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# Comparative Study Analytic and Numerical Methods for Solving Non-Linear Black-Scholes Equation with European Call Option

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**Abstract:** *In this work, we apply He's variotional iteration method for obtaining analytic solutions to non-linear Black-Scholes equation with boundary conditions for European option pricing problem. The analytical solution of the equation is calculated in the form a convergent power series with easily computable components. The powerful VIM method is capable of handling both linear and non-linear equations in direct manner. And, three approximate numerical methods of the non linear Black-Scholes equation with European call option are defined using finite differences, finite difference equations with alternative derivation, and Euler method of finite difference equations with alternative derivation. The results obtained expilicit finite difference method and finite difference method with alternative derivation and Euler method of finite difference method with alternative derivation and the results gave a good agreement with the previous methods [4, 5, 6, 10].*

**Keywords:** *He's Variational iteration method, Black-Scoles equation, European call option,Free boundary problem, Finite difference, Euler method.*

## 1. Introduction

Finance is one of the most rapidly changing and fastest growing areas in the corporate business world. Because of this rapid chance, modern financial instruments have become extremely complex. As stock prices all over the world dramatically rise and fall, investors are continually in search for financial instruments to reduce the variability of their portfolio values. Consequently the volatility in the market receives much interest from market participants and researchers.

New mathematical models are essential to implement and rice these new financial instruments. The world of corporate finance once managed by business student is now controlled by mathematicians and computer scientists.

Financial securities have become essential tools for corporations and investors over past few decades. Option pricing theory has made a great leap forward since the development of the Black-Scholes option pricing model by Fisher Black and Myron Scholes in [1] and previously by Robert Merton in [2].

Recently, many scientists have paid more attention on new methods for solving option valuation. Caurtadon [3] and Wilmott et. Al. [4] used finite difference methods for option valuation. Barone-Adesi [5], Barone-Adesi and Elliot [6], Geske and Johnson [7], McMillan [8], Barone-Adesi and Whaley [9], Gülkaç [10] developed an accurate analytical approximation method. Many authors have applied several different methods to various applications [11-20].

In this paper, we will use He's variational iteration method (VIM), proposed by He [21-28] is one of the methods which has received much attention. It has been shown by many authors to be a powerful mathematical tool for solving various kinds of functional equation [29-35]. And, three approximate numerical solution of the Black-Scholes equation are defined. In the present methods, first, an approximate numerical solution of Black-Scholes equation is defined using expilicit finite difference equation, and the matrix method of analysis of the stability of the method is also investigated. Second, an approximate numerical solution of the Black-Scholes equation is defined using finite difference equations with alternative derivation and convergence of the method is also investigated. Third, an approximate numerical solution of the problem is defined using Euler Method for finite difference equations with alternative derivation.

## 2. Black-Scholes Equations

The Black-Scholes model is one of the most important concepts in modern financial theory. It was developed in 1973 by Fischer Black and Myron Scholes [1] and Robert C. Merton [2] and is still widely used today, and regarded as one of the best ways of determining fair prices of options.

In the finance, the style or family of an option is a general term denoting the class into which the option falls, usually defined by the dates on which the option may be exercised. The vast majority of options are either European or American options.

A European option may be exercised only at the expiration date of the option, i.e. at a single pre-defined point in time.

An American option on the other hand may be exercised at any time before the expiration date.

For both, the pay off-when it occurs is via:

Max[(S-K),0], for a call option.

Max[(K-S),0], for a put option where K is strike price and S is spot price of the underlying asset.

Famous linear Black- Scholes equation,

$$0 = \mathrm{V_t} + \frac{1}{2}\sigma^2 \mathrm{S}^2 \mathrm{V_{ss}} + r\mathrm{SV_s} - r\mathrm{V} \tag{1}$$

$$\mathrm{V(S, T)=0} \tag{2}$$

$$\mathrm{V(S,T)} \approx 0 \text{ as } \mathrm{S} \to \infty \tag{3}$$

$$\mathrm{V(S, T)=max(S-K, 0)} \tag{4}$$

Where $\mathrm{S : S(t) > 0}$ and $\mathrm{t} \in (0, \mathrm{t})$, provides both an option pricing formula for a European option and a hedging portfolio that replicates the contingent claim assuming that [10]

- σ, the volatility of the underlying asset;
- K, the exercise (strike) price;
- T, the expiry;
- r, the risk-free interest rate.

It is easy to imagine that the qualificatory suppositions mentioned in the linear Black-Scholes equation are never fulfilled in reality. Due to transaction costs, large investor preferences and incomplete markets they are likely to become unrealistic and the classical model results in strongly nonlinear.

In this paper, we will be interested in non-linear Black-Scholes equation for European options with a constant trend σ and no constant modified volatility function;

$\tilde{\sigma}^2 := \tilde{\sigma}^2(t, S, V_S, V_{SS})$ .

