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Higher Five Dimensional Plane Gravitational Waveswith Wet Dark Energy in Bimetric Relativity

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Abstract: -*In this paper, Z = ($z-t$) plane gravitational wave in higher dimensions is studied with the matter Wet Dark Energy in the framework of Rosen's Bimetric theory of gravitation [1] and observed that the Wet Dark Energy does not survive in this theory. Only a vacuum model can be constructed. Also some physical properties of the Models are discussed.*

Keywords: *Plane gravitational waves, Wet Dark Energy, Bimetric relativity.*

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1. Introduction

It is well known that the cosmological models based on General relativity contain an initial singular state (the big bang) from which the universe expands. The singular state can be avoided if the behavior of matter and radiation is described by the quantum theory. Un-fortunately nobody has given a way to do this satisfactorily. A satisfactory physical theory should be free from singularities because the presence of singularity means a break-down of physical laws provided by the theory. Naturally, taking into consideration these singularities in general relativity one looks carefully at the foundation of general relativity and thinks whether modification can be made to improve it. With this Motivation, Rosen [1] proposed a bimetric theory of gravitation incorporating the covariance and equivalence principles. It is based on simple form of Lagrangian and has a simpler mathematical structure than that of the general theory of Relativity. In this theory at each point of space-time, there exist two metric tensors: a Riemannian metric tensor g_{ij} and the background flat space-time metric tensor γ_{ij}. The metric tensor g_{ij} determines the Riemannian geometry of the curved space time which plays the same role as given in the Einstein's general relativity and it interacts with matter. The background metric tensor γ_{ij} refers to the geometry of the empty (free from matter and radiation) universe and describes the inertial forces. This metric tensor γ_{ij} has no direct physical significance but appears in the field equations.

Therefore it interacts with g_{ij} but not directly with matter .One Can regard γ_{ij} as giving the geometry that would exist if there were no matter. Moreover, the bimetric theory also satisfied the covariance and equivalence principles: the formation of general relativity.The theoryagrees with the present observational facts pertaining to general relativity.

Thus at every point of space-time there are two line elements:

$$ds^2 = g_{ij}dx^i dx^j \tag{1.1}$$

And $$d\sigma^2 = \gamma_{ij}dx^i dx^j \tag{1.2}$$

Where ds is the interval between two neighbouring events as measured by means of a clock and measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable.

One can regard it as describing the geometry that would exist if no matter were present. Plane gravitational waves are usually discussed as a special case of the well-established plane fronted gravitational waves with parallel rays, the so called pp- waves. The method of specialization is quite technical, e.g. the curvature tensor must be complex recurrent with a recurrence vector which is collinear with a real null vector. H Takeno (1961) [3] propounded a rigorous discussion of plane gravitational waves, defined various terms by formulating a meaningful mathematical version and obtained numerous results.

A fairly general case of "plane" gravitational wave is represented by the metric

$$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - dz^2 + dt^2 \qquad (1.3)$$

both for weak field approximation and for exact solutions of Einstein field equations.

Reformulating Takeno's (1961) [3] definition of plane wave, we will use here,
Z = $z-t$ type plane gravitational waves by using the line elements,

$$ds^2 = -A\ \ dx^2 + dy^2 + du^2\ \ -C\ \ dz^2 - dt^2 \qquad (1.4)$$

Mohseni, Tucker and Wang [4] have studied the motion of spinning test particles in plane gravitational waves.

S Kessari, D Singh et al [5], analyzed the motion of electrically neutral massive spinning test particle in the plane gravitational and electromagnetic wave background. The theory of plane gravitational waves have been studied by many investigators Takeno [6]; Pandey [7]; Lal and Shafiullah [8]; Lu Huiqing [9]; Bondi, H.et.al.[10],Torre,C.G.[11]; Hogan, P.A.[13]; and they obtained the solutions .

Further, Sahoo P.K. [14] have studied Inhomogeneous plane symmetric string cosmological models in Bimetric theory, Bhoyar and Deshmukh [15] have studied Z = (z-t) type plane fronted waves and electromagnetic waves with massless scalar plane wave and massive scalar plane waves in Peres space-time. Also Sahoo, Behera, Tripathy et al [16] have investigated inhomogeneous cosmological models in Bimetric theory of Gravitation.

Also Deo and Ronghe [17],[18] ; Deo and Suple[20] ,[21]have studied plane gravitational waves and obtained the solutions.

In this paper, we will study Z = $z-t$ type plane gravitational wave with wet dark energy and will observe the result in the context of Bimetric theory of relativity.

