# Movement across a Long Strike-Slip Fault and Stress Accumulation in the Lithosphere-Asthenosphere System with Layered Crust Model

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**Abstract:** A long vertical strike -slip fault is taken to be situated in a layered elastic crust in welded contact with linear viscoelastic half-space representing the lithosphere –asthenosphere system in the present paper. A movement of the strike -slip nature across the fault occurs when the accumulated stress due to various tectonic reasons e.g. mantle convection etc, exceeds the local friction and cohesive forces across the fault. The movement is assumed to be slipping in nature, expressions for displacements, stresses and strains are obtained by suitable mathematical methods. A detailed study of these expressions may give some ideas about the nature of stress accumulation in the system, which in turn will be helpful in formulating an earthquake prediction programme.

Keywords: Aseismic period, layered crust, Mantle convection, Plate movements, Stress accumulation

# **1. INTRODUCTION**

Observations indicate that the lithospheric properties vary as we move downwards from the free surface of the earth. Therefore in spite of taking an elastic layer over a viscoelastic half-space to represent the lithosphere-asthenosphere system we have considered two elastic layers, welded contact with a viscoelastic half-space to represent the lithosphere-asthenosphere system.

A pioneering work involving static ground deformation in elastic media were initiated by [31-32]. [4-7], [15-16]. References [23], [24], [29] did wonderful works in analyzing the displacement, stress and strain for dip-slip movement. Later some theoretical models in this direction have been formulated by a number of authors like [2], [3], [8],[9], [11], [13], [18-21], [22], [25], [26], [28], [33], [41], [42],[43]. Reference; [30] has discussed various aspects of fault movement in his book. Reference; [12] has discussed stress accumulation near buried fault in lithosphere-asthenosphere system. The work [10] can also be mentioned.

In most of these works the medium were taken to be elastic and /or viscoelastic, some authors preferred layered model with elastic layer(s) over elastic or viscoelastic half space.

In the present case we consider a long strike-slip fault situated in a 3-layered elastic-viscoelastic half space in which two elastic layers are in welded contact with a viscoelastic half-space as is mentioned above. The medium is taken to be under the influence of tectonic forces due to mantle convection or some related phenomena. The fault is assumed to undergo a slipping movement when the stresses in the region exceed certain threshold values.

Analytical expressions for displacements, stresses and strains are obtained both before and after the fault movement using appropriate mathematical technique involving integral transformation, Green's function.

## **2. FORMULATION**

We consider a vertical long strike-slip fault F, width D situated in a elastic layer. A Cartesian coordinate system is used with a suitable point O on the strike of the fault as the origin,  $Y_1$  axis along the strike of the fault and  $Y_2$  axis is taken to be perpendicular to the fault plane, and  $Y_3$  axis pointing downwards as shown in Figure 1 below, so that the fault is given by

*F*: 
$$(y_2 = 0, 0 \le y_3 \le D)$$
.

Let  $u^k$ ,  $e^k_{ij}$ ,  $\tau^k_{ij}$  are displacement, strain and stress tensor respectively, k=1, 2, 3 for 1<sup>st</sup>, 2<sup>nd</sup> layer and half-space respectively.



**Fig1.** Section of the model by the plane  $y_1=0$ .

### 2.1. Constitutive Equations

For an elastic and viscoelastic Maxwell type medium the constitutive equations are taken as:

$$\tau^{1}_{12} = \mu_{1}(e^{1}_{12}) = \mu_{1}\left(\frac{\partial u^{1}}{\partial y_{2}}\right)$$
(1.1)

$$\tau^{1}_{13} = \mu_{1}(e^{1}_{13}) = \mu_{1}\left(\frac{\partial u^{1}}{\partial y_{3}}\right)$$
(1.2)

 $(-\infty < y_2 < \infty, 0 \le y_3 \le h_1)$ 

$$\tau^{2}_{12} = \mu_{2}(e^{2}_{12}) = \mu_{2}\left(\frac{\partial u^{2}}{\partial y_{2}}\right)$$
(1.3)

$$\tau^{2}_{13} = \mu_{2}(e^{2}_{13}) = \mu_{2}\left(\frac{\partial u^{2}}{\partial y_{3}}\right)$$
(1.4)

