International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 2, Issue 6, June 2014, PP 554-566
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Effects of Variable Viscosity and Thermal Conductivity on Flow and Heat Transfer over a Stretching Surface with Variable Heat Flux in Micropolar Fluid in Presence Magnetic Field

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Abstract: *The effect of temperature dependent viscosity and thermal conductivity on magneto hydrodynamic flow and heat transfer of an incompressible micropolar fluid over a stretching surface with variable heat flux is studied where the viscosity and thermal conductivity are assumed to be inverse linear functions of temperature. The partial differential equations governing the flow and heat transfer of the problem are transformed into dimensionless form of ordinary differential equations by using similarity substitutions. The governing boundary value problems are then solved numerically using shooting method. The effects of various parameters viz. viscosity parameter, thermal conductivity parameter and velocity exponent parameter m, heat flux exponent parameter n, coupling constant parameter, Prandtl number, and magnetic parameter on velocity, micro-rotation and temperature field are obtained and presented graphically. The coefficient of skin-friction and Nusselt number are also computed and presented graphically.*

Keywords: *Micropolar fluid, variable viscosity and thermal conductivity, heat transfer, MHD Flow.*

1. INTRODUCTION

Eringen formulated the micropolar fluid theory in 1966[7] as an extension of the Navier-Stokes model of classical hydrodynamics to facilitate the description of the fluids with complex molecules. The micropolar fluids are usually defined as isotropic, polar fluids in which deformation of molecules is neglected. Physically, a micropolar model can represent fluids whose molecules can rotate independently of the fluid stream flow and its local vortices.

The study of fluid motion caused by a stretching surface is important in polymer extrusion process, paper production, glass blowing, metal spinning and drawing plastic films etc. The quality of final product depends on the rate of heat transfer at the stretching surface. The heat transfer from a stretching surface is of interest in many practical applications. Such situations arise in the manufacturing process of plastic and rubber sheets where it often necessary to blow a gaseous medium through the unsolidified material. Several investigations have made theoretical and experimental studies of micropolar flow over a stretching surface in the presence of a transverse magnetic field during the last decades. Investigation had been made on thermal Boundary-Layer on a power law stretched surface with suction or injection by Ali [1]. Chamkha [2] studied the unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching sheet immersed in a porous medium. Effect of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet was investigated by Chen [3]. Stretching with a power-law velocity hydromagnetic flow over a surface was studied by Chiam [4]. Elbashebeshy [6] investigated the heat transfer over a stretching surface immersed in an incompressible Newtonian fluid with variable surface heat flux. Grubka [10] investigated heat transfer characteristic of a continuous stretching surface with variable temperature. Heat transfer over a stretching surface with uniform or variable heat flux in micropolar fluids was studied by Ishak *et al*. [12]. They found that the local Nusselt number is higher for micropolar fluids as compared to Newtonian fluids. Variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet was studied by Salem [15].

The main objective of our present work is to extend the work of Ishak *et al*. [12] to study the effects of variable viscosity and thermal conductivity on the flow and heat transfer over a stretching surface with variable heat flux in micropolar fluids in presence of magnetic field. Viscosity and thermal conductivity are assumed to be inverse linear functions of temperature. The governing partial differential equations are reduced in to ordinary differential equations by similarity transformations. The problem is then solved numerically using Runge-kutta shooting algorithm with iteration process.

2. Mathematical Formulation

We consider a steady, two dimensional laminar flow of an incompressible micropolar fluid on a continuous, stretching surface with velocity $U_w(x) = ax^m$and variable surface heat flux $q_w(x) = bx^n$ where a, b are constants and m, n are velocity exponent parameter and heat flux exponent parameter respectively. Also a magnetic field of constant intensity is assumed to be applied normal to the surface and the electrical conductivity of the fluid is assumed to be small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. The applied magnetic field is primary in the y −direction and is a function of x only. Under these assumptions we consider the governing equations of the problem as follows.

2.1 Basic Equations

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right) + \frac{\kappa}{\rho}\left(\frac{\partial N}{\partial y} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B^2}{\rho}u \tag{2}$$

Angular Momentum equation:

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{1}{\rho j}\frac{\partial}{\partial y}\left(\gamma\frac{\partial N}{\partial y}\right) - \frac{\kappa}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \tag{3}$$

Energy Equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) \tag{4}$$

Micro−inertia density Equation:

$$u\frac{\partial j}{\partial x} + v\frac{\partial j}{\partial y} = 0 \tag{5}$$

The equation of continuity being identically satisfied by velocity component u and v which are the velocity components along the $x-$ axis and $y-$ axis respectively. $(0, 0, N)$ is the micro rotation profile. We assume that micro-rotation density, viscosity and thermal conductivity and spin-gradient viscosity are functions of the co-ordinates x and y. Following Gorla [8] we assume that $\gamma = \left(\mu_\infty + \frac{\kappa}{2}\right)j = \mu_\infty\left(1 + \frac{K_1}{2}\right)j$, where $K_1 = \frac{\kappa}{\nu_\infty \rho}$, coupling constant parameter.

The problem is governed by the coupled non-linear equations of which the boundary conditions are:

$$\left.\begin{array}{ll} u = U_w,\ v = 0,\ \frac{\partial T}{\partial y} = -\frac{q_w}{\lambda_\infty},\ j = 0,\ N = -\frac{1}{2}\frac{\partial u}{\partial y} & \text{at } y = 0 \\ u \to 0,\ T \to T_\infty,\quad N \to 0 & \text{at } y \to \infty \end{array}\right] \tag{6}$$

Following Lai and Kulacki [13] we assume that the viscosity and thermal conductivity are linear functions of temperature, i.e.,

$$\left.\begin{array}{l}\frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\delta(T-T_\infty)]\text{, or }\frac{1}{\mu}=c(T-T_c)\text{ where }c=\frac{\delta}{\mu_\infty}\text{ and }T_c=T_\infty-\frac{1}{\delta}\\ \frac{1}{\lambda}=\frac{1}{\lambda_\infty}[1+\xi(T-T_\infty)]\text{, or }\frac{1}{\lambda}=d(T-T_r)\text{ where }d=\frac{\xi}{\lambda_\infty}\text{ and }T_r=T_\infty-\frac{1}{\xi}\end{array}\right] \tag{7}$$