2. Field Equations in Bimetric Relativity

Rosen N. has proposed the field equations of Bimetric Relativity from variation principle as

$$K_i^j = N_i^j - \frac{1}{2}Ng_i^j = -8\pi\kappa T_i^j \qquad (2.1)$$

Where $$N_i^j = \frac{1}{2}\gamma^{\alpha\beta}\left[g^{hj}g_{hi}\,|_\alpha\right]|_\beta \qquad (2.2)$$

$$N = N_\alpha^\alpha \quad \kappa = \sqrt{\frac{g}{\gamma}} \qquad (2.3)$$

and $g = \left|g_{ij}\right| \qquad ,\gamma = \left|\gamma_{ij}\right| \qquad (2.4)$

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij}

And, T_i^j the energy momentum tensor for wet dark energy is given by

$T_i^{\,j} = T_i^{\,j}{}_{wdf} = \left(\rho_{wdf} + p_{wdf}\right) u_i u^j - p_{wdf}\, g_i^{\,j}$ (2.5)

together with $g_i^{\,j} u_i u^j = 1$, $u_5 u^5 = 1$ where u_i is the five-velocity vector of the fluid having p and ρ as proper pressure and energy density respectively.

In co-moving coordinate system we have

$T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p_{wdf}$, $T_5^5 = \rho_{wdf}$ and $T_i^{\,j}{}_{wdf} = 0$ for $i \neq j$

3. Z= $z-t$ TYPE HIGHER FIVE DIMENSIONAL PLANE GRAVITATIONAL WAVE WITH WET DARK ENERGY

For plane gravitational wave Z = $z-t$, we have the line element as,

$$ds^2 = -A(dx^2 + dy^2 + du^2) - C(dz^2 - dt^2) \tag{3.1}$$

Where A= A (Z), C = C (Z) and Z= $z-t$

Corresponding to equation (3.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -(dx^2 + dy^2 + du^2 + dz^2) + dt^2 \tag{3.2}$$

Since γ_{ij} is the Lorentz metric i.e. (-1,-1,-1,-1, 1), therefore γ- covariant derivative becomes the ordinary partial derivative.

Using equations (2.1) to (2.5) with (3.1) and (3.2), we get,

$$\frac{1}{4}\left[\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) - \left(\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A}\right)\right] + \frac{1}{2}\left[\left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) - \left(\frac{\dot{C}^2}{C^2} - \frac{\ddot{C}}{C}\right)\right] = -16\pi\kappa\, p_{wdf} \tag{3.3}$$

$$\frac{3}{4}\left[\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) - \left(\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A}\right)\right] = -16\pi\kappa\, p_{wdf} \tag{3.4}$$

$$\frac{3}{4}\left[\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) - \left(\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A}\right)\right] = 16\pi\kappa\rho_{wdf} \tag{3.5}$$

where the overhead primes and dots denotes differentiation of the metric potentials with respect to z and t resp.

To solve the field equations (3.3) – (3.5), we note that there are three equations connecting four unknowns, A, C, p_{wdf} and ρ_{wdf} . So one relation connecting these variablesis needed.

Here assume the relation between the metric potentials such as Bhattacharya and Karade[12]

$C = \alpha A$, $\alpha \neq 0$ is a constant. (3.6)

on solving (3.3) to (3.5) we obtain,

$$p_{wdf} + \rho_{wdf} = 0 \tag{3.7}$$

In view of the reality conditions i.e. $p > 0$, $\rho > 0$ must hold.

The above conditions (3.6) is satisfied only when

$$p = 0, \rho = 0 \ . \tag{3.8}$$

This means that the physical parameters, viz proper pressure (p), energy density (ρ) are identically zero.

Thus higher five dimensional plane gravitational wave with wet dark energy in biometric relativity does not survive and hence only vacuum model is obtained.

Using (3.7), the vacuum field equations are

$$\frac{1}{4}\left[\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)\right]+\frac{1}{2}\left[\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=0 \qquad (3.9)$$

$$\frac{3}{4}\left[\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)\right]=0 \qquad (3.10)$$

Thus field equations yields

$$\left[\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)\right]=0 \qquad (3.11)$$

By using Method of separation of Variables, (3.11) gives us the solution

$$A\ \ Z\ \ = e^{\left[\frac{l}{2}\ \ z^2+t^2\ \ +l_1 z+l_2 t\right]} \qquad (3.12)$$

With the help of (3.6) we get

$$C\ \ Z\ \ = \alpha\, e^{\left[\frac{l}{2}\ \ z^2+t^2\ \ +l_1 z+l_2 t\right]} \qquad (3.13)$$

where, $l\ , l_1\ and\ l_2$ are the constants of integration.

Thus substituting the value of (3.12) and (3.13) in (3.1), we get the vacuum line element as

$$ds^2 = -\ \mathbf{exp}\left[\frac{l}{2}\ \ z^2+t^2\ \ +l_1 z+l_2 t\right]\ dx^2+dy^2+du^2$$

$$-\alpha\,\mathbf{exp}\left[\frac{l}{2}\ \ z^2+t^2\ \ +l_1 z+l_2 t\right]\ dz^2-dt^2$$

(3.14)

The model (3.14) has no singularities either at z = 0 or at t = 0 and it does have singularities as $z \longrightarrow \infty\ and\ t \longrightarrow \infty$

4. Conclusion

In the study of **Z =** $z-t$ type plane gravitational waves, there is nil contribution of wet dark energy inBimetric theory of relativity respectively. It is observed that the matter fieldlike wetdark energy cannot be a source of gravitational field in the Rosen's bimetric theory but only vacuum model exists.

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