International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 3, Issue 10, October 2015, PP 1-4
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Some Results Using Non Compatible Mappings in Fuzzy Metric Space

S. K. Malhotra[1]

Department of Mathematics,
Govt. Science and Commerce College Benzir,
Bhopal (M.P.), India
skmalhotra75@gmail.com

Vineeta Singh[2]

Department of Applied Math. & Comp. Sci.,
Samrat Ashok Technological Institute,
Vidisha (M.P.), India
vinitachaturvedi@rediffmail.com

Abstract: *In this paper, we have proved the common fixed point theoremsin fuzzy metric space for noncompatible maps using the notion of R-Weakly commutativity.*

Keywords: *Fuzzy Metric Space, compatible mapping, Non compatible mapping, R-weakly commuting mapping.*

1. Introduction

The concept of Fuzzy sets was given by Zadeh[1].Then it was introduced in different ways by Deng[2], Erceg[3], Kaleva and Seikkala[4], Kramosil and Michalek[5]. George and Veeramani[6], Grabiec[7] extended the fixed point theorem of Banach[8] and Edelstein[9] for fuzzy metric space.Mishra et al[10]obtained the common fixed point theorems for compatible maps on fuzzy metric space.Aamri and Moutawakil [11] introduced the E.A. property for non-compatible maps in fuzzy metric space.

Definition1.1: Let X be any set. A fuzzy set Ain X is a function with domain X and values in [0,1].

Definition 1.2:A mapping $*: [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with unit 1 such that

$a * b \leq c * d$ for $a \leq c, b \leq d$

Examples of t-norms $a * b = min\{a, b\}$(minimum t-norm)

$a * b = ab$(product t-norm)

$a * b = max\{a + b - 1, 0\}$ (Lukasiewicz t-norm)

Definition 1.3:A 3-tuple $(X, M, *)x$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty]$ satisfying the following conditions:

(f1) $M(x, y, t) > 0$

(f2) $M(x, y, t) = 1$ if and only if $x = y$

(f3) $M(x, y, t) = M(y, x, t)$;

(f4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(f5) $M(x, y, .): (0, \infty) \to (0, 1]$ is continuous.

$x, y, z \in X$ and $t, s > 0$

Then M is called a fuzzy metric on X and $M(x, y, t)$ denotes the degree i.e. of nearness between x and y with respect to t.

Definition1.4: [] Two mappings A and S of a fuzzy metric space$(X, M,*)$ into itself are R-weakly commuting provided there exists some real number R such that$M(ASx, SAx, t) \geq M(Ax, Sx, t/R)$ for each $x \in X$and $t > 0$.

Definition 1.5: Compatible and Non compatible mappings: Let A and S be mapping from a fuzzy metric space $(X, M,*)$ into itself. Then the mappings are said to be compatible if

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1, \forall\, t > 0,$$

whenever$\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X$$

from the above definition it is inferred that A and S are non compatible maps from a fuzzy metric space $(X, M,*)$ into itself if

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X$$

but either

$\lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$,or the limit does not exist.

2. Main Results

Theorem 2.1: Let f and g be noncompatible point wise R-weakly commutingselfmappings of type (Ag) of a fuzzy metric space $(X, M,*)$ satisfying

(i) $fX \subset gX$,

(ii) $M(fx, fy, kt) \geq M(gx, gy, t), k \geq 0$,and$M$

(iii) $M(fx, f2x, t) > \max\{M(gx, gfx, t), M(fx, gx, t), M(f2x, gfx, t), M(fx, gfx, t),$

$$M(gx, f2x\,, t), M(g2x, fx, t)\},$$

whenever $fx \neq f^2x$.

If the range of f or g is a complete subspace of X, then f and g have a commonfixed point .

Proof:Since f and g are satisfying property $(E.A.)$ then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = P$$

For some $P \in X$,since $P \in fx$ and $fX \subset Gx$ there exists some point u in X such that $P = gu$.

Where$P = \lim_{n\to\infty} gx_n$if $fu \neq gu$

The inequality

$$M(Fxn, fu, kt) \geq M(gx, gu, t)$$

Letting $n \to \infty$ yields

$$M(gu, fu, kt) \geq M(gu, gu, t)$$

Hence $fu = gu$ since f and g are R-weakly commuting then there exists $R > 0$ such that

$$M(fgu, gfu, t) \geq M(fu, gu, t/R) = 1$$

That is $fgu = gfu$ and $ffu = fgu = gfu = ggu$ if$fu \neq ffu$ then we get put $x = u$

$M(fu, ffu, t) > max\{M(gu, gfu, t), M(fu, gu, t), M(ffu, gfu, t), M(fu, gfu, t), M(gu, ffu, t),$
$M(ggu, fu, t)$

$$= M(fu, ffu, t)$$

Is a contradiction,hence $fu = ffu$ and$Fu = ffu = fgu = gfu = ggu$.

Hence fu is common fixed point of f and$g$.

Theorem 2.2 : Let f and g be noncompatible pointwise R-weakly commuting self mappings of type (Ag) of a fuzzy metric space $(X, M,*)$ satisfying

(i) $fX \subset gX$,

(ii) $M(fx, fy, kt) \geq M(gx, gy, t), k \geq 0$,and$M$

(iii)$M(fx, f2x, t) > max\{M(gx, gfx, t), M(fx, gx, t), M(f2x, gfx, t), M(fx, gfx, t),$

$$M(gx, f2x, t), M(g2x, fx, t)\},$$

whenever $fx \neq f2x$.