The continuity equation is satisfied by introducing a stream function ψ such that

$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x}. \tag{8}$$

Further we introduce the following similarity transformations:

$$\left.\begin{array}{l}\eta=\left[\frac{(m+1)U_W}{2xv_\infty}\right]^{1/2}y,\ \psi=\left[\frac{2xv_\infty U_W}{(m+1)}\right]^{1/2}f(\eta),\ \ N=U_w\left[\frac{(m+1)U_W}{2xv_\infty}\right]^{1/2}h(\eta),\\ j=\frac{2xv_\infty}{(m+1)U_W}i(\eta),\ T=T_\infty+\frac{q_W}{\lambda_\infty}\left[\frac{2xv_\infty}{(m+1)U_W}\right]^{1/2}\theta(\eta),\ \ B=B_0\left[\frac{x}{U_W}\right]^{-1/2}\end{array}\right] \tag{9}$$

Using (7) and (9) we get

$$\nu=-\nu_\infty\frac{\theta_c}{\theta-\theta_c},\lambda=-\lambda_\infty\frac{\theta_r}{\theta-\theta_r}\quad\Big] \tag{10}$$

Substituting these in (2)—(5) we get the following ordinary differential equations

$$\left(1+K_1\frac{\theta_c-\theta}{\theta_c}\right)f'''=\left[K_1h'-Mf'+\left(ff''-\frac{2m}{m+1}f'^2\right)\right]\frac{\theta-\theta_c}{\theta_c}+\frac{\theta'f''}{\theta-\theta_c} \tag{11}$$

$$\left(1+\frac{1}{2}K_1\right)i'h''=\left(\frac{3m-1}{m+1}hf'-h'f\right)i+K_1(2h+f'')-\left(1+\frac{1}{2}K_1\right)i'h' \tag{12}$$

$$\theta''=P_r\left(f\theta'-\frac{2n-m+1}{m+1}f'\theta\right)\frac{\theta-\theta_r}{\theta_r}+\frac{\theta'^2}{\theta-\theta_r} \tag{13}$$

$$2(1-m)if'-(m+1)fi'=0 \tag{14}$$

The corresponding boundary conditions are

$$\left.\begin{array}{l}\eta=0: f=0, f'=1, \theta'=-1, i=0,\ h=-\frac{1}{2}\ f''\\ \eta\to\infty:\ \ f=0,\ \ f'=0,\ \ \theta=0,\ \ h=0\end{array}\right] \tag{15}$$

The physical quantities of interest in this problem are the skin –friction coefficient c_f and Nusselt number Nu which indicate physically wall shear stress and rate of heat transfer respectively. For micropolar boundary layer flow, the wall skin friction τ_w is given by

$$\tau_w=\left[(\mu+k)\frac{\partial u}{\partial y}+kN\right]_{y=0} \tag{16}$$

The skin –friction coefficient c_f can be defined and derived as

$$c_f=\frac{2\,\tau_W}{\rho U_0{}^2}=\sqrt{2}\left(\frac{\theta_c}{\theta_c-\theta(0)}+\frac{K_1}{2}\right)Re^{-\frac{1}{2}}f''(0) \tag{17}$$

The heat transfer from the plate is given by

$$q_w=-\lambda\left[\frac{\partial T}{\partial y}\right]_{y=0}$$

The Nusselt number is obtained as

$$Nu = \frac{q_w x}{\lambda_\infty (T_W - T_\infty)} = \sqrt{\frac{m+1}{2}} Re^{\frac{1}{2}} \frac{1}{\theta(0)} \tag{18}$$

3. Results and Discussion

The equations (11) — (14) together with the boundary conditions (15) are solved for various combination of the parameters involved in the equations using an algorithm based on the shooting method and presented results for the dimensionless velocity distribution, dimensionless micro-rotation distribution, dimensionless temperature distribution with the variation of different parameters. Initially solution was taken for constant values of M=1.00, Pr=0.70, K_1 =0.10, $n = 1, m = 0.10$ with the viscosity parameter θ_c ranging from -15.00 to -1.00 at certain value of $\theta_r = -10.00$. Similarly solutions have been found with varying the thermal conductivity parameter θ_r ranging from -15.00 to -1.00 at certain value of $\theta_c = -10.00$ keeping the other values remaining same. Solutions have also been found for different values of magnetic parameter (M), Prandtl number (P_r), velocity exponent parameter m, heat flux exponent parameter n, and the coupling constant parameter (K_1).

The variations in velocity distribution, micro-rotation distribution, temperature distribution and micro-inertia density are illustrated in figures (1) — (16) with the variation of different parameters. Variations in velocity distribution are shown in figures (1) — (5). From figure (1) we have observed that velocity decreases with the increasing values of the magnetic parameter M. It is due to the fact that the application of transverse magnetic field will result a resistive force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. It is also observed that the velocity is maximum near the plate and decreases away from the plate and finally takes asymptotic value. From figure (2) we have observed that velocity decreases with the increasing values of the viscosity dependent temperature θ_c. From figure (3) we have observed that velocity increases with the increasing values of the thermal conductivity dependent temperature θ_r. From figure (3) we have observed that velocity decreases with the increasing values of the velocity exponent parameter m. From figure (5) it is clear that velocity increases with increasing value of coupling constant parameter $K1$.

Figures (6) — (9) represent the variations in micro-rotation distribution with the variation of different parameters. From figures (6) — (8) we have observed that micro-rotation decreases and after certain distance from the wall it increases with the increasing values of the parameters θ_c, M and m while from figure (9) we have seen that micro-rotation increases and after certain distance from the wall it decreases with the increasing values of the parameter K_1.

Figures (10) — (14) represent the variations in temperature distribution with the variation of the parameters θ_r, M, n, m and P_r .We have observed that temperature increases with increasing values of the parameter M while temperature decreases with increasing values of the parameters θ_r, n and P_r . From figure (13) we have observed that temperature increases and after certain distance from the wall it decreases with the increasing value of the parameter m. From figure (14) we have seen that temperature decreases with increasing values of the parameters P_r. It is due to the fact that with the increasing value of the Prandtl number kinematic viscosity of the fluid increases and therefore diffusion of momentum increases while thermal diffusivity decreases.