 $(-\infty \!\! < \!\! y_2 \!\! < \!\! \infty, \, h_1 \! \le \! y_3 \! \le h_1 \! + \! h_2)$ 

$$\left(\frac{1}{\eta_3} + \frac{1}{\mu_3}\frac{\partial}{\partial t}\right)\tau^{3}_{12} = \frac{\partial}{\partial t}(e^{3}_{12}) = \frac{\partial}{\partial t}\left(\frac{\partial u^3}{\partial y_2}\right)$$
(1.5)

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$$\left(\frac{1}{\eta_3} + \frac{1}{\mu_3}\frac{\partial}{\partial t}\right)\tau^3{}_{13} = \frac{\partial}{\partial t}(e^3{}_{13}) = \frac{\partial}{\partial t}\left(\frac{\partial u^3}{\partial y_3}\right)$$
(1.6)

 $(-\infty \le y_2 \le \infty, h_2 \le y_3 \le \infty, t \ge 0)$ 

Where  $\eta_k$  is the effective viscosity and  $\mu_k$  is the effective rigidity of the material for k=1,2,3.

### 2.2. Stress Equation of Motions

The stresses satisfy the following equations: (assuming quasistatic deformation for which the inertia terms are neglected).

$$\frac{\partial}{\partial y_2}(\tau^{k_{12}}) + \frac{\partial}{\partial y_3}(\tau^{k_{13}}) = 0$$
(1.7)

(Assuming the body forces do not change during the investigation).

#### 2.3. The Boundary Conditions

We take t=0 representing an instant when the medium is in aseismic state:

as 
$$|y_{2}| \rightarrow \infty, y_{3} \ge 0, t \ge 0$$
  
 $\tau^{k_{12}}(y_{2}, y_{3}, t) = \tau_{\infty}(t)$  (1.8)  
 $e^{1}_{12}(y_{2}, y_{3}) = (e^{1}_{12})_{0\infty} + f(t)$   
 $e^{2}_{12}(y_{2}, y_{3}) = (e^{2}_{12})_{0\infty} + f(t)$   
 $e^{3}_{12}(y_{2}, y_{3}, t) = (e^{3}_{12})_{0\infty} + f(t)$ 

Where,  $(e^{k_{12}})_{0\infty} = \lim_{y_2 \to \pm\infty} (e^{k_{12}})_0$  for k=1,2,3 and  $(e^{k_{12}})_0$  is the initial value of  $(e^{k_{12}})_1$ .

f(t) is a continuous function of t, such that f(0)=0. The same function f(t) is taken for all the strain functions to ensure the boundary conditions.

On the free surface  $y_3 = 0, (-\infty < y_2 < \infty, t \ge 0)$ 

$$\tau^{1}_{13}(y_2, y_3, t) = 0 \tag{1.9}$$

Also, as  $y_3 \rightarrow \infty(-\infty < y_2 < \infty, t \ge 0)$ 

$$\tau^{3}_{13}(y_2, y_3, t) = 0 \tag{1.10}$$

for 
$$-\infty < y_2 < \infty$$
,  $y_3 = h_1$ ,

$$\tau^{1}_{12}(y_2, y_3, t) = \tau^{2}_{12}(y_2, y_3, t)$$
(1.11)

$$\mathbf{u}^{1} = \mathbf{u}^{2} \tag{1.12}$$

At 
$$y_3 = h_1 + h_2, -\infty < y_2 < \infty, t \ge 0$$

$$\tau^{2}_{12}(y_{2}, y_{3}, t) = \tau^{3}_{12}(y_{2}, y_{3}, t)$$
(1.13)

$$u^2 = u^3$$
 (1.14)

Where  $\tau_{\infty}$  (t) is the shear stress maintained by mantle convection and other tectonic phenomena far away from the fault which is connected with f(t) by means of the following relationship

$$\tau_{\infty}(t) = \mu \int_{0}^{t} \frac{u}{u + \frac{\mu}{\eta}} f(t - u) du + \tau_{\infty}(0) e^{-\frac{\mu}{\eta}t}$$

#### 2.4. The Initial Conditions

Let $(u^k)_{0,}(\tau^k_{ij})_0$  and  $(e^k_{ij})_0$  i,j=1,2,k=1,2,3. be the value of  $(u^k)_{ij}$ ,  $(\tau^k_{ij})_{ij}$  and  $(e^k_{ij})_{ij}$  at time t=0 which are functions of  $y_2$ ,  $y_3$  and satisfy the relations (1.1)-(1.14).

