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A Concise and Precise Treatise on Solution of One Dimensional Telegrapher's Equation

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Abstract: *The general initial-boundary-value problem for Transmission lines was formulated. Its concise and convenient general solution was derived analytically by using the Laplace transform method. The obtained solution was verified with the existing solutions for particular values. Moreover, a couple of particular examples were also presented in order to validate the Laplace transform method. The obtained results were also compared with the existing results. The present approach was seemed to be simple and did not involve complex mathematics. Therefore it would be useful and helpful for engineering students. The tendency of change in voltage with respect to physical parameters; C, G, L, R, x, and t as well as boundary parameter* α *was examined. It was found that the dependency of physical and boundary parameters on voltage intensifies as we move away from the source (x=0) as well as time increases for all aforementioned physical parameters. In addition, at any time t=to,* $u(x, t_o) \alpha e^{\alpha x}$ *for fixed values of parameters C, G, L, R, t and* α*. Furthermore the voltage decays as physical parameters (C, G, L, R) rise whereas the voltage will intensify with the boundary parameter* α*..*

Keywords: *Telegrapher's equation; Laplace transform, partial differential equations, Telegraph wire, Leakage, transmission line.*

1. INTRODUCTION

The ordinary electrical cable is incapable to transport the high frequency current and voltage wave. Therefore a special cable is constructed to support such the power and communication signals, which is called transmission lines (TLs). They have extensive use of

1) supporting the production of uniform impedance (also called characteristic impedance) to prevent the reflection,

2) carrying the electromagnetic wave signals very efficiently with minimal losses and reflections,

3) carrying communication signals of high frequencies with minimal signal losses,

4) carrying currents in the radio frequency range or higher with minimal power losses.

The power losses, at microwave frequencies and above, in transmission lines become excessive and wave guides are used. Therefore, the TLs are also said to be guided transmission cable (or media), however, it is also considered as a type of TLs by some sources [1]. Furthermore other types of TLs are coaxial cable, optical fiber, ladder-line, strip-line, micro-strip, computer network media and etc.

Consequently TLs are highly used for

1) computer network connection,

2) supporting the high speed computer data buses,

3) communicating the data through optical fiber,

4) distributing cable television signal and communicating the signals through telegraph

5) studying the neural network in the brain [2].

The equation of state for elucidating the voltage and current, with respect to spatially and temporally, in TLs is called the Telegrapher's equation (TE). It was initially modeled by Oliver Heaviside in 1880 [1]. Moreover, the detailed historical background had been surveyed eminently by many authors like [2].

In the present attempt, we present the concise and precise way of derivation of TE and then its solution. Other salient features of this attempt are:

1) The general solution of general one dimensional TE-boundary value problem (BVP) is presented in a very convenient way.

2) The effect of coefficients of TE on voltage, V, and current, I, is observed,

3) The effect of boundary condition parameter on voltage, V, or current, I, is also examined.

4) The derived (general) analytical solution is well compared with the existing particular solutions.

This paper comprises four sections. The basic analysis and derivation of TE is presented in section 1. Section 2 contains its precise general solution while calculated-results are given in section 3. Finally, a conclusion is made in section 4.

Many valuable contributions had been made on solving the Telegrapher's equation numerically like [3-5] and analytically [6-11]. We are interested to solve the Telegrapher's equation by using the single Laplace transform method (to make the way to find solution convenient, concise and precise for learners) despite of [11] whom used the double Laplace transform.

2. BASIC ANALYSIS

Consider an infinitesimal segment, dx, of (one-dimensional) Telegraph-wire as an electrical circuit, as shown in figure 1, having potential differences V(x,t) and V(x+dx,t), respectively, at points P and Q whereas I(x,t) is the current at any point x at any time t.