Figures (15) — (16) represent the variations in micro-inertia density with the variation of the parameters θ_r and M. It is observed that micro-inertia density increases with the increasing values of the parameter M while it decreases with the increasing values of θ_r.

4. Figures

Figures (15) — (16) represent the variations in micro-inertia density with the variation of the parameters θ_r and M. It is observed that micro-inertia density increases with the increasing values of the parameter M while it decreases with the increasing values of θ_r.

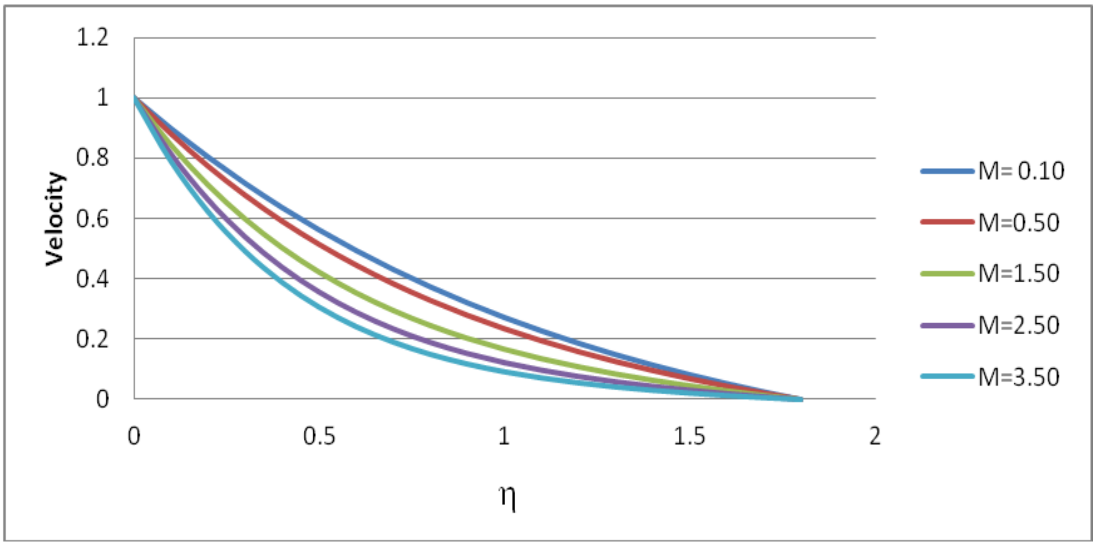

Fig.1 *Variation of velocity with M*

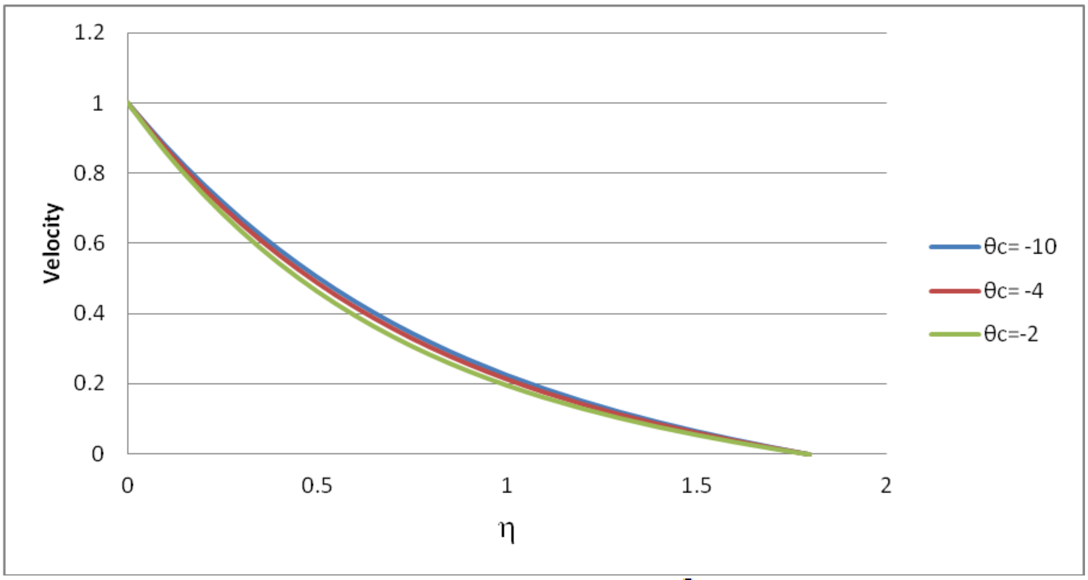

Fig.2 *Variation of velocity with* θ_c

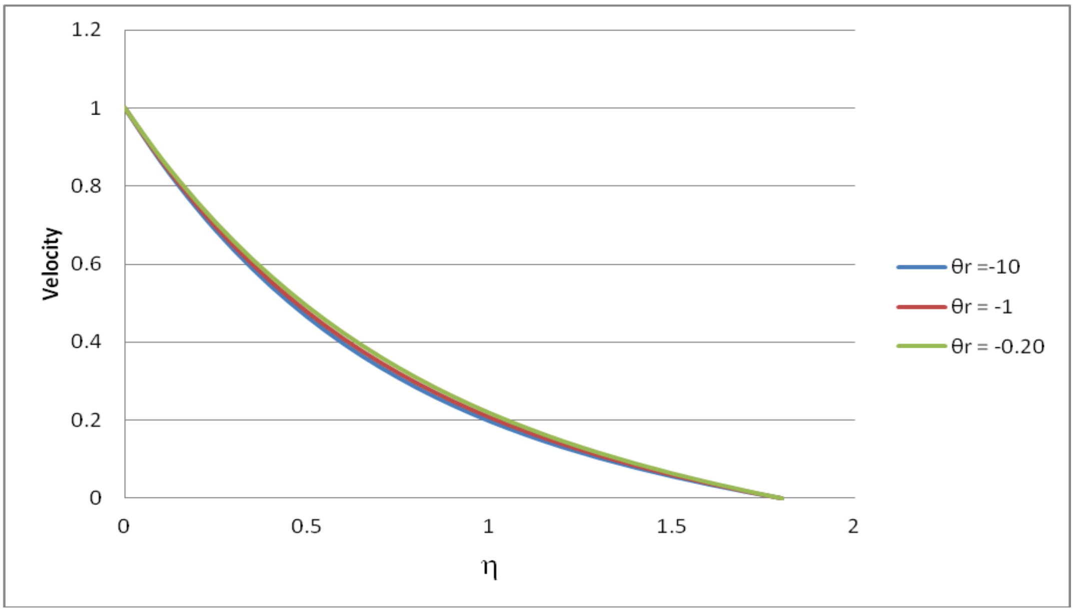

Fig. 3 *Variation of velocity with* θ_r

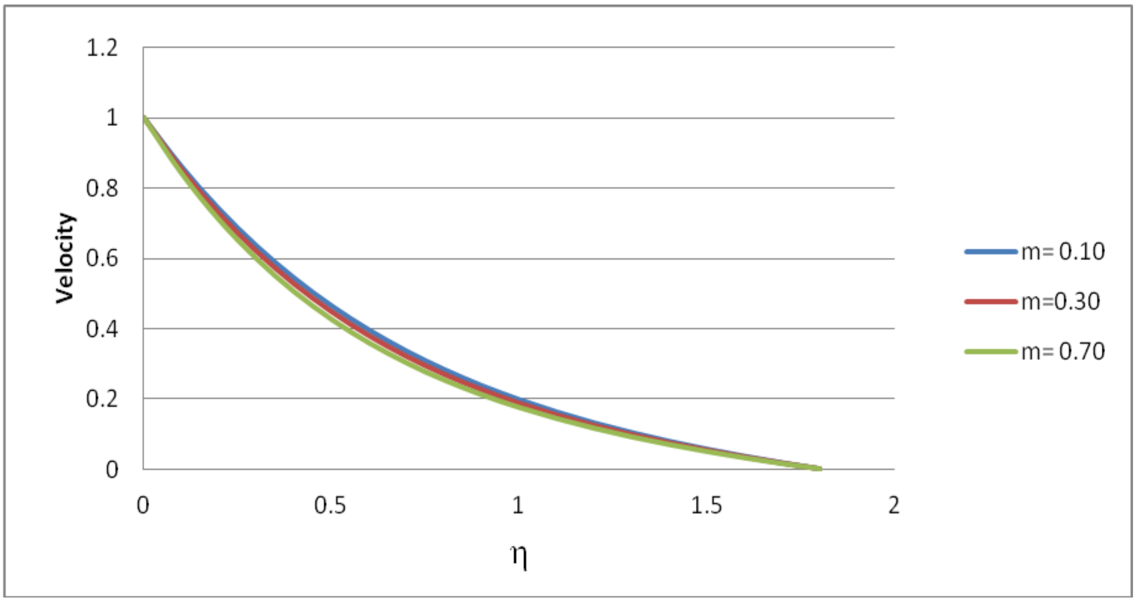