#### 3. SOLUTIONS IN THE ABSENCE OF ANY FAULT DISLOCATION

The boundary value problem given by (1.1)-(1.14), can be solved(as shown in the Appendix-1) by taking Laplace transformation with respect to time 't' of all the constitutive equations and the boundary conditions. On taking the inverse Laplace transformation we get the solutions for displacements, stresses as:

$$u^{1}(y_{2}, y_{3}, t) = (u^{1})_{0} + y_{2} f(t) + \left(\frac{h_{1}\tau_{h_{1}}t}{\eta_{2}}\right)$$

$$\tau^{1}_{12}(y_{2}, y_{3}, t) = (\tau^{1}_{12})_{0} + \mu_{1} f(t)$$

$$\tau^{1}_{13}(y_{2}, y_{3}, t) = (\tau^{1}_{13})_{0}$$

$$e^{1}_{12}(y_{2}, y_{3}, t) = (e^{1}_{12})_{0} + f(t)$$

$$e^{1}_{13}(y_{2}, y_{3}, t) = (e^{1}_{13})_{0}$$

$$u^{2}(y_{2}, y_{3}, t) = (u^{2})_{0} + y_{2} f(t) + \left(\frac{y_{3}\tau_{h_{1}}t}{\eta_{2}}\right)$$

$$\tau^{2}_{12}(y_{2}, y_{3}, t) = (\tau^{2}_{12})_{0} + \mu_{2} f(t)$$

$$\tau^{2}_{13}(y_{2}, y_{3}, t) = (\tau^{2}_{13})_{0}$$
(A)  
From the above solution we find that  $\tau^{1}_{12}$  increases with time and tends to  $\tau_{w}(t)$  as  $t$  tends to  $\infty$ 

From the above solution we find that  $\tau_{12}^{t}$  increases with time and tends to  $\tau_{\infty}(t)$  as t tends to  $\infty$ , while  $\tau_{13}^{t}$  remains the same as $(\tau_{13}^{t})_{0}$ . We assume that the geological conditions as well as the characteristic of the fault is such that when the stress-component  $\tau_{12}^{t}$  reaches some critical value, say  $\tau_{c} < \tau_{\infty}(t)$  the fault F undergoes a sudden slip.

For bounded stress and strains, the slip function should satisfy the following conditions as discussed in [20].

 $(C_1)$  Its value will be maximum on the free surface.

 $(C_2)$  The magnitude of the creep will decrease with  $y_3$  as we move downwards and ultimately tends to zero near the lower edge of the fault.

 $(y_2 = 0, y_3 = D)$ 

If  $g(x_3)$  be the slip function, it should satisfy the above conditions.

#### 4. SOLUTIONS AFTER THE FAULT MOVEMENTS

We assume that after a time  $T_1$ , the stress component  $\tau_{12}^1$  (which is the main driving force for the strike-slip motion of the fault) exceeds the critical value  $\tau_c$ , and the fault *F* slips, characterized by a dislocation across the fault given by.

$$[(u^{1})]_{F} = Ug(y_{3})H(t_{1})$$

Where,  $H(t_1)$  is the Heaviside function and

 $[(u^{1})]_{F}$  = the discontinuity of u<sup>1</sup> across *F* given by:

$$[(u^{1})]_{F} = \lim_{(y'_{2} \to 0^{+})} (u^{1}) - \lim_{(y'_{2} \to 0^{-})} (u^{1})$$

 $(y_2 = 0, 0 \le y_3 < D)$ , U is a constant.

We solve the resulting boundary value problem by modified Green's function method following [14-15], [26-27] and correspondence principle (As shown in the Appendix-1) and get the solution for displacements, stresses and strain as :

Case1.

When P is any point on the fault in  $\mu_1$  medium and Q is any point in the  $\mu_1$  medium.

$$u^{1}(y_{2}, y_{3}, t) = (u^{1})_{0} + y_{2}f(t) + U[H(t - T_{1})/2\pi]\phi_{1}(y_{2}, y_{3})$$
  

$$\tau^{1}_{12}(y_{2}, y_{3}, t) = (\tau^{1}_{12})_{0} + \mu_{1}f(t) + \mu_{1}U[H(t - T_{1})/2\pi]\phi_{2}(y_{2}, y_{3})$$
  

$$\tau^{1}_{13}(y_{2}, y_{3}, t) = (\tau^{1}_{13})_{0} + \mu_{1}U[H(t - T_{1})/2\pi]\phi_{3}(y_{2}, y_{3})$$