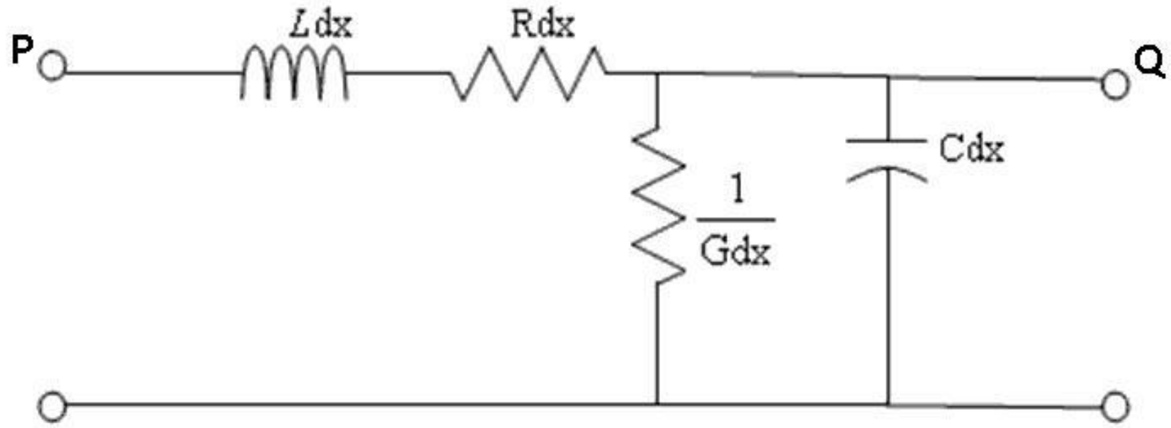

Figure 1. *Flow Schematic*

On using the Kirchhoff's voltage law, we can write

$$\frac{\partial V}{\partial x} = -\left[RI + L\frac{\partial I}{\partial t} \right] \tag{1}$$

and on using the Kirchhoff's current law, we have

$$\frac{\partial I}{\partial x} = -\left[GV + C\frac{\partial V}{\partial t} \right] \tag{2}$$

where *R, L, C* and *G* are, respectively, the (distributed) resistance, the inductance, the capacitance and the leakage factor (conductance to the ground).

Next differentiate Eq. (1) w.r.t. "*t*" and Eq. (2) w.r.t. "*x*", we yield, after simplification,

$$\frac{\partial^2 I}{\partial x^2} = a\frac{\partial^2 I}{\partial t^2} + b\frac{\partial I}{\partial t} + cI \tag{3}$$

Similarly if we differentiate Eq. (2) w.r.t. "t" and Eq. (1) w.r.t. "x", we yield,

$$\frac{\partial^2 V}{\partial x^2} = a\frac{\partial^2 V}{\partial t^2} + b\frac{\partial V}{\partial t} + cV \tag{4}$$

where

$$a=LC,\ \ b=LG+RC \text{ and } c=RG. \tag{5}$$

In general Eqs. (3)-(4) can be concatenated as:

$$\frac{\partial^2 u}{\partial x^2} = a\frac{\partial^2 u}{\partial t^2} + b\frac{\partial u}{\partial t} + cu \tag{6}$$

such that $u \in \{I, V\}$.

3. General Analytical Solution

Consider boundary value problem (BVP) comprising Eq. (6) along with the following boundary conditions

$$u(x,0) = e^{\alpha x}, \qquad \frac{\partial}{\partial t}u(x,0) = -e^{\alpha x} \tag{7}$$

$$u(0,t) = C_1 e^{K_1 t} + C_2 e^{K_2 t}, \qquad \frac{\partial}{\partial x}u(0,t) = \alpha\left[C_1 e^{K_1 t} + C_2 e^{K_2 t}\right] \tag{8}$$

where

$$\left.\begin{aligned} &C_1 = \frac{K_o - \beta + 2}{2K_o}, \quad C_2 = \frac{K_o + \beta - 2}{2K_o}, \quad K_1 = -\frac{\beta + K_o}{2}, \\ &K_2 = -\frac{\beta - K_o}{2}, \quad K_o = \sqrt{\beta^2 - 4\gamma}, \quad \beta = \frac{b}{a}, \quad \gamma = \frac{c - \alpha^2}{a} \end{aligned}\right\}. \tag{9}$$

On applying Laplace transform both sides of Eq. (6) with the introduction of Eq. (7), we get

$$\frac{\partial^2 \breve{u}}{\partial x^2} - f^2 \breve{u} = -a(s + \beta - 1)e^{\alpha x} \tag{10}$$

where $f = as^2 + bs + c$ whereas $\breve{u}$ is the Laplace transform of u. Hence, the general solution of Eq. (10) will be

$$\breve{u} = Ae^{-fx} + Be^{fx} + \frac{(s + \beta - 1)e^{\alpha x}}{s^2 + \beta s + \gamma} \tag{11}$$

Here the constants of integration, A and B, can be determined as A= 0 = B by using conditions given in Eq. (8). Accordingly Eq. (11) will reduce to

$$\breve{u} = \frac{(s + \beta - 1)e^{\alpha x}}{s^2 + \beta s + \gamma} \tag{12}$$

On applying inverse Laplace transform on above equation, we get

$$u(x,t) = \left[C_1 e^{K_1 t} + C_2 e^{K_2 t}\right] e^{\alpha x} \tag{13}$$

which is an exact solution of BVP (Eqs. 6 − 9). Let us consider a couple of following examples to justify the general solution, Eq. (13) of one-dimensional Telegrapher's BVP under consideration.

