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Cordial Labeling in Context of Path Union of Vertex Switching of Special Graphs

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Abstract: *A function from vertex set of a graph to set {0, 1}, which assigns the label |f (u) − f (v)| for each edge uv is called a cordial labeling of the graph if the number of vertices labeled 0 and number of vertices labeled 1 differ by at most 1, and similar condition is satisfied by edges of the graph. In this paper we discuss cordial labeling of cycle graph, wheel graph and petersen graph in context of two graph operations namely path union and vertex switching.*

Keywords: *cordial graph, vertex switching, path union, cycle, wheel, Petersen.*

AMS Subject classification number: *05C78.*

1. Introduction

Let f be a function from vertex set V of a graph G to the set {0, 1} and for each edge e = uv, assign the label |f (u) − f (v)|. Then f is called a cordial labeling of graph G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1, and the number of edges labeled 0 and the number of edges labeled 1 differ at most by 1. In this paper Cn denotes cycle with n vertices and Wn denotes wheel with n + 1 vertices (Wn = Cn + K1). Further Petersen graph is a 3-regular graph with 10 vertices and 15 edges which is denoted by P (5, 2).

2. Literature Survey and Previous Work

The concept of cordial graphs was introduced by Cahit[3]. Cahit[4] proved that complete bipartite graphs Km,n are cordial for all m and n and wheel graph Wn is cordial if and only if n ≡ 3(mod4). Vaidya et al.[11] proved that star of petersen graph, the graph obtained by joining two copies of petersen graph by a path of arbitrary length and the graph obtained by joining two copies of wheel graph by a path of arbitrary length are cordial graphs. Andar et al.[1],[2] proved that helms, closed helms, flowers, multiple shells are cordial. A dynamic sur-vey of graph labeling is published and updated every year by Gallian[5].

3. Main Results

Theorem 3.1 The graph obtained by path union of vertex switching of cycle Cn is cordial.

Proof: Let (Cn)v1 denote the vertex switching of cycle Cn with respect to an arbitrary vertex v1 and let G be the path union of k copies G1, G2, . . . , Gk of (Cn)v1 , where |Gi| = n, for each i. Let $\{u_{i1}, u_{i2}, u_{i3}, . . . ,u_{in}\}$ denote the consecutive vertices of graph Gi, where u_{i1} is the switched vertex, i = 1, 2, . . . , k. We define labeling function f : V (G) → {0, 1} as follows.

Case 1: n ≡ 0(mod4)

$f (u_{ij}) = 0$; if j ≡ 0, 1(mod4)

= 1; if j ≡ 2, 3(mod4), 1 ≤ j ≤ n, 1 ≤ i ≤ k

Case 2: n ≡ 1, 2, 3(mod4)

When i is odd,

$f (u_{ij}) = 0$; if j ≡ 0, 1(mod4)

= 1; if j ≡ 2, 3(mod4), 1 ≤ j ≤ n, 1 ≤ i ≤ k

When i is even,

$f(u_{ij}) = 0$; if $j \equiv 2, 3 \pmod 4$

$= 1$; if $j \equiv 0, 1 \pmod 4$, $1 \le j \le n$, $1 \le i \le k$

The labeling pattern defined in above cases satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$ in each case which is shown in Table 1. Hence the graph under consideration is cordial graph.

Let $n = 4a + b$, $k = 4c + d$, where $n, k \in N$.

Table 1. *Table for Theorem 3.1*

b	d	vertex conditions	edge conditions
0	0,1,2,3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
1,3	0,2	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$
	1,3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1) + 1$
	3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
2	0,1,2,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$

Illustration 3.1 Cordial labeling for the path union of four copies the graph obtained by vertex switching of cycle C_6 is shown in Fig. 1 as an illustration for the Theorem 3.1. It is the case related to $n \equiv 2 \pmod 4$.

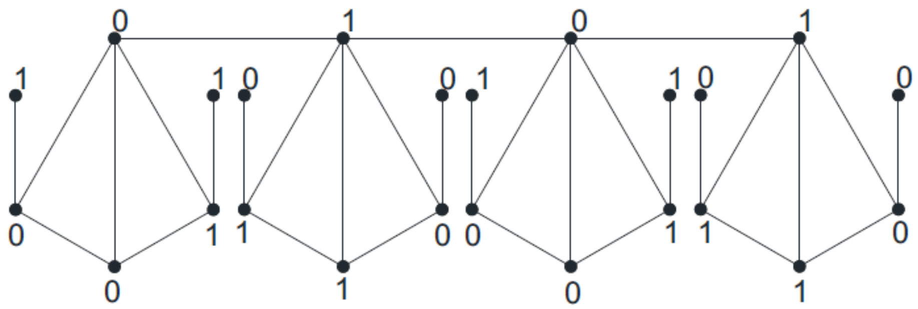

Figure 1. *Cordial labeling of path union of four copies of vertex switching of cycle C_6*

Theorem 3.2 The graph obtained by path union of vertex switching of any vertex of wheel Wn (except apex vertex) is cordial.