Where,

$$\phi_1(y_2, y_3) = \int_F g(x_3) G(P,Q) dx_3$$

$$\phi_2(y_2, y_3) = \frac{\partial}{\partial y_2} \phi_1(y_2, y_3)$$
$$\phi_3(y_2, y_3) = \frac{\partial}{\partial y_3} \phi_1(y_2, y_3)$$

$$G(P,Q)=\mu_1\partial/\partial x_2G_1(P,Q)$$

$$G_{1}(P,Q) = \left(\frac{1}{2\pi \times \mu_{1}}\right) \{\log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - y_{3})^{2}} + \log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} + y_{3})^{2}} \} + \frac{4\mu_{1}\mu_{2}}{(\mu_{1} + \mu_{2})^{2}} \sum_{m=1}^{\infty} \left(\frac{\mu_{2} - \mu_{3}}{\mu_{2} + \mu_{3}}\right)^{m} \left(\frac{\mu_{2} - \mu_{1}}{\mu_{2} + \mu_{1}}\right)^{m-1} \times \sum_{r=1}^{m} {m \choose r} (\alpha - \beta)^{r} e^{-\beta t} \frac{t^{r-1}}{(r-1)!} \times \left\{\log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m-1)h_{1} - y_{3})^{2}} + \log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m-1)h_{1} + y_{3})^{2}} \right\}$$

Similarly

$$\mathbf{u}^{2}(\mathbf{y}_{2},\mathbf{y}_{3},\mathbf{t}) = (\mathbf{u}^{2})_{0} + (\mathbf{y}_{2}/\mu_{2}) \left[ (\tau_{\infty}(t) - \tau_{\infty}(0) + \left(\frac{\mu_{2}}{\eta_{2}}\right) \int_{0}^{t} \tau_{\infty}(\tau) d\tau \right] + \left(\frac{y_{3}\tau_{h_{1}}t}{\eta_{2}}\right)$$

$$+U[H(t-T_{1})/2\pi]\psi_{1}(y_{2}, y_{3})$$
  

$$\tau^{2}_{12}(y_{2}, y_{3}, t) = \tau_{\infty}(t) - [\tau_{\infty}(0) - (\tau^{2}_{12})_{0}] e^{-\frac{\mu_{2}}{\eta_{2}}t} + \mu_{2}U[H(t-T_{1})/2\pi]\psi_{2}(y_{2}, y_{3})$$
  

$$\tau^{2}_{13}(y_{2}, y_{3}, t) = (\tau^{2}_{13})_{0} e^{-\frac{\mu_{2}}{\eta_{2}}t} + \tau_{h_{2}}(1 - e^{-\frac{\mu_{2}}{\eta_{2}}t}) + \mu_{2}U[H(t-T_{1})/2\pi]\psi_{3}(y_{2}, y_{3})$$

Where

$$\psi_1(y_2, y_3) = \int_F g(x_3) G(P,Q) dx_3$$
$$\psi_2(y_2, y_3) = \frac{\partial}{\partial y_2} \psi_1(y_2, y_3)$$
$$\psi_3(y_2, y_3) = \frac{\partial}{\partial y_3} \psi_1(y_2, y_3)$$

 $G(P, Q) = \frac{\mu_1 \partial}{\partial x_2} G_1(P, Q)$ 

Where P is any point on the fault in  $\mu_1$  medium and Q is any point in the  $\mu_2$  medium.