According to these circumstances (1) equation becomes the following non-linear Black-Scholes equation with the terminal and boundary conditions:

$$0 = V_t + \frac{1}{2}\tilde{\sigma}^2(t, S, V_s, V_{ss}) S^2 V_{SS} + rSV_S - rV \tag{5}$$

Where $dS = \sigma S dt + \tilde{\sigma}^2 S dW$ , $S > 0$ , $t \in (0, T)$ (6)

$$V(S,t) = (S-K)^{+} \quad \text{for} \quad 0 \le S < \infty \tag{7}$$

$$V(0,t) = 0 \quad \text{for} \quad 0 \le t \le T \tag{8}$$

$$V(S,t) \approx S - Ke^{-r(T-t)} \quad \text{as} \quad S \to \infty \tag{9}$$

In order to able to solve equation (5) with terminal and boundary conditions (7, 8, 9), we perform the following variable transformation [36, 4]:

$$\mathrm{x} = \ln\left(\frac{\mathrm{S}}{\mathrm{K}}\right), \; \tau = \frac{1}{2}\sigma^2 (T-t), \qquad u(x,t) = e^{-x\frac{V(S,t)}{K}} \tag{10}$$

Since

$S = Ke^{x}$ and $V = uS$ differentiation yields: (11)

$$V_t = u_\tau \tau_t S = -\frac{1}{2}\sigma^2 S u_\tau \tag{12}$$

$$V_S = u_x x_S S + u = u_x + u \tag{13}$$

$$V_{SS} = u_{xx} x_S + u_x x_S = \frac{1}{S}(u_{xx} + u_x) \tag{14}$$

Substituting these derivatives into equation (5) leads to

$$0 = -\frac{1}{2}\sigma^2 S u_\tau + \frac{1}{2}\tilde{\sigma}^2 S(u_{xx} + u_x) + rS(u_x + u) - rS \tag{15}$$

And a final multiplication by $-\dfrac{2}{S\sigma^2}$ gives

$$0 = u_\tau - \frac{\tilde{\sigma}^2}{\sigma^2}(u_{xx} + u_x) - Du_x \tag{16}$$

Where $D = \dfrac{2r}{\sigma^2}$ and $\tilde{\sigma}^2$ depends on the volatility model, $x \in R$ and $0 \le \tau \le \tilde{T} = \dfrac{\sigma^2 T}{2}$.

Now $u(x,\tau)$ solves (16) on the transformed domain $x \in R$, $0 \le \tau \le \tilde{T}$ subject to the following initial and boundary conditions resulting from (17, 18, 19).

$$u(x,0) = (1 - e^{-x})^{+} \quad \text{for} \quad x \in R \tag{17}$$

$$u(x,\tau) = 0 \quad \text{as} \quad x \to \infty \tag{18}$$

$$u(x,\tau) \approx 1 - e^{-D\tau - x} \quad \text{as} \quad x \to \infty \tag{19}$$

## 3. METHODS

### 3.1 He's Variational Iteration Method (VIM)

Consider the differential equation

$$Lu(x,t) + Nu(x,t) = g(x,t) \tag{20}$$

L and N are respectively linear and nonlinear operators, and g(x, t) a known analytical function. In [21, 28], He proposed the variational iteration method where a correction functional for equation (20) can be written as

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\tau)(L u_n(x,\tau) + N\tilde{u}_n(x,\tau) - g(x,\tau))d\tau \tag{21}$$

Where $\lambda$ is a general Lagrange multiplier [37] which can be identified optimally via variation theory, and $\tilde{u}_n$ is considered as a restricted variation, i.e. $\delta\tilde{u}_n = 0$. In this method, we first determine the Lagrange multiplier $\lambda$ that will be identified optimally via integration by parts. The successive approximations $u_{n+1}$, $n \geq 0$, of the solution $u$ will be readily obtained upon using the determined Lagrange multiplier and using the initial approximation $u_0$. Consequently, the solution is given by

$$u\ (x) = \lim_{n\to\infty} u_n(x,t). \tag{22}$$

### 3.2 The Black-Scholes Equation with VIM

To clarify the basic ideas of He's variational iteration method, we consider equation (16) with the initial and boundary conditions (17, 18, 19).

According to variational iteration method (VIM), we derive a correct functional as follows:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda\left(\frac{\partial u_n(x,\tau)}{\partial\tau} - F\left[\frac{\partial^2\tilde{u}_n(x,t)}{\partial x^2} + \frac{\partial\tilde{u}\ (x,\tau)}{\partial x}\right] - D\frac{\partial\tilde{u}}{\partial x}\right)d\tau \tag{23}$$

Where $F = \dfrac{\tilde{\sigma}^2}{\sigma^2}$. Making the above correction functional stationary, we have

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta \int_0^t \lambda\left(\frac{\partial u_n(x,\tau)}{\partial\tau}\right)d\tau\ , \tag{24}$$

this yields the following stationary conditions:

$$\lambda'(\tau) = 0, \tag{25}$$

$$1 + \lambda(\tau) = 0\Big|_{t=\tau} \tag{26}$$

$$\lambda = -1. \tag{27}$$

Substituting this value into Eq. (23) results the iterant formula:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left(\frac{\partial u_n(x,\tau)}{\partial\tau} - F\left[\frac{\partial^2 u_n(x,t)}{\partial x^2} + \frac{\partial u\ (x,\tau)}{\partial x}\right] - D\frac{\partial u}{\partial x}\right)d\tau \tag{28}$$

