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Some Non Weak Neighbourhood Magic Graphs Obtained from Path

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Abstract: *Let G be a graph. Let f: V∪E→ $\{1,2,\dots,v+\epsilon\}$ be a one to one map. f is called a weak neighbourhood magic labeling if $\sum_{v\in N[u]} f(v)$ is constant. In this paper, some non weak neighbourhood magic graphs obtained from path are studied.*

Keywords: *Neighbourhood, path, duplication, magic, labeling*

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1. Introduction

The graphs considered here are finite, undirected and without loops or multiple edges. Terms not defined here are used in the sense of Harary[1].The symbols *V(G)* and *E(G)* denote respectively the vertex set and edge set of a graph.

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex (edge) labeling. There are several types of graph labeling and a detailed survey is found in [2].

V.Swaminathan and A.Subramanian introduced the concept of weak neighbourhood magic labeling in [3].They studied the weak neighbourhood magic labeling and non weak neighbourhood magic labeling of some standard graphs.

1.1 Definition

Let G be a graph. Let *f*: V∪E→ $\{1,2,\dots,v+\epsilon\}$ be a one to one map. f is called a weak neighbourhood magic labeling if $\sum_{v\in N[u]} f(v)$ is constant.

The following are some results in weak neighbourhood magic labeling.

1.2 Previous Results[3]

i. K_n has weak neighbourhood magic labeling
ii. C_3 has weak neighbourhood magic labeling
iii. C_n has no weak neighbourhood magic labeling if n≥ 4
iv. P_n $(n \geq 3)$ has no weak neighbourhood magic labeling
v. $K_{m,n}$ $(m \text{ or } n \geq 2)$ has no weak neighbourhood magic labeling

In this paper, the non weak neighbourhood magic labeling of the graphs obtained from the path are studied.

2. Results

2.1 Definition [4]

Duplication of a vertex v of a graph G produces a new graph G' by adding a new vertex v' such that $N(v')$ = $N(v)$

2.2 Theorem

The graph obtained by duplicating an arbitrary vertex of the path P_n has no weak neighbourhood magic labeling

Proof: Let G be the graph obtained by duplicating an arbitrary vertex of the path P_n.

Let $V(G) = \{u, v_i; 1 \leq i \leq n\}$ where u is the duplicated vertex. Suppose f is a weak neighbourhood magic labeling of G.

Case (i): Suppose i = 1 or n. Without loss of generality, let i =1.

Let $f(u) + f(v_2) = K$ (Say). So $f(v_1) + f(v_2) = K$

Then $f(u) + f(v_2) = f(v_1) + f(v_2)$ and hence $f(u) = f(v_1)$.

Since f is one-one, we have $u = v_1$.This is a contradiction.

Case (ii): Suppose $1 < i < n$.

Let $f(v_i) + f(v_{i-1}) + f(v_{i+1}) = K$ (Say).So $f(u) + f(v_{i-1}) + f(v_{i+1}) = K$

Then $f(v_i) + f(v_{i-1}) + f(v_{i+1}) = f(u) + f(v_{i-1}) + f(v_{i+1})$

Implies $f(u) + f(v_2) = f(v_i) + f(v_2)$

Implies $f(u) = f(v_i)$. Since f is one-one, we have $u = v_i$.This is also a contradiction.

Hence G has no weak neighbourhood magic labeling.

2.3 Theorem

The graph obtained by duplicating all the vertices of the path P_n has no weak neighbourhood magic labeling.

Proof: Let G be the graph obtained by duplicating all the vertices of the path P_n.

Let V (G) = $\{v_i, u_i; 1 \leq i \leq n\}$ where u_i is the new vertex obtained by duplicating the vertex v_i. Suppose f is a weak neighbourhood magic labeling of G.

Let $f(v_i) + f(v_{i+1}) + f(u_{i+1}) = K$ (Say).So $f(u_{i+1}) + f(v_i) + f(v_{i+2}) = K$

Then $f(v_i) + f(v_{i+1}) + f(u_{i+1}) = f(u_{i+1}) + f(v_i) + f(v_{i+2})$

Implies $f(v_{i+1}) = f(v_{i+2})$

Since f is one-one, $v_{i+1} = v_{i+2}$.This is a contradiction. Hence the graph obtained by duplicating all the vertices of the path P_n has no weak neighbourhood magic labeling.

2.4 Definition [4]

Duplication of an edge $e = uv$ of a graph G produces a new graph G' by adding an edge $e' = u'v'$ such that $N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$

2.5 Theorem

The graph obtained by duplicating an arbitrary edge of the path P_n has no weak neighbourhood magic labeling

Proof: Let G be the graph obtained by duplicating an arbitrary edge of the path P_n.

Let $V(G) = \{u_j, u_{j+1}, v_i; 1 \leq i \leq n\}$ where $u_j u_{j+1}$ is the new edge obtained by duplicating the edge $v_j v_{j+1}$ .Suppose f is a weak neighbourhood magic labeling of G.