Fig.4 *Variation of velocity with* m

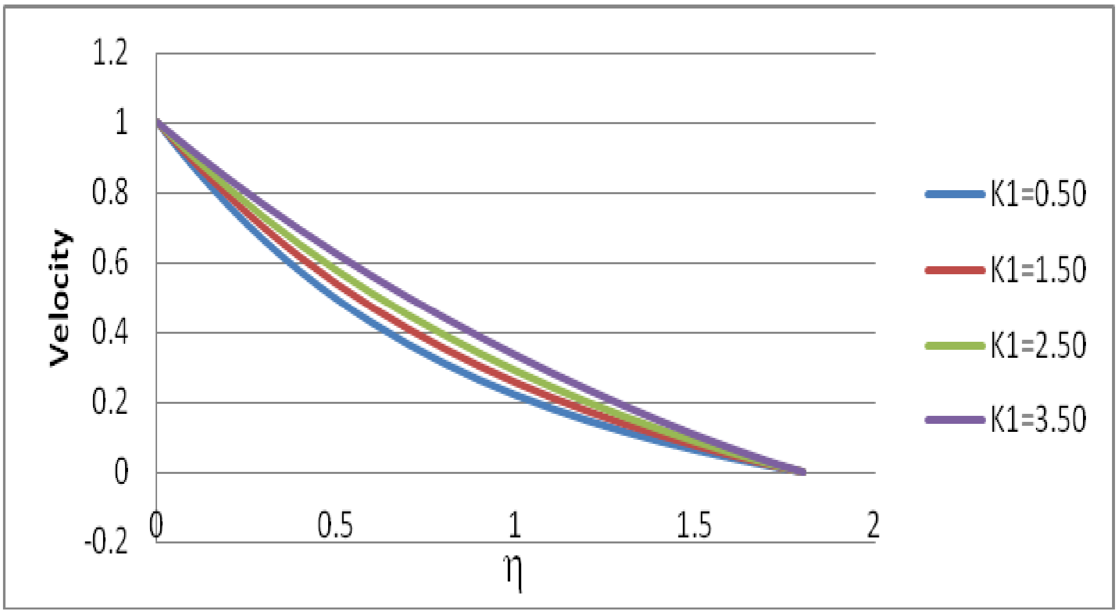

Fig.5 *Variation of velocity with* K_1

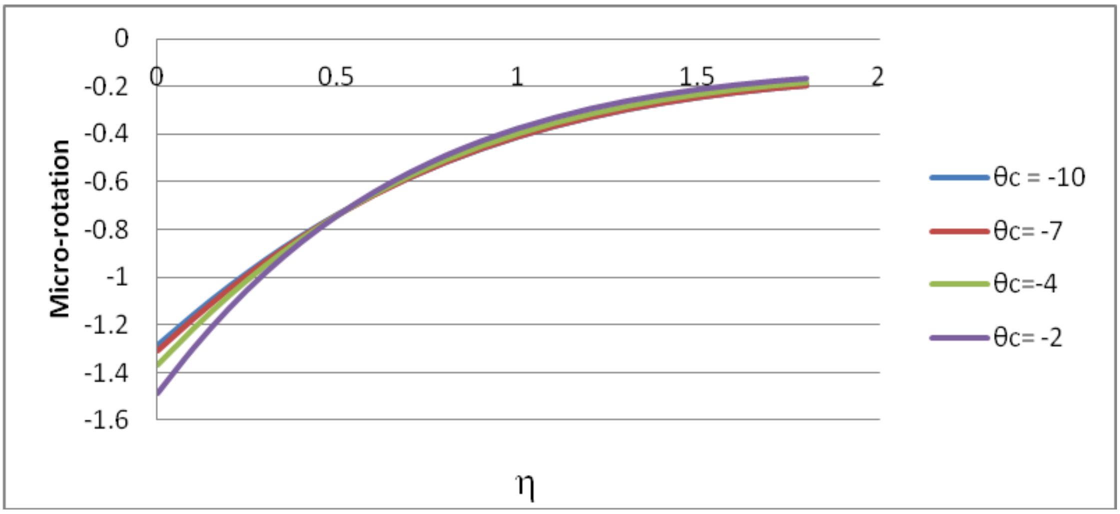

Fig.6 *Variation of Micro-rotation with* θ_c

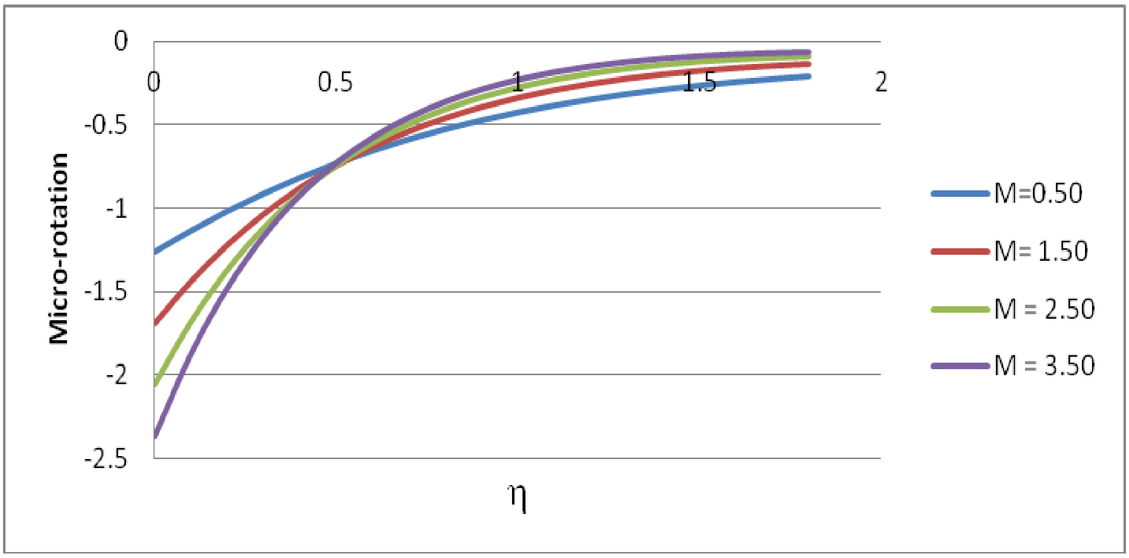

Fig. 7 *Variation of Micro-rotation with M*

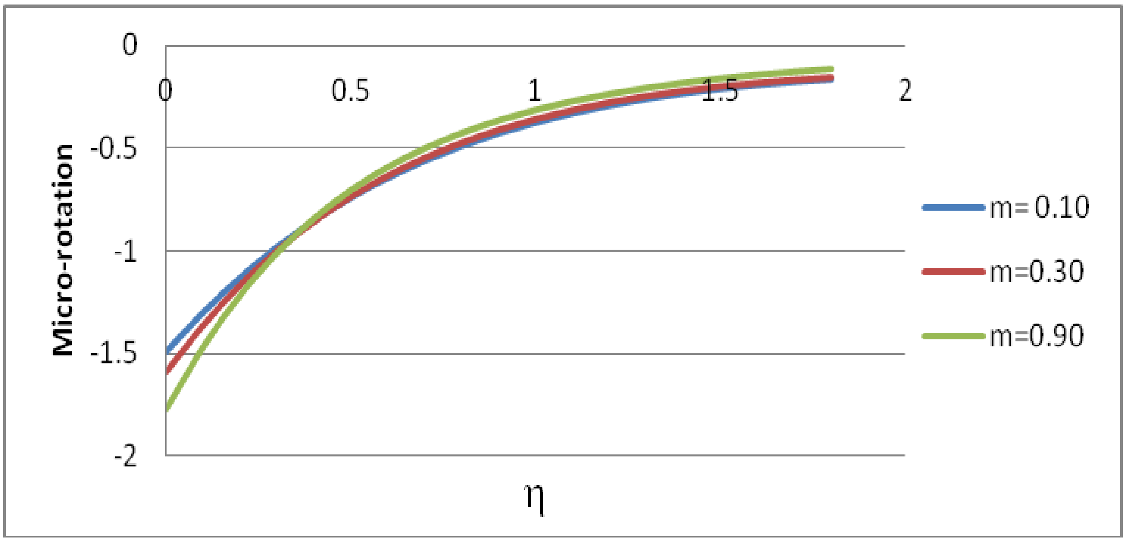

Fig.8 *Variation of Micro-rotation with* m

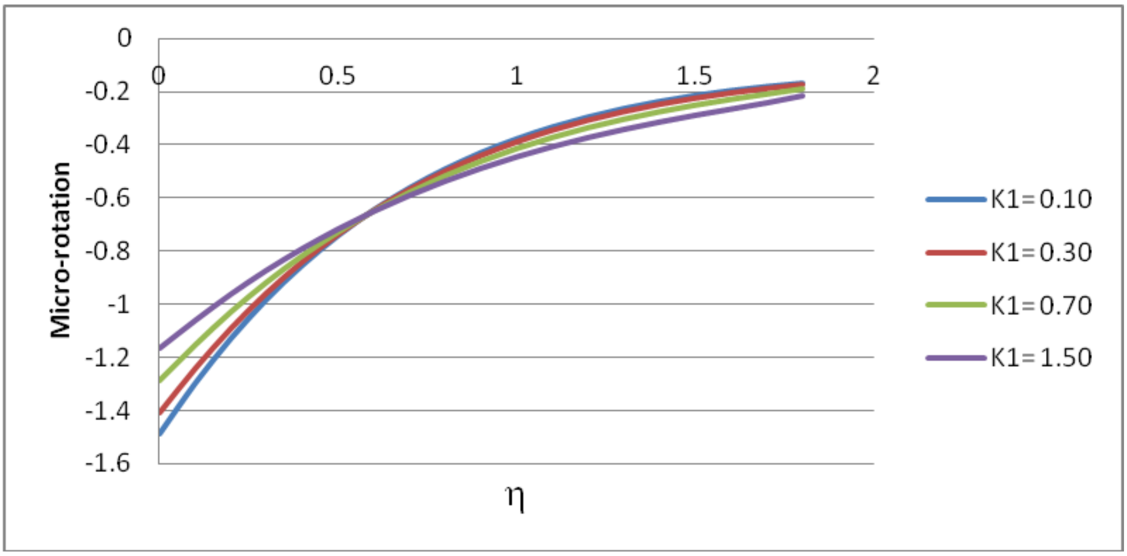

Fig.9 *Variation of Micro-rotation with* K_1

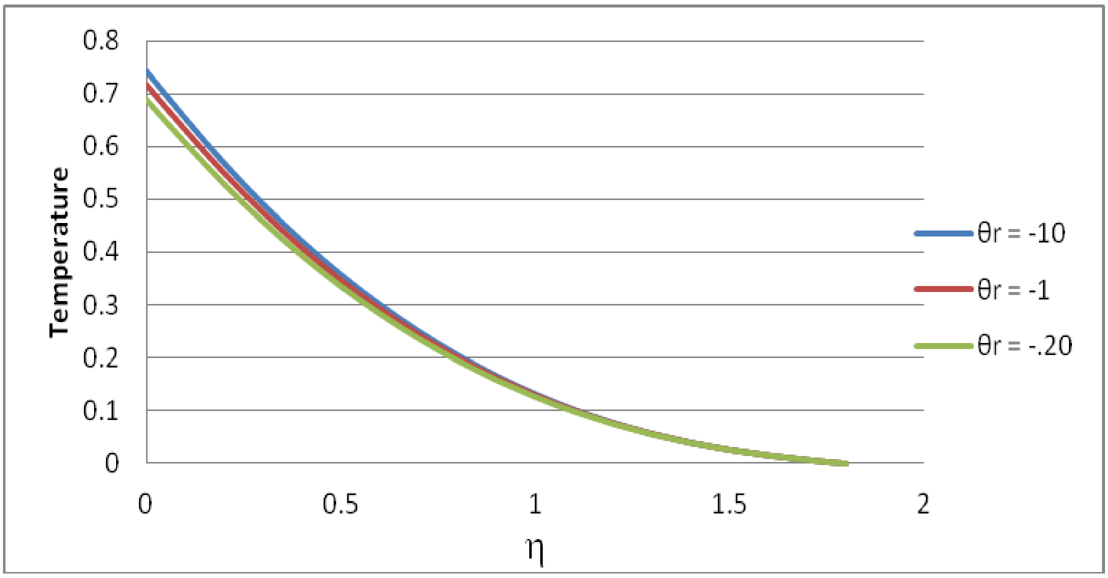

Fig.10 *Variation of temperature with* θ_r

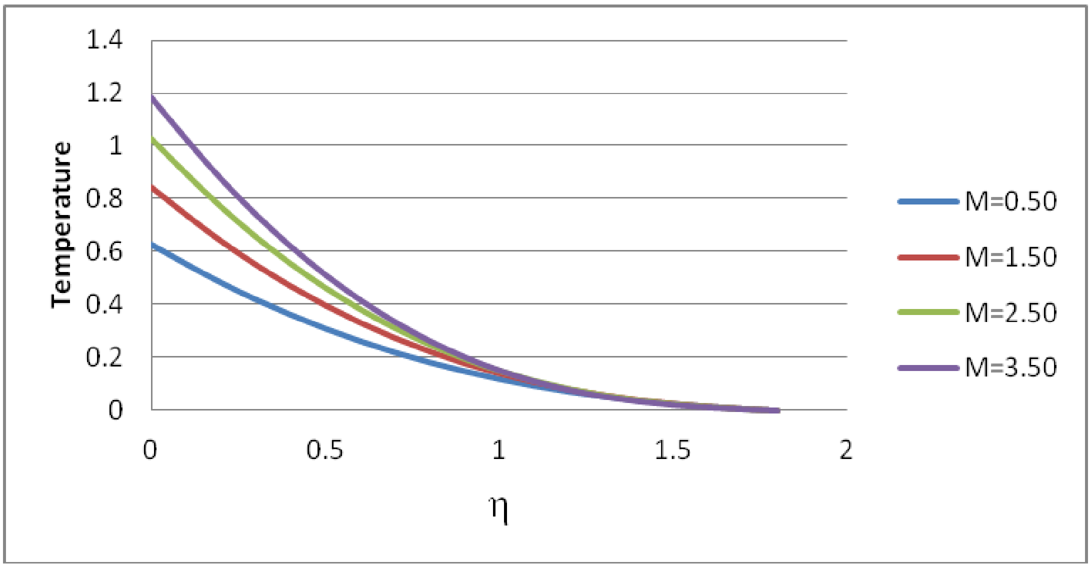

Fig.11 *Variation of temperature with M*

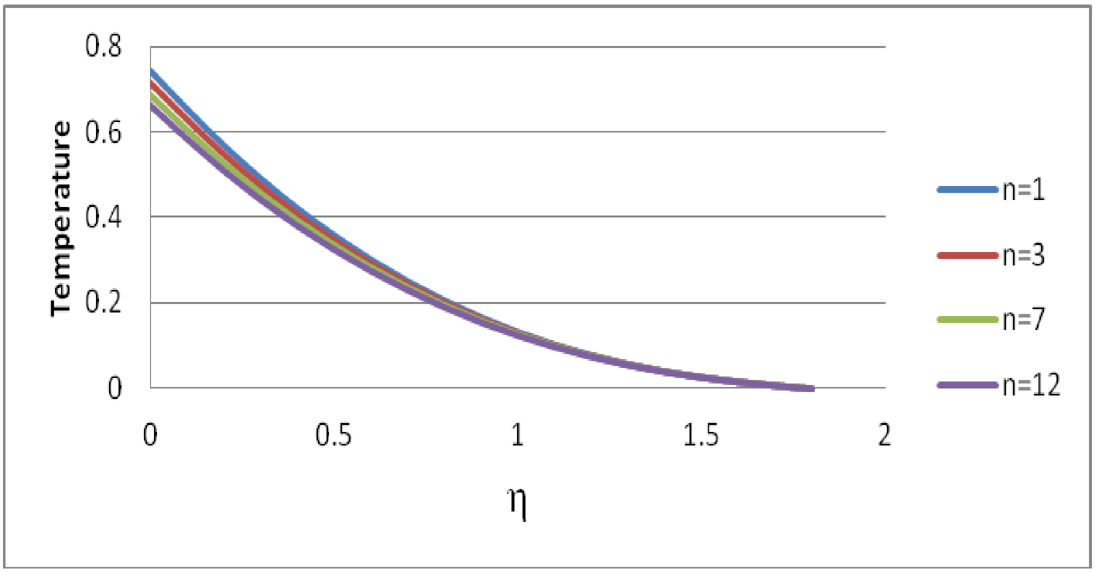

Fig.12 *Variation of temperature with* n

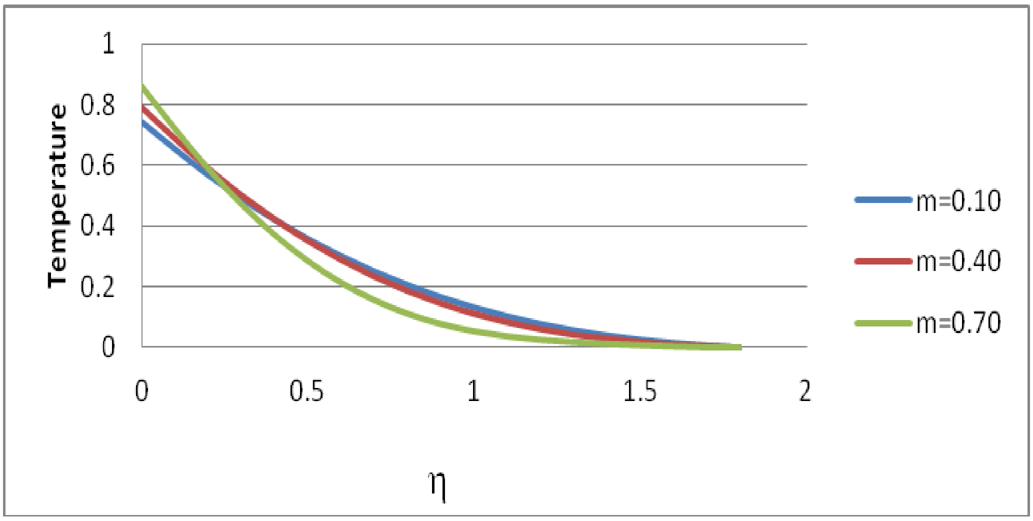

Fig.13 *Variation of temperature with* m

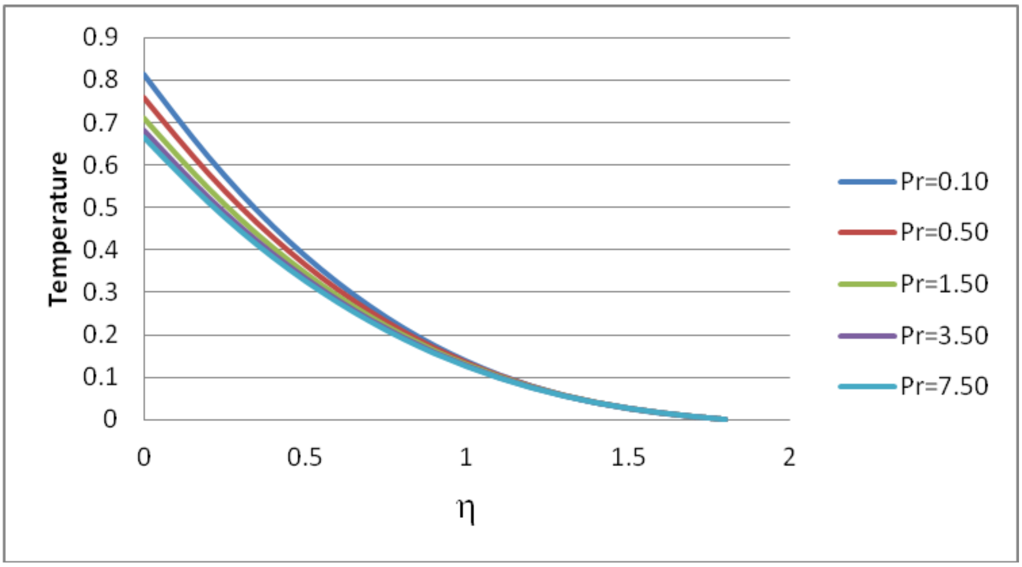