We obtain the following successive approximations:

$$u_0(x,t) = 1 - e^{-x} \tag{29}$$

$$u_1(x,t) = 1 - e^{-x}\left[1 + (2F - D)t\right] \tag{30}$$

$$u_2(x,t) = 1 - e^{-x}\left[1 + (2F - D)t + (2F - D)^2\frac{t^2}{2}\right] \tag{31}$$

$$u_3(x,t) = 1 - e^{-x}\left[1 + (2F - D)t + (2F - D)^2\frac{t^2}{2} + (2F - D)^3\frac{t^3}{2.3}\right] \tag{32}$$

$$u_n(x,t) = 1 - e^{-x}\left[1 + (2F - D)t + (2F - D)^2\frac{t^2}{2} + \ldots + (2F - D)^n\frac{t^n}{2.3\ldots n}\right] \tag{33}$$

The VIM admits the use of

$$u = \lim_{n\to\infty} u_n, \tag{34}$$

This gives the exact solution,

$$u\ (x,t) = 1 - e^{-x+(2F-D)t}\ . \tag{35}$$

Obtained upon using the Taylor expansion of $e^{-x+(2F-D)t}$

## 4. Approximate Numerical Methods

### 4.1 First Method of Solution

Using the usual forward and central difference approximation for the time and spatial derivatives in equation (16) takes the following form:

$$u_{i,j+1} = rFu_{i-1,j} + (1 - 2rF - r(F-D)h)u_{i,j} + (rF + r(F-D)h)u_{i+1,j} \tag{36}$$

where $F = \dfrac{\tilde{\sigma}^2}{\sigma^2}$, $x = ih, \tau = jk$, and $h = x_{i+1} - x_i$, and $r = k/h^2$ .

*4.1.1 Stability of the First Method*

To investigate the stability analysis of equation (36) it is convenient to use matrix analysis method [38]. Equation (36) can be written the following form:

$$u_{j+1} = Au_j \tag{37}$$

Where

$$A = \begin{bmatrix} 1-2rF-r(F-D)h & rF+r(F-D)h & & & \\ r & 1\text{-}2rF\text{-}r(F\text{-}D)h & rF+r(F-D)h & & \\ & r & 1-2rF-r(F-D)h & rF+r(F-D)h & \\ & & \ddots & & \\ & & r & & 1-2rF-r(F-D)h \end{bmatrix} \tag{38}$$

is of order (N-1). The eigenvalue $\mu$ of $A$.

Brauer's Theorem: Let $P_s$ be the sum of the module of the terms along the sth row excluding the diagonal element $a_{ss}$. Then every eigenvalue of $A$ lies inside or on the boundary of at least one of the circles $a_{ss}$, $|\mu - a_{ss}| = P_s$ [38].

The finite difference equations will be stable when the module of every eigenvalue of $A$ does not exceed one, that is when

$$|\mu - (1 - 2rF - r(F-D)h| \le r + rF + r(F-D)h \tag{39}$$

$$\mu \le 1 + r - rF\ , \tag{40}$$

Every positive D, and every positive F=1,2,… for r>0 $\mu \le 1$, therefore the equations are unconditionally stable as $\mu \le 1$ for all values of D, F and r.

### 4.2 Second Method of Solution

Consider the equation (16) where $u$ satisfies the initial condition $u(x,0) = g(x), 0 \le x \le X$ and has known boundary values at $x = 0$ and $X, \tau > 0$. If the $x$ derivative at (x, $\tau$ ) is replaced by

$\frac{1}{\mathrm{h}^2}\{u(x-h,\tau)-2u(x,\tau)+u(x+h,\tau)\}+O(h^2)$ and $x$ considered as a constant eqn. (16) can be written as the ordinary differential equation [38]

$$\frac{\mathrm{du}}{\mathrm{d}\tau}=\frac{F}{h^2}\{u(x-h,\tau)-2u(x,\tau)+u(x+h,\tau)\}+\frac{F-D}{h}\{u(x+h,\tau)-u(x,\tau)\} \tag{41}$$

It then follows that the values $V_i(\tau)$ approximating $u_i(\tau)$ will be exact solutions values of the system of (M-1) ordinary differential equations

$$\frac{\mathrm{dV_1}}{\mathrm{d}\tau}=\frac{\mathrm{F}}{\mathrm{h}^2}(V_0-2V_1+V_2)+\frac{F-D}{h}\{V_2-V_1\}$$

$$\frac{\mathrm{dV_2}}{\mathrm{d}\tau}=\frac{\mathrm{F}}{\mathrm{h}^2}(V_1-2V_2+V_3)+\frac{F-D}{h}\{V_3-V_2\}$$