Example 1

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u \tag{14}$$

with respect to initial and boundary conditions:

$$u(x,0) = e^x, \qquad \frac{\partial}{\partial t} u(x,0) = -e^x \tag{15}$$

$$u(0,t) = e^{-t}, \qquad \frac{\partial}{\partial x} u(0,t) = e^{-t} \tag{16}$$

On applying Laplace transform both sides of Eq. (14) with the introduction of Eq. (15), we get,

$$\frac{\partial^2 \breve{u}}{\partial x^2} - \left(s^2 + s + 1\right)\breve{u} = -se^x \tag{17}$$

The general solution of Eq. (17) is

$$\breve{u} = Ae^{\left(-x\sqrt{s^2+s+1}\right)} + Be^{\left(x\sqrt{s^2+s+1}\right)} + \frac{e^x}{s+1} \tag{18}$$

Consistently, *A* and *B* can be determined as A= 0 = B by using conditions given in Eq.(16). Accordingly Eq. (18) will reduce to

$$\breve{u} = \frac{e^x}{s+1} \tag{19}$$

On applying inverse Laplace transform on above equation, we get

$$u(x,t) = e^{x-t} \tag{20}$$

This is an exact solution of Telegrapher's equation given in Eq. 14. Furthermore, we can get same (i.e. solution Eq. 20) just on taking $(a, b, c, \alpha) = (1, 1, 1, 1)$ in Eq. (13) as well as it is in coincidence with the solution given in [10] and [11].

. **Example 2**

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + 4\frac{\partial u}{\partial t} + 4u \tag{21}$$

with respect to initial and boundary conditions:

$$u(x,0) = e^x, \qquad \frac{\partial}{\partial t} u(x,0) = -e^x \tag{22}$$

$$u(0,t) = e^{-t}, \qquad \frac{\partial}{\partial x} u(0,t) = e^{-t} \tag{23}$$

On applying Laplace transform both sides of Eq. (21) with the introduction of Eq. (22), we get,

$$\frac{\partial^2 \breve{u}}{\partial x^2} - \left(s^2 + 4s + 4\right)\breve{u} = -(s+3)e^x \tag{24}$$

The general solution of Eq. (24) is

$$\breve{u} = Ae^{-x(s+2)} + Be^{x(s+2)} + \frac{e^x}{s+1} \tag{25}$$

On applying inverse Laplace transform on above equation, we yield

$$u(x,t) = Ae^{-2x}H(t-x)\delta(t-x) + Be^{2x}H(t+x)\delta(t+x) + e^{x-t} \tag{26}$$

Here $H(.)$ and $\delta(.)$, respectively, are the Heaviside unit step and the Dirac delta functions, whereas A and B are the constants of integration and will be determined later. Furthermore, Eq. (26) can be simplified as

$$u(x,t) = (A+B)e^{-2t} + e^{x-t} \tag{27}$$

On the introduction of dirichlet condition of Eq. (23), while Neumann condition of Eq. (23) satisfies automatically, we get A + B = 0. Therefore Eq. (27) reduces to

$$u(x,t) = e^{x-t} \tag{28}$$

This is an exact solution of Telegrapher's equation given in Eq. (21). Moreover, Eq. (13) for *(a, b, c, α) = (1, 4, 4, 1),* is also compared exactly with solution given in [8].

Next consider following a couple of examples in order to support the fact that the one-dimensional Telegrapher's equation solved with different initial and boundary conditions can be solved with the help of the Laplace transform in a very convenient manner.