Fig.14 *Variation of temperature with Pr*

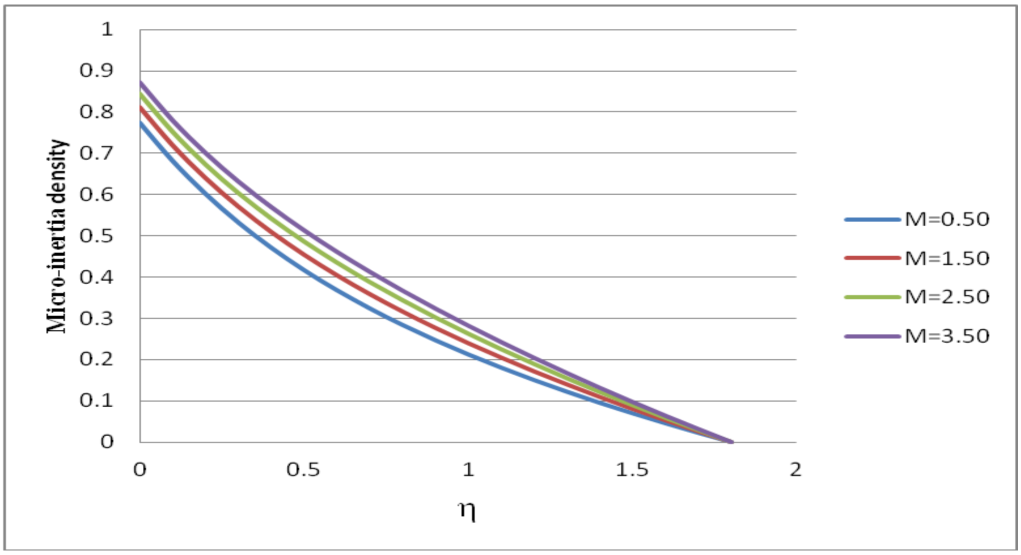

Fig. 15 *Variation of micro-inertia density with M*

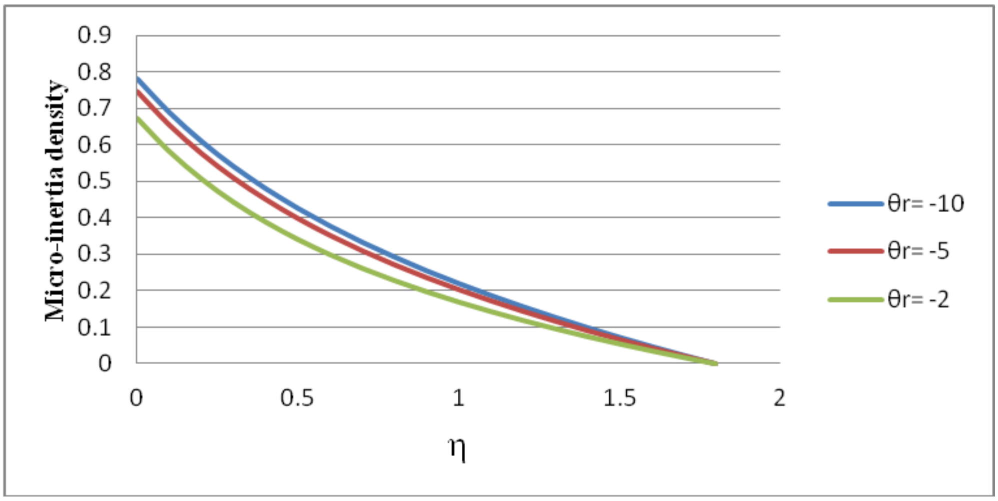

Fig. 16 *Variation of micro-inertia density with* θ_r

5. Numerical values of f'(0), h'(0), $\theta'(0)$, c_f , Nu and Tables

Finally effect of the above mentioned parameters on the values of f'(0), g'(0), h'(0), θ '(0), $C_f$and Nu are shown in the tables (1) — (4). The behavior of these parameters is self evident from the tables and hence any further discussions about them seem to be redundant.

Table 1

Re=0.10, θ_r=-10.00, n=1.00 , m=0.10 , M=0.60 , Pr=0.70 , K1=0.10					
θ_c	f'(0)	h'(0)	θ '(0)	c_f	Nu
-10.00	-1.34453	-0.83269	0.740029	-1.91874	0.427318
-5.00	-1.39688	-0.86983	0.74299	-1.84378	0.425615
-1.00	-1.74411	-1.12228	0.763275	-1.47993	0.414304