•

•

•

$$\frac{\mathrm{dV_{M\text{-}1}}}{\mathrm{d}\tau}=\frac{\mathrm{F}}{\mathrm{h}^2}(V_{M-2}-2V_{M-1}+V_M)+\frac{F-D}{h}\{V_M-V_{M-1}\}$$

Or this equation systems can be written as

$$\frac{\mathrm{dV_i}}{\mathrm{d}\tau}=\frac{F}{h^2}V_{i-1}-(\frac{2F}{h^2}+\frac{F-D}{h})V_i+(\frac{F}{h^2}+\frac{F-D}{h})V_{i+1} \tag{42}$$

Where $V_0$ and $V_M$ are known boundary values. These can be written in matrix form as

$$\frac{d}{d\tau}\begin{bmatrix}V_1\\V_2\\.\\.\\V_{M-2}\\V_{M-1}\end{bmatrix}=\begin{bmatrix}-\frac{2F}{h^2}-\frac{F-D}{h} & \frac{F}{h^2}+\frac{F\text{-}D}{h} & & & \\ \frac{F}{h^2} & -\frac{2F}{h^2}-\frac{F\text{-}D}{h} & \frac{F}{h^2}+\frac{F\text{-}D}{h} & & \\ & . \quad . \quad . & & & \\ & & . \quad . \quad . & & \\ & & \frac{F}{h^2} & -\frac{2F}{h^2}-\frac{F\text{-}D}{h} & \frac{F}{h^2}+\frac{F\text{-}D}{h} \\ & & & \frac{F}{h^2} & -\frac{2F}{h^2}-\frac{F\text{-}D}{h}\end{bmatrix}\begin{bmatrix}V_1\\V_2\\.\\.\\V_{M-2}\\V_{M-1}\end{bmatrix}+\frac{F}{h^2}\begin{bmatrix}V_0\\0\\.\\.\\0\\V_M\end{bmatrix}\text{i.}$$

e. as $\frac{dV(\tau)}{d\tau}=AV(\tau)+b$ (43)

the solution of the ordinary scalar differential equation

$\frac{dV}{d\tau}=AV+b$ where $A$ and $b$ are independent $\tau$ and $V(\tau)$ satisfies the initial condition $V(0)=g$, easily shown, by the method of seperation of variables, to be

$$V(\tau)=-\frac{b}{A}+(g+\frac{b}{A})\exp(A\tau) \tag{44}$$

$$V(\tau)=-A^{-1}b+\{\exp(\tau A)\}(g+A^{-1}b) \tag{45}$$

$$V(\tau+k) = -A^{-1}b + \{\exp((\tau+k)A)\}(g + A^{-1}b) \qquad (46)$$

$$V(\tau+k) = -A^{-1}b + \{\exp(\tau A)\}\{\exp(kA)\}(g + A^{-1}b) \qquad (47)$$

$$V(\tau+k) = -A^{-1}b + \{\exp(kA)\}(V(\tau) + A^{-1}b) \qquad (48)$$

if all boundary values are zero, (see eqn. (8)),

$$V(\tau+k) = \{\exp(kA)\}V(\tau)\,. \qquad (49)$$

The boundary values can always be eliminated if we are concerned more, say, with stability than with a particular numerical solution.

*4.2.1 Stability of Second Method*

To investigate the stability analysis of equation (43) it is convenient to use matrix analysis method [38].

The eigenvalue $\mu$ of $A$. By Gerschgorin's theorem the modulus of largest eigenvalues cannot exceed the largest sum of moduli of term along any row or column of A [38]; hence,

$$|\mu_{\max}| \le \frac{F}{h} + \frac{F}{h} + (F-D) - \frac{2F}{h} - (F-D) \qquad (50)$$

$$|\mu_{\max}| \le 0 \qquad (51)$$

proving that this method converges for all positive F, D, h and k values.

### 4.3 Third Method of Solution

Equation (49) can be written as Euler method, if all boundary values are zero,

$$u_{i+1}(\tau) = u_i(\tau) + f(x_i, \tau_j)h, \text{ where } f(x_i,\tau_j) = Au(x,\tau) \qquad (52)$$

$$u_{i+1}(\tau) = u_i(\tau) + Au_i h \qquad (53)$$

This gives the iterative solution,

$$u_{i+1}(\tau) = [I + Ah]u_i\,. \qquad (54)$$

*4.3.1 Stability of Third Method*

To investigate the stability analysis of equation (52) it is convenient to use matrix analysis method [38].

$$I + hA = \begin{bmatrix} 1-\frac{2F}{h}-(F-D) & \frac{F}{h}+(F-D) & & & \\ \frac{F}{h} & 1-\frac{2F}{h}-(F-D) & \frac{F}{h}+(F-D) & & \\ & \cdot\ \cdot\ \cdot & & & \\ & & \cdot\ \cdot\ \cdot & & \\ & & \frac{F}{h} & 1-\frac{2F}{h}-(F-D) & \frac{F}{h}+(F-D) \\ & & & \frac{F}{h} & 1-\frac{2F}{h}-(F-D) \end{bmatrix}$$

By Gershchgorin's theorem, if $\xi_{\max}$ largest eigenvalue is of matrix (I+hA)

$$\left|\xi_{\max}\right| \le \frac{F}{h} + \frac{F}{h} + (F-D) + 1 - \frac{2F}{h} - (F-D) \le 1 \tag{55}$$

$$\left|\xi_{\max}\right| \le 1 \,. \tag{56}$$

Proving that this iteration converges for all positive values of F, D, h and k.