Example 1

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} + u \tag{29}$$

with respect to initial and boundary conditions:

$$u(x,0) = e^x, \qquad \frac{\partial}{\partial t}u(x,0) = -2e^x \tag{30}$$

$$u(0,t) = e^{-2t}, \qquad \frac{\partial}{\partial x}u(0,t) = e^{-2t} \tag{31}$$

On applying Laplace transform both sides of Eq. (29) with the introduction of Eq. (30), we get,

$$\frac{\partial^2 \breve{u}}{\partial x^2} - \left(s^2 + 2s + 1\right)\breve{u} = -se^x \tag{32}$$

The general solution of Eq. (32) is

$$\breve{u} = Ae^{-x(s+1)} + Be^{x(s+1)} + \frac{e^x}{s+2} \tag{33}$$

On applying inverse Laplace transform on above equation, we get

$$u(x,t) = Ae^{-x}H(t-x)\delta(t-x) + Be^{x}H(t+x)\delta(t+x) + e^{x-2t} \tag{34}$$

Furthermore Eq. (34) can be simplified as:

$$u(x,t) = (A+B)e^{-t} + e^{x-2t} \tag{35}$$

On the introduction of dirichlet condition of Eq. (31), while Neumann condition of Eq. (31) satisfies automatically, we get A + B = 0. Therefore Eq. (35) reduces to

$$u(x,t) = e^{x-2t} \tag{36}$$

which is an exact solution of Telegrapher's initial-boundary value problem given in Eq. (29) and compared exactly with solution mentioned in [6], [8] and [10].

Example 2

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + 4\frac{\partial u}{\partial t} + 4u \tag{37}$$

with respect to initial and boundary conditions:

$$u(x,0) = 1 + e^{2x}, \qquad \frac{\partial}{\partial t}u(x,0) = -2 \tag{38}$$

$$u(0,t) = 1 + e^{-2t}, \qquad \frac{\partial}{\partial x}u(0,t) = 2 \tag{39}$$

If we apply Laplace transform both sides of Eq. (37) with the introduction of Eq. (38), we have,

$$\frac{\partial^2 \breve{u}}{\partial x^2} - \left(s^2 + 4s + 4\right)\breve{u} = -(s+4)e^{2x} - s - 2 \tag{40}$$

Therefore the general solution of Eq. (40) will be

$$\breve{u} = Ae^{-x(s+2)} + Be^{x(s+2)} + \frac{e^{2x}}{s} + \frac{1}{s+2} \tag{41}$$

Eq. (41) will take the form after applying inverse Laplace transform;

$$u(x,t) = Ae^{-2x}H(t-x)\delta(t-x) + Be^{2x}H(t+x)\delta(t+x) + e^{-2t} + e^{2x} \tag{42}$$

On simplifying Eq. (42) further, we get

$$u(x,t) = (A+B+1)e^{-2t} + e^{2x} \tag{43}$$

On the introduction of dirichlet condition of Eq. (39), while Neumann condition of Eq. (39) satisfies automatically, we get A + B = 0. Therefore Eq. (43) deforms as:

$$u(x,t) = e^{-2t} + e^{2x} \tag{44}$$

This is an exact solution of Telegrapher's BVP mentioned in Eq. (37). Consistently it compared exactly with solution given in [6], [8] and [10].

4. Results and Discussion

As we mentioned in the previous section that *u(x, t)* may be voltage or current, but for the purpose of convenience and concentration we fix *u* as voltage in the forthcoming results and discussion. The calculated results, on the basis of Eq. (13), were carried out for different values of *C, G, L, R, x, t* and

α. The selected range of these parameters are $C \in [4.9, 5.4]$μF/km, $G \in [6.8, 168]$μ/Ω−km, $L \in [0.0004, 46750]$mH/km, $R \in [849, 1150]$Ω/km, $x \in [0, 2]$km, $t \in [0, 1]$sec and $\alpha \in [0, 1]$. The effects of these parameters on voltage u(x, t) was investigated and obtained-results were simulated in figures 2 to 5.

Effect of *C* on *u*(x, t)