Table 2

Re=0.10, θ_r=-15.00, θ_c=-2.00, n=1.00 , m=0.10 , M=0.60 , K1=0.10					
P_r	f'(0)	h'(0)	θ '(0)	c_f	Nu
0.70	-1.54057	-0.97291	0.751254	-1.67034	0.420993
1.00	-1.51453	-0.95413	0.642941	-1.6421	0.491846
4.60	-1.41617	-0.88412	0.286209	-1.53545	1.104884
7.00	-1.39604	-0.86994	0.227542	-1.51363	1.389754

Table 3

Re=0.10, θ_r=-15.00, θ_c=-2.00, n=1.00 , m=0.10 , Pr=0.70 , K1=0.10					
M	f'(0)	h'(0)	θ '(0)	c_f	Nu
0.6	-2.43883	-2.77781	0.435854	-2.64425	0.725535
1	-2.50646	-2.85842	0.436118	-2.71758	0.725098
1.4	-2.57324	- 2.93803	0.127298	-2.789980	0.724667

Table 4

Re=0.10, θ_r=-10.00, θ_c=-2.00, m=0.10 , M=0.60 , Pr=0.70 , K1=0.10					
n	f'(0)	h'(0)	θ '(0)	c_f	Nu
0.10	-1.59046	-1.00913	0.961544	-1.72442	0.328875
1.10	-1.53639	-0.96989	0.734242	-1.6658	0.430686
2.3	-1.4984	-0.94257	0.584468	-1.62461	0.541052

6. Conclusion