$$G_{1}(P,Q) = \left(\frac{1}{2\pi \times \mu_{1}}\right) \{\log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - y_{3})^{2}} + \log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} + y_{3})^{2}} \} + \sum_{m=1}^{\infty} \left(\frac{\mu_{2} - \mu_{3}}{\mu_{2} + \mu_{3}}\right)^{m} \left(\frac{\mu_{2} - \mu_{1}}{\mu_{2} + \mu_{1}}\right)^{m-1} \times \sum_{r=1}^{m} \binom{m}{r} (\alpha - \beta)^{r} e^{-\beta t} \frac{t^{r-1}}{(r-1)!} \times \left\{\log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m-1)h_{1} - y_{3})^{2}} + \log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m-1)h_{1} + y_{3})^{2}} \right\} + \sum_{m=1}^{\infty} \left(\frac{\mu_{2} - \mu_{3}}{\mu_{2} + \mu_{3}}\right)^{m} \left(\frac{\mu_{2} - \mu_{1}}{\mu_{2} + \mu_{1}}\right)^{m} \times \sum_{r=1}^{m} \binom{m}{r} (\alpha - \beta)^{r} e^{-\beta t} \frac{t^{r-1}}{(r-1)!} \times \left\{\log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m-1)H - y_{3})^{2}} + \log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m-1)H + y_{3})^{2}} \right\} \right\}$$
And  $\alpha = \mu_{1} \mu_{2} \eta_{2} \eta_{3} \frac{\eta_{2} - \eta_{3}}{\mu_{2} - \mu_{3}}, \beta = \mu_{1} \mu_{2} \eta_{2} \eta_{3} \frac{\eta_{2} + \eta_{3}}{\mu_{2} + \mu_{3}}, \gamma = \frac{\mu_{1}\mu_{2}}{\eta_{2}(\mu_{2} - \mu_{1})}, \delta = \frac{\mu_{1}\mu_{2}}{\eta_{2}(\mu_{2} + \mu_{1})}$ 

$$H=h_{1}+h_{2}.$$
(B)

 $H=h_1+h_2$ .

We take the slip function,

$$g(x_3) = \left(1 - \frac{3}{D^2}x_3^2 + \frac{2}{D^3}x_3^3\right), \text{ satisfying the conditions stated in } (C_1) - (C_2).$$

#### 5. RESULTS AND DISCUSSIONS

From the above solution (A) we find that  $\tau^{1}_{12}$  increases with time and tends to  $\tau_{\infty}(t)$  as t tends to  $\infty$ , while  $\tau_{13}^1$  remains the same as  $(\tau_{13}^1)_0$ . But in reality before  $\tau_{12}^1$  becoming to  $\tau_{\infty}(t)$ ,  $\tau_{12}^1$  will achieve the critical value  $\tau_c$  of the region of consideration and a movement across the fault will occur (slip) leading to an earthquake. Then after the reestablishment of the aseismic state the solutions for the displacements, stresses are given by (B). The solutions have three parts, first part is the initial one, second few terms are the effect of the tectonic forces, the terms starting with U[\*] are the effects of the fault movement and the terms with the series are due to the presence of the layers. The infinite series being convergent, therefore the displacement and the stresses are also bounded through the media.

#### 6. APPENDIX-1

#### **6.1. Solutions After the Fault Movement**

We assume that after a time T<sub>1</sub> the stress component  $\tau_{12}^1$  (which is the main driving force for the strike-slip motion of the fault) exceeds the critical value  $\tau_c$ , the fault F slips. Immediately after the slip across the fault, a disturbance in the region sets in leading to an earthquake. It will take some time (of the order of a few seconds to a few minutes) for the disturbance to die out and aseismic state to re-established. We leave out this small time-span during which the above equations are not valid and consider the model afresh with a new time origin t = 0 (say) when the

aseismic state is re-established in the system. Then we have an additional condition characterizing the dislocation in  $u^1$  due to the slipping movement as:

$$[(u^{1})]_{F} = Ug(x_{3})H(t_{1})$$
(2.1)

where,  $H(t_1)$  is the Heaviside function and  $[(u^1)]_F$ . The discontinuity of  $u^1$  across F is given by  $[(u^1)]_F = \lim_{n \to \infty} (u^1) - \lim_{n \to \infty} (u^1)$ 

$$(y_2 \to 0+) \qquad (y_2 \to 0-) \qquad (2.2)$$

$$(y_2 = 0, 0 \le y_3 < D).U$$
 is constant

Taking Laplace transformation in (2.1) we get,

$$[(u^{1})]_{F} = Ug(x_{3})$$
(2.3)

The fault slip commences across F after time  $T_1$ ,

Therefore,  $[(u^1)]_F = 0$ 

For  $t_1 \le 0$ , where  $t_1 = t - T_1$ , *F* is located in the region ( $y_2 = 0, 0 < y_3 < D$ ).