In order to seek the dependency of voltage, u(x, t), on the capacitance C, it was plotted for fixed values of (G,L,R,α, t) = (118.074μ/Ω−km, 0.4675mH/km, 999.41Ω/km, 1, 1sec), for three values of C namely C = {4.957, 5.157, 5.457}μF/km as shown in figure 2(a). It is not amazing to observe from the figure that voltage, *u(x, t)* decays as capacitance enhances. This figure also depicts that voltage, *u(x, t)*, grows exponentially with *x*, which is not only coincidence with observed results of [6] but also in agreement with $u(x,1) = C_3 e^{\alpha x}$ where $C_3 = C_1 e^{K_1} + C_2 e^{K_2}$, which is obtained from Eq. (13). Furthermore $u(0,1) = C_3$ and at any time $t = t_o$, $u(0,t_o) = C_4$ where $C_4 = C_1 e^{t_o K_1} + C_2 e^{t_o K_2}$. It reflects that the voltage, $u(x, t_o)$, has analogous variational-trend as that of *u(x, 1)*. It means that the variational-trend of *u(x, t)* is similar for all values of time. In addition, we can conclude that $u(x,t_o) \propto e^{\alpha x}$ for fixed values of parameters *C, G, L, R, t* and α. Furthermore the dependency of *C* on *u* intensifies as we move away from the sending end ($x=0$) as well as time increases.

Effect of *G* on *u*(x, t)

The effect of *G* on voltage, *u(x, t)*, is presented in figure 2(b) when *G* = (4.957, 5.157, 5.457)μF/km for fixed values of (C,L,R, α, t) = (5.157μF/km, 0.4675mH/km, 999.41Ω/km, 1, 1sec). Figure 2(b) signifies that voltage, *u(x, t)* increases as *x* increases similar to the previous case but it decreases with *G* for all values of *C, L, R,* and α. At any time, $t = t_o$, $u(x,t_o) \propto e^{\alpha x}$ for fixed values of parameters *C, G, L, R, t* and α, analogous to as predicted earlier in the above case (effect of *C*).

Effect of *R* on *u*(x, t)

Voltage *u(x, t)* depends on resistance R significantly. This fact is observed and displayed in figure 2(c) for fixed values of (C,G,L,α, t) = (5.157μF/km, 118.074μ/Ω−km, 0.4675mH/km, 1, 1sec), for three values of *R* as mentioned in the caption of figure 2(c). Figure 2(c) depicts that voltage, *u(x, t)* increases with *x* and *R* for all values of *C, G, L* and α. Once again we get same situation, at any time, $t = t_o$, $u(x,t_o) \propto e^{\alpha x}$ for fixed values of parameters *C, G, L, R, t* and α

Effect of *L* on *u*(x, t)

In contrast of previous cases, the dependency of Voltage, *u(x, t)* on inductance *L* is not significant for small values of *L* (i.e., $0 < L < 1$ approximately). If $L >> 1$ then change in voltage occurs significantly. This fact is simulated in figure 2(d). Moreover, the variation of voltage *u(x, t)* at (*x, t*) = (0, 1) with respect to *L* is examined for fixed values of (C,G,L,α, t) = (5.157μF/km, 118.074μ/Ω−km, 0.4675mH/km, 1, 1sec). This fact is observed and displayed in figure 3 for fixed values of (C, G, L, α) = (5.157μF/km, 118.074μ/Ω−km, 0.4675mH/km, 1, 1sec). It describes that as *L* decreases, voltage at (*x, t*) = (0, 1) decreases, which is in coincidence with $u(0,1) = C_3 = C_1 e^{K_1} + C_2 e^{K_2}$ as obtained from Eq. 13.

Effect of α on *u*(x, t)

Figure 4 shows the variation of boundary parameter α versus voltage *u*(x, t) for fixed values of (C,G,L,R,α) = (5.157μF/km, 118.074μ/Ω−km, 0.4675mH/km, 999.41/km, 1), when t = 0, 0.5 and 1. It can be observed that voltage *u*(x, 1) increases as α increases as well as it also increases as time, t, increases. Consistently same observation was made as predicted for earlier described cases i.e., at any time, $t = t_o$, $u(x,t_o) \propto e^{\alpha x}$ for fixed values of parameters *C, G, L, R, t* and α.