In this study, the effects of variable viscosity and thermal conductivity on the flow and heat transfer over a stretching surface with variable heat flux in micropolar fluids in presence of magnetic field are examined. The results demonstrate clearly that the viscosity and thermal conductivity parameters along with the other parameters such as magnetic parameter (M), Prandtl number (P_r), velocity exponent parameter m, heat flux exponent parameter n, and the coupling constant parameter (K_1) have significant effects on velocity, temperature, concentration and micro-rotation distributions within the boundary layer. Thus assumption on constant properties may cause a significant error in flow problem.

7. Nomencleatures

$\lambda =$ Thermal conductivity

λ_∞ = Thermal conductivity of the ambient fluid

μ = Dynamic viscosity

μ_∞ = Dynamic viscosity of the ambient fluid

ν =Kinematic viscosity

ν_∞ = Kinematic Viscosity of the ambient fluid

κ =Vortex viscosity

c_p =Specific heat

η = Dimensionless co-ordinates

u = Velocity in the x −direction

f =Dimensionless velocity

h =Dimensionless microrotation

θ =Dimensionless temperature

θ_c = Dimensionless reference temperature corresponding to viscosity parameter

θ_r = Dimensionless reference temperature corresponding to thermal conductivity parameter

T = Temperature

T_∞ = Ambient temperature

T_w = Wall temperature

j =Micro rotation density

σ =Electrical conductivity

ρ= Density

$Re = \frac{U_w x}{\nu_\infty}$, local Reynolds number

$P_r = \frac{\nu_\infty \rho c_p}{\lambda_\infty}$, Prandtl number

$M = \frac{2\sigma B_0{}^2}{\rho(m+1)}$, Hartmann number

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