We try to find the solution as,

$$u^{k} = (u^{k})_{1} + (u^{k})_{2},$$
  

$$\tau^{k}_{12} = (\tau^{k}_{12})_{1} + (\tau^{k}_{12})_{2}, \tau^{k}_{13} = (\tau^{k}_{13})_{1} + (\tau^{k}_{13})_{2},$$
(2.4)

Where  $(u^k)_1$ ,  $(\tau^k_{ij})_1$ , are continuous everywhere in the model and are given by (A). While the second part  $(u^k)_2$ ;  $(\tau^k_{ij})_2$  are obtained by solving modified boundary value problem as stated below. The resulting boundary value problem can now be stated as: $(u^k)_2$  satisfies 2D Laplace equation as,

$$\nabla^2 (\overline{u^k})_2 = 0 \tag{2.5}$$

Where,  $(\overline{u^k})_2$  is the Laplace transformation of  $(u^k)_2$ , with the modified boundary condition  $\bar{\tau}^k_{12}(y_2, y_3, s) = 0$  as  $|y_2| \to \infty, y_3 \ge 0$  (2.6)

And the other boundary conditions are as before.

Where s is the Laplace transform variable defined as

$$(\overline{u^{k}})_{2}(s) = \int_{0}^{\infty} e^{-st} (u^{k})_{2}(t)dt$$
(2.7)

We solve the above boundary value problem by modified Green's function method following [14-15], [26-27], and the correspondence principle.

Let  $Q(y_2, y_3)$  be any point in the  $\mu_k$  medium and  $P(x_2, x_3)$  be any point in the fault, then we have,

$$(\overline{u^{k}})_{2}(Q) = \int_{F} U(P)g(x_{3})[G(P,Q)]dx_{3}$$
(2.8)

(a) If P is any point on the fault and Q is in the  $\mu_1$  medium, then,

$$G(P,Q) = \mu_1 \partial / \partial x_2 G_1(P,Q)$$

Where,

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$$\begin{split} G_{1}\left(P,Q\right) &= \left(\frac{1}{2\pi \times \mu_{1}}\right) \{\log \sqrt{\left(x_{2} - y_{2}\right)^{2} + \left(x_{3} - y_{3}\right)^{2}} + \log \sqrt{\left(x_{2} - y_{2}\right)^{2} + \left(x_{3} + y_{3}\right)^{2}} \} \\ &+ \frac{4\mu_{1}\mu_{2}}{\left(\mu_{1} + \mu_{2}\right)^{2}} \sum_{m=1}^{\infty} \left[\frac{\mu_{2} - \overline{\mu_{3}}}{\mu_{2} + \overline{\mu_{3}}}\right]^{m} \times \left[\frac{\mu_{2} - \mu_{1}}{\mu_{2} + \mu_{1}}\right]^{m-1} \times \left\{\log \sqrt{\left(x_{2} - y_{2}\right)^{2} + \left(x_{3} - \left(2m - 1\right)h_{1} - y_{3}\right)^{2}} + \log \sqrt{\left(x_{2} - y_{2}\right)^{2} + \left(x_{3} - \left(2m - 1\right)h_{1} - y_{3}\right)^{2}} + \log \sqrt{\left(x_{2} - y_{2}\right)^{2} + \left(x_{3} - \left(2m - 1\right)h_{1} + y_{3}\right)^{2}} \right\}} \end{split}$$
  
Therefore,  $\left(u^{1}\right)_{2}(Q) = U(t_{1})\phi(y_{2}, y_{3})H(t_{1})$ 

Where,  $H(t_1)$  is the Heaviside step function, which gives the displacement at any points  $Q(y_2, y_3)$ .

Where, 
$$\phi(y_2, y_3) = \int_F g(x_3)G(P,Q)dx_3$$

We also have, 
$$(\bar{\tau}_{12})_2 = \frac{U\mu_1}{(2 \times \pi)} \phi_2(y_2, y_3)$$

Where, 
$$\phi_2(y_2, y_3) = \frac{\partial}{\partial y_2} \phi(y_2, y_3)$$

$$(\bar{\tau}^{1}_{13})_{2} = \frac{U\mu_{1}}{(2 \times \pi)}\phi_{3}(y_{2}, y_{3})$$

Where, 
$$\phi_3(y_2, y_3) = \frac{\partial}{\partial y_3} \phi(y_2, y_3)$$
 (2.9)

