-1)^2} + \frac{T\cosh((p+1)T)}{(p+1)} - \frac{T\cosh((p+1)T)}{(p+1)^2}}{2}$$

3.2.4. Find $R_{sh}(te^{-t})$.

Solution: We know that

$$R_{sh}(t) = \frac{T\cosh(pT)}{p} - \frac{\sinh(pT)}{p^2}$$

And $R_{sh}(e^{-at}.f(t)) = \dfrac{F_S((p+a),T) + F_S((p-a),T) + F_C((p-a),T) - F_C((p+a),T)}{2}$

$\Rightarrow R_{sh}(te^{-t})$

3.2.5.

$$= \frac{\frac{T\cosh((p+1)T)}{(p+1)} - \frac{\sinh((p+1)T)}{(p+1)^2} + \frac{T\cosh((p-1)T)}{(p-1)} - \frac{T\cosh((p-1)T)}{(p-1)} + \frac{T\cosh((p-1)T)}{(p-1)} + \frac{T\cosh((p-1)T)}{(p-1)^2} + \frac{T\cosh((p+1)T)}{(p+1)} - \frac{T\cosh((p+1)T)}{(p+1)^2}}{2}$$

Find R_{sh} (t sin(t))

Solution: We know that

$$R_{sh}(t) = \frac{T\cosh(pT)}{p} - \frac{\sinh(pT)}{p^2}$$

And R_{sh} (sin(t).f(t)) $= \dfrac{F_C((p+ia),T) - F_C((p-ia),T}{2i}$

$\Rightarrow$ R_{sh} (sin(t).f(t)) =

$$\frac{\dfrac{T\cosh((p+i)T)}{(p+i)} - \dfrac{\sinh((p+i)T)}{(p+i)^2} - \dfrac{T\cosh((p-i)T)}{(p-i)} + \dfrac{\sinh((p-i)T)}{(p-i)^2}}{2i}$$

3.2.6. Find R_{sh} (t cos(t)).

Solution: We know that

$R_{sh}(t) = \dfrac{T\cosh(pT)}{p} - \dfrac{\sinh(pT)}{p^2}$

And R_{sh} (cos(at). f(t)) $= \dfrac{F_S((p+ia),T) + F_S((p-ia),T)}{2}$

$\Rightarrow$$R_{sh}$ (t cos(t)).= $$\frac{\dfrac{T\cosh((p+i)T)}{(p+i)} - \dfrac{\sinh((p-i)T)}{(p+i)^2} + \dfrac{T\cosh((p-i)T)}{(p+i)} - \dfrac{\sinh((p-i)T)}{(p-i)^2}}{2}$$

4. DISCUSSION AND CONCLUSION

As like Laplace transform we observe that; linearity, scalar multiplication, scaling and shifting properties also are satisfied by using RAM Finite Hyperbolic Transform.

REFERENCES

[1]. R.A.Muneshwar[1], K.L.Bondar[2], V.S.Thosare[3], “RAM Finite Hyperbolic Transforms” , IOSR Journal of Mathematics (IOSR-JM) , Volume 10, Issue 6, Ver. IV (Nov.-Dec. 2014) ,PP 63-70.

[2]. Chandrasenkharan K., Classical Fourier Transform, Springer-Verlag , New York (1989).

[3]. Debnath L. and Thomas J., On Finite Laplace Transformation with Application, Z. Angrew. Math.und meth.56(1976),559-593.

[4]. S.B.Chavan , V.C.Borkar. , “Canonical Sine transform and their Unitary Representation”, Int. J. Contemp. Math. Science, Vol.7, 2012, No. 15,717-725.

[5]. S.B.Chavan, V.C.Borkar., “Operation Calculus of Canonical Cosine transform”, IAENG International Journal of Applied Mathematics, 2012.

[6]. S.B.Chavan, V.C.Borkar., “Some aspect of Canonical Cosine transform of generalised function”, Bulletin of Pure and Applied Sciences.Vol.29E(No.1),2010.

[7]. S.B.Chavan, V.C.Borkar., “Analyticity and Inversion for Generalised Canonical Sine transforms”, Applied Science Periodical Vol.XIV, (No 2), May 2012.

[8]. Lokenath debnath, Dombaru Bhatta, Integral Transfer and their application, Chapman and Hall/CRC, Taylor and Francis group, (2007).

[9]. Watson E.J., Laplace Transformation and Application, Van Nostrand , Reinhold, New York (1981).

[10]. S.B.Chavan, V.C.Borkar., “Some properties and Applications of Generalised Canonical Transforms”, Inter National Journal of Applied Sciences, Vol. 5, No.7, 309-314, 2011.

[11]. Wyman M., The method of the Laplace Transformation, Roy. Soc. Canada, (2)(1964),227-256.

[12]. Zemanian A.H., Generalized Integral Transformation, John Wiley and Son, New York, (1969).

[13]. Zemanian A.H., Distribution Theory and Transform Analysis, John Wiley and Son, New York, (1969).