## 5. CONCLUSION

In this paper, He's variational iteration method has been applied successfully for solving non-linear Black-Scholes equation with European call option. He's variational iteration method successfully worked to give exact solution to this problem. Also, this method provides the solution in a rapidly convergent form. He's variational iteration method gives several successive approximations through using the iteration of the correction functional. In this method, there is no specific need to handle non-linear terms. He's variational iteration method provides an efficient method for handling this nonlinear behavior. Figure1, Figure 2, Figure 3,and Figure 4 illustrates call option values of non linear Black-Scholes equation with variational iteration method. Figure 5 illustrates call option values of non linear Black-Scholes equation, finite difference with alternative derivation. Figure 6 illustrates call option values of non-linear Black-Scholes Equation, with expilicit finite difference and Figure 7 illustrates call option values of non-linear Black-Scholes Equation, with Euler method.

The second method is approximate numerical solution of the nonlinear Black-Scholes equation with European call option defined using explicit finite difference method and analysis of the stability of the second method is also investigated and explicit finite difference equations were found to be unconditionally stable for all F, D, and r>0.

The third method is defined by alternative derivatives. Computing the procedure of this method is very effective and analysis of the stability of this iterative method is also investigated and iterative method was found to stable for all positive values of F, D, h and k.

The fourth method is defined by Euler Method for finite difference method with alternative derivatives. Computing the procedure of this iterative method is also investigated and iterative method was found to stable for all positive values of F, D, h and k.

All of these methods have several advantages. First, they can evaluate option positions with the same maturity for essentially all possible asset prices simultaneously. Second, they methods are believed to be adaptive to other options value problem. And, third, they solve the optimum exercise boundary together with option prices without extra energy.

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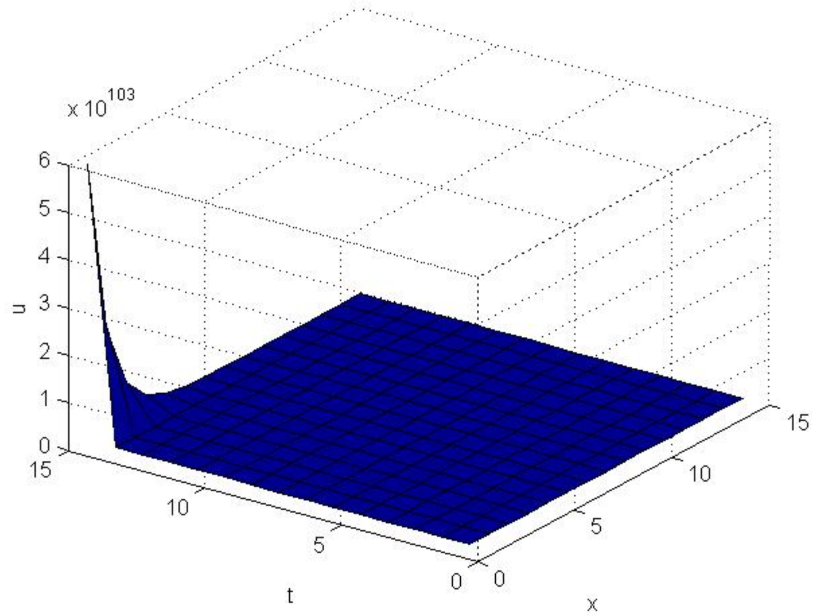