(b) If P is on the fault but Q is in the  $\mu_2$  medium, then

$$G_{1}(P,Q) = \left(\frac{1}{\pi \times (\mu_{1} + \mu_{2})}\right) \{\log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - y_{3})^{2}} + \log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} + y_{3})^{2}} \} + \sum_{m=1}^{\infty} \left[\frac{\mu_{2} - \overline{\mu_{3}}}{\mu_{2} + \overline{\mu_{3}}}\right]^{m} \times \left[\frac{\mu_{2} - \mu_{1}}{\mu_{2} + \mu_{1}}\right]^{m-1} \times \left\{\log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m - 1)H - y_{3})^{2}} + \log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m - 1)H + y_{3})^{2}} \right\} + \sum_{m=1}^{\infty} \left[\frac{\mu_{2} - \overline{\mu_{3}}}{\mu_{2} + \overline{\mu_{3}}}\right]^{m} \times \left[\frac{\mu_{2} - \mu_{1}}{\mu_{2} + \mu_{1}}\right]^{m} \times \left\{\log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m - 1)H - y_{3})^{2}} + \log \sqrt{(x_{2} - y_{2})^{2} + (x_{3} - (2m - 1)H + y_{3})^{2}}\right\} \right\}$$

$$(2.10)$$

Taking inverse Laplace transformation,

$$(u^2)_2(Q) = U(t_1)\phi^{1}(y_2, y_3)H(t_1)$$

Where,  $H(t_1)$  is the Heaviside step function, which gives the displacement at any points  $Q(y_2, y_3)$ . Where,  $\phi_1^{(1)}(y_2, y_3) = \int_F g(x_3)G(P,Q)dx_3$ 

We also have, 
$$(\bar{\tau}_{12})_2 = \frac{U \mu_1}{(2 \times \pi)} \phi_2^1(y_2, y_3)$$

Where, 
$$\phi_{12}^{1}(y_2, y_3) = \frac{\partial}{\partial y_2} \phi_{11}^{1}(y_2, y_3)$$

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$$(\bar{\tau}^{2}{}_{13})_{2} = \frac{U\mu_{1}}{(2 \times \pi)} \phi^{1}{}_{3}(y_{2}, y_{3})$$
  
Where,  $\phi^{1}{}_{3}(y_{2}, y_{3}) = \frac{\partial}{\partial y_{3}} \phi^{1}{}_{1}(y_{2}, y_{3})$  (2.11)

(c) If P is on the fault and Q is in  $\mu_3$  medium, then

$$\begin{split} G_1(P,Q) &= \left(\frac{2\mu_2}{\pi \times (\mu_1 + \mu_2)(\mu_2 + \overline{\mu_3})}\right) \{\log \sqrt{(x_2 - y_2)^2 + (x_3 - y_3)^2} + \log \sqrt{(x_2 - y_2)^2 + (x_3 + y_3)^2}) \\ &+ \sum_{m=1}^{\infty} \left[\frac{\mu_2 - \overline{\mu_3}}{\mu_2 + \overline{\mu_3}}\right]^m \times \left[\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}\right]^{m-1} \times \left\{\log \sqrt{(x_2 - y_2)^2 + (x_3 - (2m - 1)H - y_3)^2} + \log \sqrt{(x_2 - y_2)^2 + (x_3 - (2m - 1)H + y_3)^2}\right\} \end{split}$$

Taking inverse Laplace transformation,

$$(u^{3})_{2}(Q) = U\psi_{1}(y_{2}, y_{3})H(t_{1})$$
(2.12)

Where,  $H(t_1)$  is the Heaviside step function, which gives the displacement at any points  $Q(y_2, y_3)$ .

Where, 
$$\psi_1(y_2, y_3) = \int_F g(x_3) G(P,Q) dx_3$$

We also have, 
$$(\bar{\tau}_{12}^{3})_{2} = \frac{U\mu_{2}}{(2 \times \pi)} \psi_{2}(y_{2}, y_{3})$$

where, 
$$\Psi_2(y_2, y_3) = \frac{\partial}{\partial y_2} \Psi_1(y_2, y_3)$$

$$(\bar{\tau}^{3}_{13})_{2} = \frac{U\mu_{2}}{(2 \times \pi)} \psi_{3}(y_{2}, y_{3})$$

Where, 
$$\Psi_3(y_2, y_3) = \frac{\partial}{\partial y_3} \Psi_1(y_2, y_3)$$

$$\overline{\mu}_{k} = \mu_{k} \frac{s}{s + \frac{\mu_{k}}{\eta_{k}}}$$
(2.13)

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