Proof: Let $u_1, u_2, \ldots, u_n$ be successive vertices of wheel graph Wn. Let u_0 be apex vertex of Wn. The graph obtained by vertex switching of rim vertex u_i is isomorphic to the graph ob-tained by vertex switching of rim vertex u_j , $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$. Hence we require to discuss the case of vertex switching of an arbitrary vertex say u_1 of Wn. Let $(Wn)u_1$ denote the vertex switching of Wn with respect to the vertex u_1. Let G be the path union of k copies $G_1, G_2, \ldots, G_k$ of $(Wn)u_1$, where $|G_i| = n$, for each i. Let us denote the consecutive vertices of graph G_i by $\{u_{i1}, u_{i2}, u_{i3}, \ldots, u_{in}\}$, where u_{i1} is the switched vertex, $i = 1, 2, \ldots, k$.

To define labeling function $f : V(G) \to \{0, 1\}$ we consider following cases.

Case 1: $n \equiv 0 \pmod 4$

When $i \equiv 0, 1 \pmod 4$,

$f(u_0) = 0$

$f(u_{ij}) = 0$; if $i \equiv 0, 3 \pmod 4$

$= 1$; if $i \equiv 1, 2 \pmod 4$, $1 \le j \le n$, $1 \le i \le k$

When $i \equiv 2, 3 \pmod 4$,

$f(u_0) = 1$

$f(u_{ij}) = 0$; if $i \equiv 1, 2 \pmod 4$

$= 1$; if $i \equiv 0, 3 \pmod 4$, $1 \le j \le n$, $1 \le i \le k$

Case 2: n ≡ 1(mod4)

When i ≡ 1, 3(mod4),

f (u_0) = 1

f (u_{ij}) = 0; if i ≡ 0, 1(mod4)

= 1; if i ≡ 2, 3(mod4), 1 ≤ j ≤ n, 1 ≤ i ≤ k

When i ≡ 0, 2(mod4),

f (u_0) = 0

f (u_{ij}) = 0; if i ≡ 2, 3(mod4)

= 1; if i ≡ 0, 1(mod4), 1 ≤ j ≤ n, 1 ≤ i ≤ k

Case 3: n ≡ 2(mod4)

When i ≡ 0, 1(mod4),

f (u_0) = 1

f (u_{ij}) = 0; if i ≡ 2, 3(mod4)

= 1; if i ≡ 0, 1(mod4), 1 ≤ j ≤ n, 1 ≤ i ≤ k

When i ≡ 2, 3(mod4),

f (u_0) = 0

f (u_{ij}) = 0; if i ≡ 0, 1(mod4)

= 1; if i ≡ 2, 3(mod4), 1 ≤ j ≤ n, 1 ≤ i ≤ k

Case 4: n ≡ 3(mod4)

When i ≡ 1, 3(mod4),

f (u_0) = 1

f (u_{ij}) = 0; if i ≡ 1, 2(mod4)

= 1; if i ≡ 0, 3(mod4), 1 ≤ j ≤ n, 1 ≤ i ≤ k

When i ≡ 0, 2(mod4),

f (u_0) = 0

f (u_{ij}) = 0; if i ≡ 0, 3(mod4)

= 1; if i ≡ 1, 2(mod4), 1 ≤ j ≤ n, 1 ≤ i ≤ k

The labeling pattern defined above satisfies the condition |vf (0) − vf (1)| ≤ 1 and

$|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in Table 2. Hence the graph under consideration is cordial graph.

Let n = 4a + b, where n ∈ N .

Table 2. *Table for the graph G in Theorem 3.2*

b	d	vertex conditions	edge conditions
0	0,2 1 3	$v_f(0) = v_f(1)$ $v_f(0) = v_f(1) + 1$ $v_f(0) + 1 = v_f(1)$	$e_f(0) + 1 = e_f(1)$ $e_f(0) = e_f(1)$ $e_f(0) = e_f(1)$
1	0,1,2,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$
2	0,2 1 3	$v_f(0) = v_f(1)$ $v_f(0) + 1 = v_f(1)$ $v_f(0) = v_f(1) + 1$	$e_f(0) + 1 = e_f(1)$ $e_f(0) = e_f(1)$ $e_f(0) = e_f(1)$
3	0,1,2,3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$

Illustration 3.2 Cordial labeling for the path union of three copies the graph obtained by vertex switching wheel W_5 is shown in Fig. 2 as an illustration for the Theorem 3.2. It is the case related to n ≡ 1(mod4).