**Figure1.** *Call option values of non-linear Black-Scholes Equation, with variotional iteration method, F=8, D=0,005.*

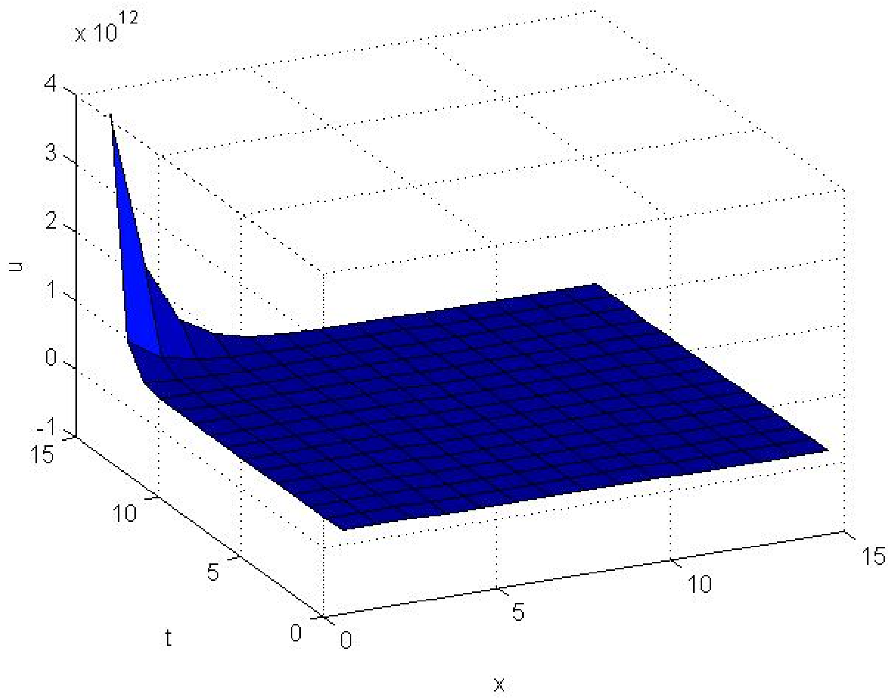


**Figure2.** *Call option values of non-linear Black-Scholes Equation, with variotional iteration method, F=1, D=0.005.*

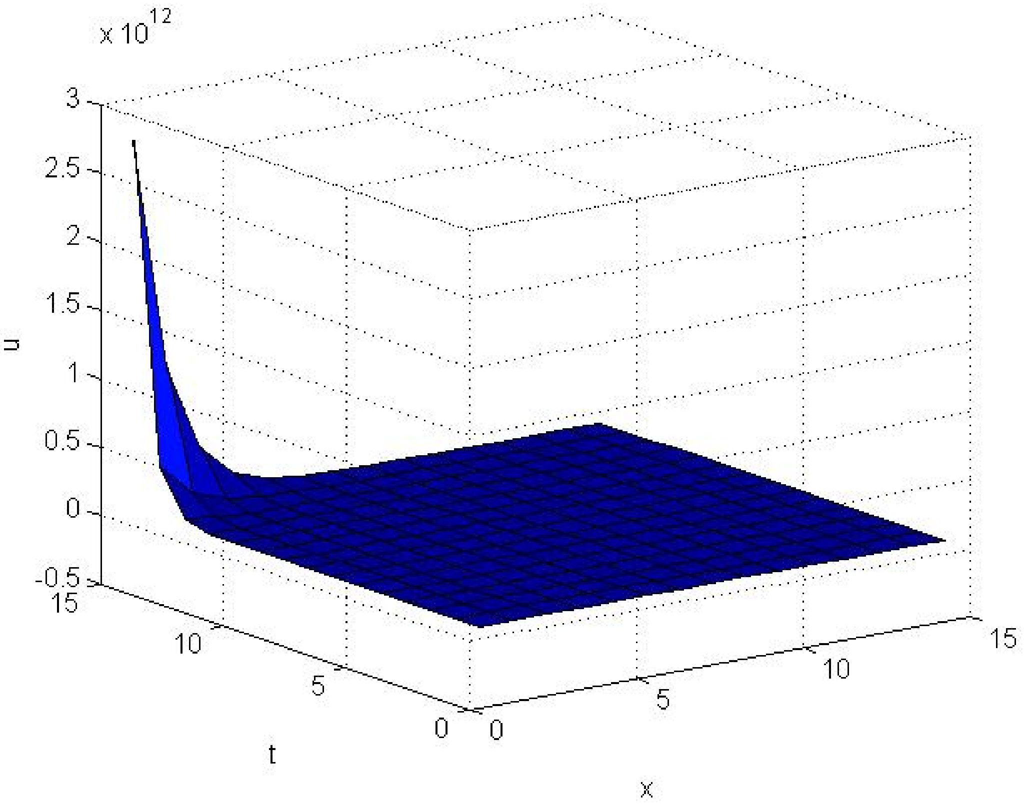


**Figure 3.***Call option values of non-linear Black-Scholes Equation, with variotional iteration method, F=1, D=0.025.