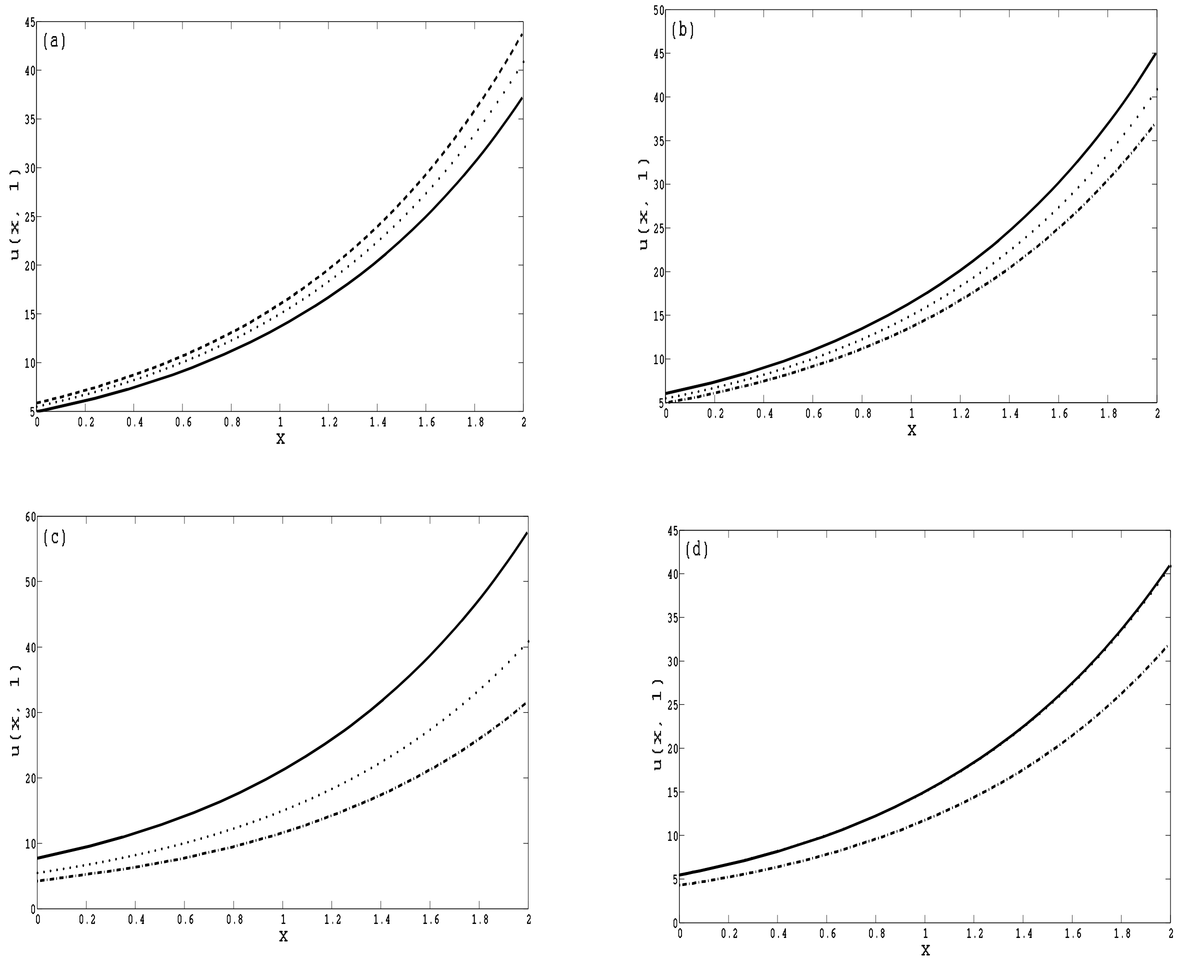

Figure 2: *Variation of u(x, 1) with x for (a) different values of C namely C = 5.457μF/km (solid curve), C = 5.157μF/km (dotted curve) and C = 4.957μF/km (dotted-dashed curve); (b) different values of G namely G = 68.074μ/Ω−km (solid curve), G = 118.074μ/Ω−km (dotted curve), and G = 168.074μ/Ω−km (dotted-dashed curve); (c) different values of R namely R = 849.41/km (solid curve), R = 999.41/km (dotted curve) and R = 1149.41/km (dotted-dashed curve); (d) different values of L namely L = 0.4675μH/km (solid curve), L = 0.4675mH/km (dotted curve) and L = 0.04675kH/km (dotted-dashed curve).*