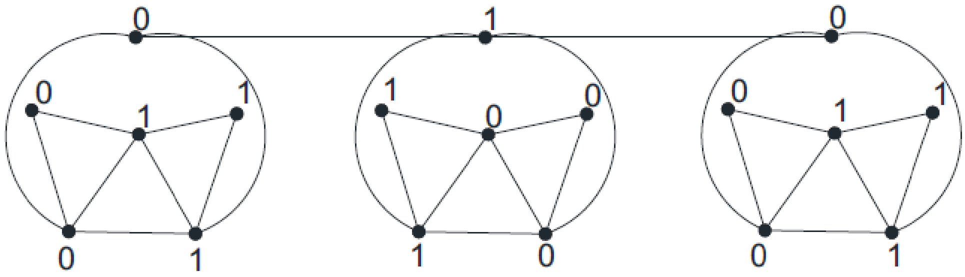

Figure 2. *Cordial labeling of path union of three copies of vertex switching of wheel W5*

Theorem 3.3 The graph obtained by path union of vertex switching of petersen graph is cor-dial.

Proof: Let P (5, 2) denote the petersen graph. Let u_1, u_2, u_3, u_4, u_5 be internal vertices and u_6, u_7, u_8, u_9, u_{10} be the external vertices of petersen graph such that ui is adjacent to u_{i+5}, i = 1, 2, 3, 4, 5. Note that the graphs obtained by vertex switching of vertex u_i is isomor-phic to the graph obtained by vertex switching of vertex u_j , i = 1, 2, . . . , 10, j = 1, 2, . . . , 10.

Hence we require to discuss the case of vertex switching of an arbitrary vertex say u_1 of P (5, 2). Let (P (5, 2))u_1 denote the vertex switching of P (5, 2) with respect to the vertex u_1. Let G be the path union of k copies G_1, G_2, . . . ,G_k of (P (5, 2))u_1 , where $|G_i|$ = n, for each i. Let {u_{i1}, u_{i2}, u_{i3}, . . . , u_{10}} denote the consecutive vertices of graph G_i, i = 1, 2, . . . , k. Here u_{i1}, u_{i2}, u_{i3}, u_{i4}, u_{i5} are internal vertices and u_{i6}, u_{i7}, u_{i8}, u_{i9}, u_{i10} are the external vertices of G_i, i = 1, 2, . . . , k.

To define labeling function f : V (G) → {0, 1} we consider the following cases.

Case 1: i = 1

f (u_{ij}) = 0; if j = 3, 4, 5, 8, 9

= 1; if j = 1, 2, 6, 7, 10

Case 2: i ≡ 0(mod4), 1 ≤ i ≤ k

f (u_i) = 0; if j = 1, 2, 6, 7, 10

= 1; if j = 3, 4, 5, 8, 9

Case 3: i ≡ 1, 2(mod4), 1 ≤ i ≤ k, i = 1

f (u_i) = 0; if j = 1, 2, 5, 8, 9

= 1; if j = 3, 4, 6, 7, 10

Case 4: i ≡ 3(mod4), 1 ≤ i ≤ k

f (u_i) = 0; if j = 3, 4, 5, 8, 9

= 1; if j = 1, 2, 6, 7, 10

The labeling pattern defined in above cases satisfies the conditions of cordial labeling which is shown in Table 3. Hence the graph under consideration is cordial graph.

Let n = 4a + b, k = 4c + d, where n, k ∈ N.

Table 3. *Table for Theorem 3.3*

d	vertex conditions	edge conditions
0	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$
1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
2	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$

Illustration 3.3 Cordial labeling for the path union of four copies the graph obtained by vertex switching of petersen graph is shown in Fig. 3 as an illustration for Theorem 3.3.

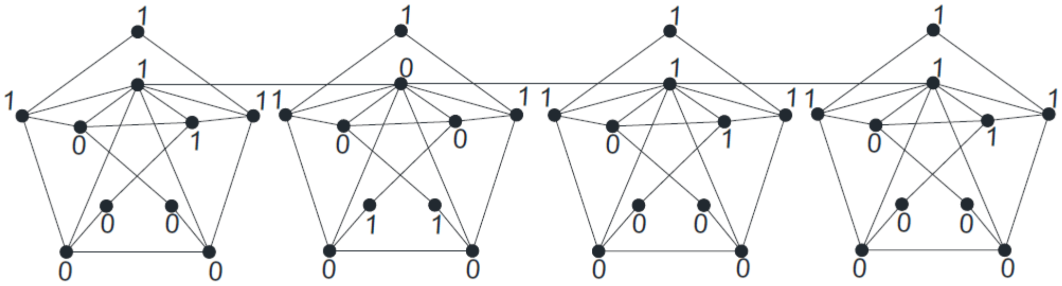

Figure 3. *Cordial labeling of path union of four copies of vertex switching of petersen graph*

4. CONCLUSION

It is already proved that petersen graph, wheel graph, flower graph, gear graph and shell graph are cordial. We have discussed cordiality of these graphs in context of path union and vertex switching with respect to any arbitrary vertex.

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