If the range of f or g is a complete subspace of X, then f and g have a common fixed point and the fixed point is the point of discontinuity.

Proof: Since f and g are noncompatible maps there exists a sequence$\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = P$$

For some $p \in X$, but either $\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$or the limit does not exist.

Since $p \in Fx$and $fX \subset Gx$, there exists some point u in X such that$P = gu$ where$p = \lim_{n\to\infty} gx_n$. If $fu \neq gu$, the inequality

$$M(fx_n, fu, kt) \geq M(gx_n, gu, t)$$

As $x = x_n, y = u$ letting $n \to \infty$ yields

$$M(gu, fu, kt) \geq M(gu, gu, t)$$

Hence $fu = gu$.

Since f and g are R-weakly commutating of type(Ag),there exists $R > 0$ such that

$$M(ffu, gfu, t) \geq M(fu, gu, t/R) = 1$$

That is $fu = gfu$ and $ffu = fgu = gfu = ggu$.

If $fu \neq ffu$ using (iii) we get put $x = u$

$M(fu, ffu, t) > max\{M(gu, gfu, t), M(fu, gu, t), M(ffu, gfu, t), M(ffu, gfu, t),$

$M(gu, ffu, t), M(ggu, fu, t)\}$

$= M(fu, ffu, t)$

It is contradiction. Hence $fu = ffu$and$Fu = ffu = fgu = gfu = ggu$.

Hence fu is a common fixed point of f and g.

The case when Fx is a complete subspace of X is similar to the above case since$fX \subset Gx$we now show that f and g are discontinuous at the common fixed point $p = fu = gu$.

If possible, suppose f is continuous.Then considering the sequence $\{x_n\}$ of (1) we get $\lim_{n\to\infty} ffx_n = fp = p$. R-weakly commutativity of type (Ag) implies that

$$M(ffx_n, gfx_n, t) \geq M(fx_n, gx_n, t/R) = 1$$

Which on letting $n \to \infty$ this yields$\lim_{n\to\infty} gfx_n = fp = p$.This in turn yields $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$.This contradicts the fact that$\lim_{n\to\infty} M(fgx_n, gfx_n, t)$ is either nonzero or nonexistent for the sequence$\{x_n\}$ of (1).Hence f is discontinuous at the fixed point.Next, suppose that g is continuous. Then for the sequence $\{x_n\}$ of (1),we get$\lim_{n\to\infty} gfx_n = gp = p$andlim$n \to \infty$ $ggxn=gp=p$. In view of these limits,the inequality

$$M(fx_n, fgx_n, kt) \geq M(gx_n, ggx_n, t)$$

Yields a contradiction unless$\lim_{n\to\infty} fgx_n = fp = gp$, but$fgx_n = gpand\lim_{n\to\infty} gfx_n = gp$ contradicts the fact that $\lim_{n\to\infty} d(fgx_n, gfx_n)$ is either nonzero or nonexistent. Thus both f and g are discontinuous at their common fixed point.Hence the theorem is proved.

3. Conclusion

In this paper we have extended some results for the common fixed point theorem using non compatible mappings in fuzzy metric space. Results are proved without exploiting the notion of continuity and without imposing any condition of t-norm.

References

[1] Zadeh L. A., Fuzzy sets inform and control, 8(1965), 338-353.

[2] Deng Z. K., Fuzzy Pseudo metric space, J. Math Anal Appl., 86(1982), 74-95.

[3] Erceg M. A., Metric space in fuzzy set theory, J. Math. Anal Appl., 86(1982), 74-95.

[4] Kaleva O. and Seikkala S., On Fuzzy metric spaces, Fuzzy sets and systems, 12(1984), 215-229.

[5] Kramosil O. and Michalek J., Fuzzy metric and statistical metric spaces, Kybernetika, 11(1975), 326-334.

[6] George A., and Veeramani P., On some results in fuzzy metric spaces, Fuzzy sets and systems, 64(1994), 395-399.

[7] Grabiec M., Fixed points in fuzzy metric space, Fuzzy sets and systems, 27(1988), 385-389.

[8] Banach S., TheoriclesoperationLineaijzes, ManograieMathematyezne, Warsaw, Poland, 1932.

[9] Edelstein M., On Fixed and periodic points under contraction mappings, J. London Math. Soc., 37(1962), 74-79.

[10] Mishra S. N., Sharma N. and Singh S. L., Common fixed points of maps on fuzzy metric spaces, International J. Math and Math. Sci., 17(1994), 253-258.

[11] Aamri M. and Maoutawakil, Some new common fixed point theorems under strict contractive conditions, J. Math. Appl., 270(2002), 181-188.

Authors' Biography

Dr. Sandeep Kumar Malhotra, working as associate professor Govt. Science and Commerce College Benzir, Benzir Bhopal in Department of Mathematics. He did Ph.d from Barkatullah University Bhopal and his area of specialization is fixed point theory. He is research supervisor in Barkaullah University and guiding to number of research scholars. His research Interest in Fixed point theory, Fuzzy metric space, abstract spaces etc….

Mrs. Vineeta Singh, Working as Assistant professor in Samrat Ashok technological institute in Department of Applied mathematics and computer science. She is a research scholar in Barkatullah University Bhopal and Postgraduate from MLB college bhopal . She is doing research in fuzzy metric spaces.