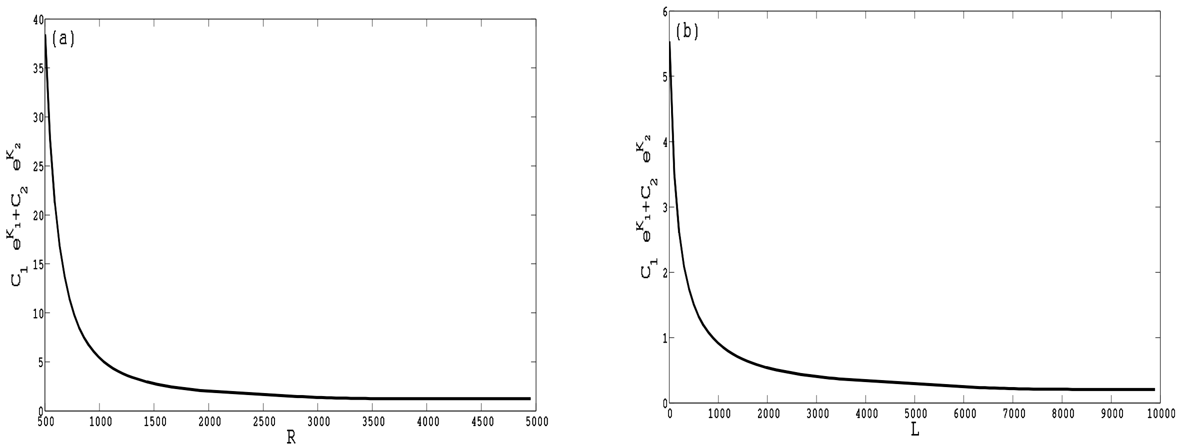

Figure 3: *Variation of u(0, 1), when (C,G) = (5.157μF/km, 118.074μ/Ω−km), (a) with R, for L = 0.4675mH/km; (b) with L, for R = 999.41/km.*

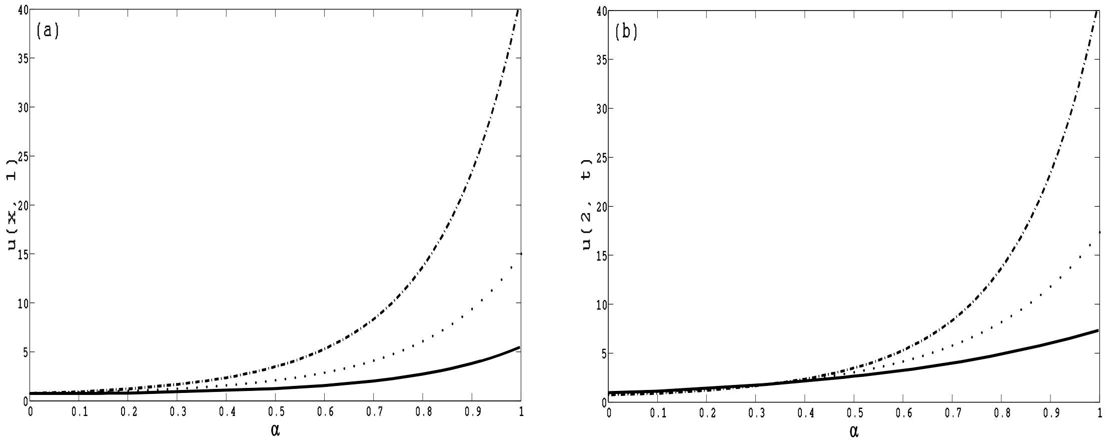

Figure 4: *Effect of α on (a) u(x 1) for x = 0 (solid curve), x = 1 (dotted curve), and x = 2 (dotted-dashed curve); (b) u(2, t) for t = 0 (solid curve), t = 0.5 (dotted curve), and t = 1 (dotted-dashed curve), when (C,G,L,R) = (5.157μF/km, 118.074μ/Ω–km, 0.4675mH/km, 999.41/km).*

5. CONCLUSION

The general initial-boundary-value problem for Transmission lines was formulated. Its concise and convenient general solution was derived analytically by using the Laplace transform method. The obtained solution was verified with the existing solutions for particular values. Moreover, a couple of particular examples were also presented in order to validate the Laplace transform method. The obtained results were also compared with the existing results. The present approach was seemed to be simple and did not involve complex mathematics. Therefore it would be useful and helpful for engineering students.

The tendency of change in voltage with respect to physical parameters; *C, G, L, R, x,* and *t* as well as boundary parameter α was examined. It was found that the dependency of physical and boundary parameters on voltage intensifies as we move away from the source (*x=0*) as well as time increases for all *C, G, L, R, x,* and *t*. In addition, at any time $t = t_o$, $u(x,t_o) \propto e^{\alpha x}$ for fixed values of parameters *C, G, L, R, t* and α. Furthermore the voltage decays as physical parameters *C, G, L, R* rise whereas the voltage will intensify with the boundary parameter α.

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