Case(i): Let a pendant edge be duplicated. Without loss of generality, let $v_1 v_2$ be the duplicated edge.

Let $f(u_2) + f(u_1) + f(v_3) = K$(Say).So $f(v_3) + f(u_2) + f(v_2) + f(v_4) = K$

Then $f(u_2) + f(u_1) + f(v_3) = f(v_3) + f(u_2) + f(v_2) + f(v_4)$

Implies $f(u_1) = f(v_2) + f(v_4)$ (1)

Similarly $f(u_1)$ $+f(u_2)$= $f(v_3) + f(u_2) + f(v_2)$ $+f(v_4)$

Implies $f(u_1)$= $f(v_3) + f(v_2)$ $+f(v_4)$ (2)

From *(1)* and *(2)* we have $f(v_3) = 0$.This is a contradiction.

Case(ii): Let the edge other than the pendant edge be duplicated. Let $v_i v_{i+1}$ be the duplicated edge. Let $u_i u_{i+1}$ be the new edge.

Let $f(u_i) + f(u_{i+1})$ $+f(v_{i-1})$ =K(Say) .So $f(v_{i-1}) + f(u_i)$ $+f(v_i)$ $+f(v_{i-2}) = K$

Then $f(u_i) + f(u_{i+1})$ $+f(v_{i-1}) = f(v_{i-1})$ $+f(u_i)$ $+f(v_i)$ $+f(v_{i-2})$

Implies $f(u_{i+1})$ = $f(v_i)$+ $f(v_{i-2})$ (3)

Also $f(u_{i+1})$ + $f(u_i)$ + $f(v_{i+2})$=K

Then $f(u_{i+1})$ + $f(u_i)$ + $f(v_{i+2})$= $f(v_{i-1})$ + $f(v_{i-2})$ + $f(v_i)$+ $f(u_i)$

Implies $f(u_{i+1}) + f(v_{i+2})$ = $f(v_{i-1})$ + $f(v_{i-2})$ + $f(v_i)$ (4)

From (3) and (4), $f(v_{i+2})$ = $f(v_{i-1})$

Since f is one-one, v_{i+2} = v_{i-1}.This is a contradiction. Hence the graph obtained by duplicating an arbitrary edge of the path P_n has no weak neighbourhood magic labeling.

2.6 Theorem

The graph obtained by duplicating all the edges of the path P_n has no weak neighbourhood magic labeling.

Proof: Let G be graph obtained by duplicating all the edges of the path P_n.

Let $V(G)$ = $\{u_j, v_j; 1 \leq j \leq n\}$ where $u_j u_{j+1}$ is the new edge obtained by duplicating the edge $v_j v_{j+1}$ .Suppose f is a weak neighbourhood magic labeling of G.

Let $f(v_{i+1})$ $+f(v_i)$ $+f(v_{i+2})$ $+f(u_i)$ $+f(u_{i+2})$ =K (Say).So $f(u_{i+1}) + f(u_i) + f(u_{i+2}) + f(v_i) + f(v_{i+2}) = K$

Implies $f(v_{i+1})$+$f(v_i)$+$f(v_{i+2})$+$f(u_i)$+$f(u_{i+2})$= $f(u_{i+1}) + f(u_i) + f(u_{i+2}) + f(v_i) + f(v_{i+2})$

Implies $f(v_{i+1}) = f(u_{i+1})$

Since f is one-one, v_{i+1} = u_{i+1}.This is a contradiction. Hence the graph obtained by duplicating all the edges of the path P_n has no weak neighbourhood magic labeling.

2.7 Definition [4]

Duplication of a vertex v_k by a new edge e = $v'_k v''_k$ in a graph G produces a new graph G' such that $N(v'_k)$ ={v_k, v''_k } and $N(v''_k)$ ={v_k, v'_k }

2.8 Theorem

The graph obtained by duplicating an arbitrary vertex by an edge of the path P_n has no weak neighbourhood magic labeling.

Proof: Let G be the graph obtained by duplicating an arbitrary vertex by an edge of the path P_n

Let $V(G)$ = $\{v'_k, v''_k, v_i; 1 \leq i \leq n\}$ where $v'_k v''_k$ is the new edge obtained by duplicating the vertex v_k.Suppose f is a weak neighbourhood magic labeling of G.

Case (i): Let a pendant vertex be the duplicated. Without loss of generality, let v_1 be the duplicated vertex.

Let $f(v_1) + f(v'_1)$ $+f(v''_1) + f(v_2)$ =K(Say).So $f(v_2)$ $+f(v_1) + f(v_3) = K$

Then $f(v_1) + f(v'_1)$ $+f(v''_1) + f(v_2) = f(v_2)$ $+f(v_1) + f(v_3)$

Implies $f(v'_1)$ $+f(v''_1)$ = $f(v_3)$ (5)

Also $f(v'_1)$ $+f(v''_1) + f(v_1) = K$

Then $f(v_1') + f(v_1'') + f(v_1) = f(v_2) + f(v_1) + f(v_3)$

Implies $f(v_1') + f(v_1'') = f(v_2) + f(v_3)$ (6)

From (5) and (6), $f(v_2) = 0$. This is a contradiction.