*

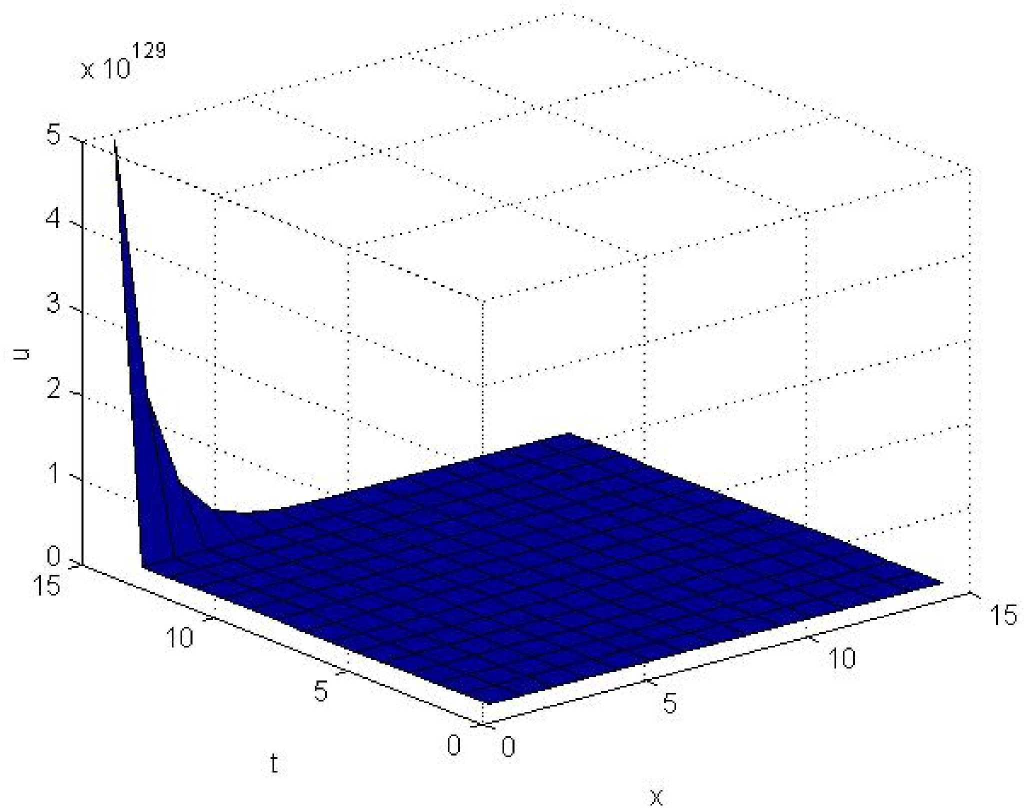


**Figure4.***Call option values of non-linear Black-Scholes Equation, with variotional iteration method, F=10, D=0.025.*

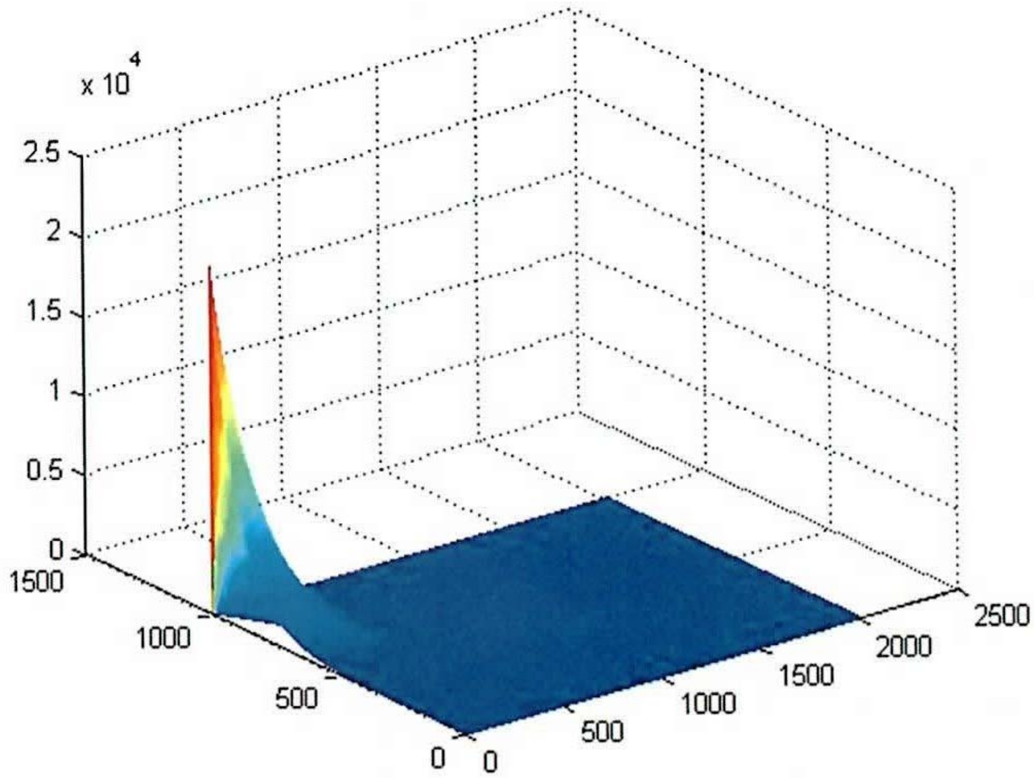


**Figure5.** *Call option values of non-linear Black-Scholes Equation, finite difference with alternative derivation, F=1, D=0.025.*