Case (ii): Let a vertex v_k other than the pendant vertex be duplicated. Let $v_k' v_k''$ be the duplicated edge.

Let $f(v_k) + f(v_{k-1}) + f(v_{k+1}) + f(v_k') + f(v_k'')$ = K(Say).So $f(v_k') + f(v_k'') + f(v_k) = K$

Then $f(v_k) + f(v_{k-1}) + f(v_{k+1}) + f(v_k') + f(v_k'') = f(v_k') + f(v_k'') + f(v_k)$

Implies $f(v_{k-1}) + f(v_{k+1}) = 0$

Implies $f(v_{k-1}) = -f(v_{k+1})$ which is impossible. Hence the graph obtained by duplicating an arbitrary vertex by an edge of the path P_n has no weak neighbourhood magic labeling.

2.9 Theorem

The graph obtained by duplicating all the vertices of the path P_n by edges has no weak neighbourhood magic labeling.

Proof: Let G be the graph obtained by duplicating all the vertices of the path P_n by edges. Let $V(G) = \{v_i', v_i'', v_i; 1 \leq i \leq n\}$ where $v_i' v_i''$ is the new edge obtained by duplicating the vertex v_i.Suppose f is a weak neighbourhood magic labeling of G.

Let $f(v_1) + f(v_1') + f(v_1'') + f(v_2) = K$(Say). So $f(v_1') + f(v_1'') + f(v_1) = K$

Then $f(v_1) + f(v_1') + f(v_1'') + f(v_2) = f(v_1') + f(v_1'') + f(v_1)$

Implies $f(v_2) = 0$ which is impossible. Hence the graph obtained by duplicating all the vertices of the path P_n by edges has no weak neighbourhood magic labeling.

2.10 Definition [4]

Duplication of an edge e = uv by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$

2.11 Theorem

The graph obtained by duplicating an arbitrary edge by a vertex of the path P_n has no weak neighbourhood magic labeling.

Proof: Let G be the graph obtained by duplicating an arbitrary edge by a vertex of the path P_n Let $V(G) = \{w, v_i; 1 \leq i \leq n\}$ where w is the new vertex obtained by duplicating the edge $v_i v_{i+1}$.Suppose f is a weak neighbourhood magic labeling of G.

Case(i): Let a pendant edge be duplicated. Without loss of generality, let $v_1 v_2$ be the duplicated edge.

Let $f(w) + f(v_1) + f(v_2) = K$(Say).So $f(v_2) + f(w) + f(v_1) + f(v_3) = K$

Then $f(w) + f(v_1) + f(v_2) = f(v_2) + f(w) + f(v_1) + f(v_3)$

Implies $f(v_3) = 0$ which is impossible.

Case(ii): Let the edge other than the pendant edge be duplicated. Let $v_i v_{i+1}$ be the duplicated edge.

Let $f(w) + f(v_i) + f(v_{i+1}) = K$(Say).So $f(v_i) + f(v_{i+1}) + f(w) + f(v_{i-1}) = K$

Then $f(w) + f(v_i) + f(v_{i+1}) = f(v_i) + f(v_{i+1}) + f(w) + f(v_{i-1})$

Implies $f(v_{i-1}) = 0$ which is also impossible.Hence the graph obtained by duplicating an arbitrary edge by a vertex of the path P_n has no weak neighbourhood magic labeling.

2.12 Theorem

The graph obtained by duplicating all the edges of the path P_n by vertices has no weak neighbourhood magic labeling.

Proof: Let G be the graph obtained by duplicating all the edges of the path P_n by vertices. Let $V(G) = \{v_{i,}u_j; 1 \le i \le n, 1 \le j \le n-1\}$ where $u_i$is the new vertex obtained by duplicating the edge $v_i v_{i+1}$.Suppose f is a weak neighbourhood magic labeling of G.

Let $f(v_i) + f(u_i) + f(v_{i+1}) = K$(Say).So $f(v_{i+1})$+$f(v_i)$+$f(u_i)$+$f(u_{i+1})$+$f(v_{i+2})$=K

Then $f(u_{i+1})$+$f(v_{i+2}) = 0$.Implies $f(u_{i+1}) = -f(v_{i+2})$ which is impossible. Hence the graph obtained by duplicating all the edges of the path P_n by vertices has no weak neighbourhood magic labeling.

3. Conclusion

In this paper, some non weak neighbourhood magic graphs obtained by duplicating the elements of a path are studied. Efforts will be made in future to generate weak neighbourhood magic graphs from non weak neighbourhood magic graphs.

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