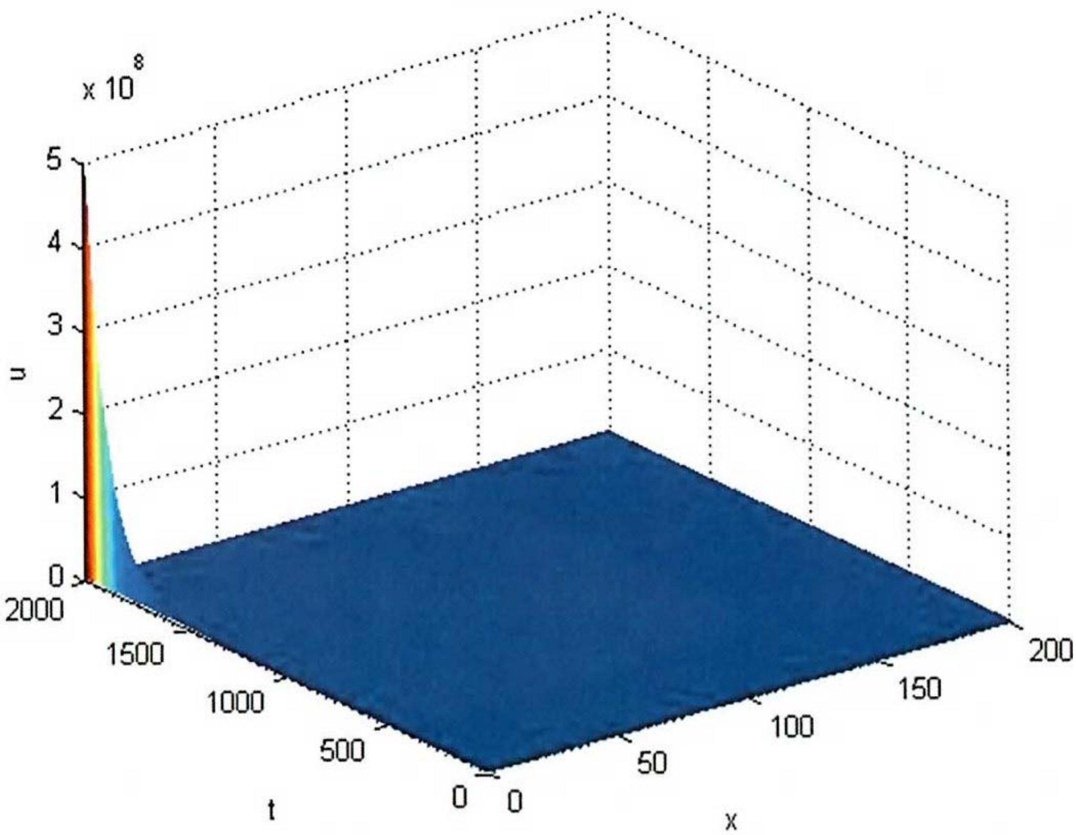


**Figure6.** *Call option values of non-linear Black-Scholes Equation, with expilicit finite difference, F=1, D=0.005.*

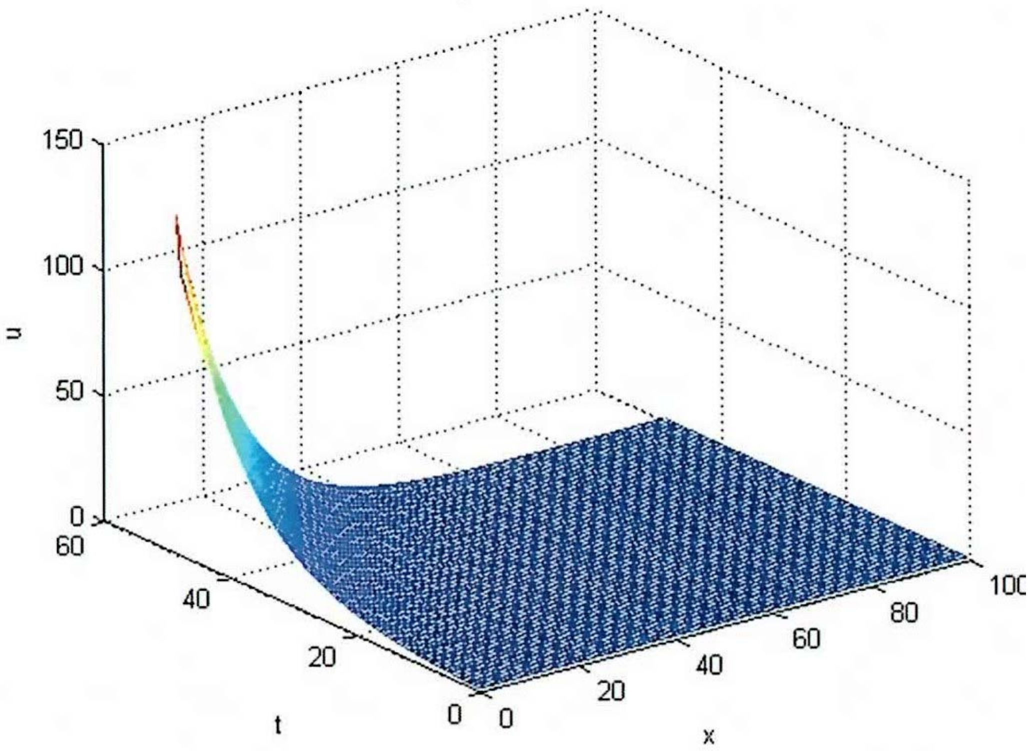


**Figure7.** *Call option values of non-linear Black-Scholes Equation, with Euler method, F=1, D=0.005.*

## AUTHOR'S BIOGRAPHY

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