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State Space Solution to the Unsteady Slip Flow of a Micropolar Fluid between Parallel Plates

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Abstract: *In this work, the unsteady motion of an incompressible micropolar fluid between two infinite parallel plates under the effect of slip boundary conditions for both velocity and microrotation is considered. The motion of the fluid is generated by applying a time dependent pressure gradient between the two plates. The Laplace transform technique and the state space approach are utilized to obtain the analytical solution in the Laplace domain. The inverse Laplace transform is evaluated numerically. The velocity and microrotation functions are represented graphically and the effects of the slip, and micropolarity parameters on the flow field are discussed.*

Keywords: *Micropolar fluid; Slip condition; Laplace transform; State space approach.*

1. Introduction

The motion of the classical viscous fluids is controlled accurately by Navier-Stokes equations. However, these equations cannot describe adequately the motion of some types of fluids with microstructure, such as muddy water, chemical suspensions, lubricants, etc. Eringen [1] has introduced his theory of micropolar fluids to recover the inadequacy of Navier-Stokes equations describing the correct behavior of such fluids taking the motion of the microelements into consideration. Physically, micropolar fluids represent fluids consisting of randomly oriented particles suspended in a viscous medium. In the theory of micropolar fluids, there are two vectors characterizing the motion of the fluid; the classical velocity vector describes the motion of the macro-volume element and the microrotation vector represents the motion of the micro-volume element. The study of micropolar fluids can model many physical applications in the engineering and biological fields such as animal blood flow, chemical suspensions, liquid crystals and lubricants [1, 2].

The no-slip boundary condition has been used extensively in fluid dynamics. However, the slip condition proposed by Navier [3], seems to be more realistic. It assumes the possibility of fluid slippage along the surface of the boundary. The slip condition assumes that the tangential velocity of the fluid particles relative to the solid boundary at a point on the boundary is proportional to the tangential stress acting at the point of contact. The constant of proportionality is named slip coefficient and is assumed to depend only on the nature of the fluid and solid boundary. The slip condition has been utilized by several researchers in their works in the classical Newtonian fluids [4-7] and in the micropolar and microstretch fluids [8-11].

The unsteady unidirectional Poiseuille flow of a micropolar fluid between two parallel plates with no-slip and no-spin boundary conditions was investigated by Faltas et al [12]. Ashmawy [8] studied the problem of Couette flow of an incompressible micropolar fluid using slip condition. The state space approach was employed by Devakar and Iyengar [13] to discuss the Stokes' first problem of a micropolar fluid with no-slip and no-spin conditions. They also used the same technique in [14, 15] to discuss the motion of a micropolar fluid between two parallel plates taking the classical no-slip and no-spin boundary conditions into consideration.

In this work, we apply the state space approach to investigate the unsteady motion of a micropolar fluid flow between two parallel plates. The slip conditions are applied and the motion of the fluid is produced by applying a time dependent pressure gradient.

2. Formulation of the Problem

The unsteady motion of an isothermal incompressible micropolar fluid flow, in the absence of body forces and body couples, is governed by the following differential equations;

$$\nabla \cdot \vec{q} = 0, \tag{2.1}$$

$$\rho \frac{d\vec{q}}{dt} = -\nabla p + \kappa \nabla \times \vec{c} - (\mu + \kappa) \nabla \times \nabla \times \vec{q}\,, \tag{2.2}$$

$$\rho j \frac{d\vec{q}}{dt} = -2\kappa \vec{c} + \kappa \nabla \times \vec{q} - \gamma \nabla \times \nabla \times \vec{c} + (\alpha + \beta + \gamma) \nabla (\nabla \cdot \vec{c}), \tag{2.3}$$

where the scalar quantities ρ and j are, respectively, the fluid density and gyration parameters. The vectors $\vec{q}$ and $\vec{c}$ are representing the velocity and microrotation, respectively, and p is the fluid pressure at any point. The material constants (μ, κ) represent the viscosity coefficients and $(\alpha_0, \beta, \gamma)$ represent the gyro-viscosity coefficients.

The stress and couple stress tensors are given by

$$t_{ij} = (\lambda q_{r,r} - p)\delta_{ij} + \mu q_{i,j} + (\mu + \kappa) q_{j,i} - \kappa \varepsilon_{ijk} c_k\,, \tag{2.4}$$

$$m_{ij} = \alpha c_{r,r} \delta_{ij} + \beta c_{i,j} + \gamma c_{j,i}\,, \tag{2.5}$$

where δ_{ij} and ε_{ijk} are, respectively, the Kronecker delta and the alternating tensor.

We now consider the flow of a micropolar liquid between two infinite parallel plates separated by a distance b. The two plates are stationary and the fluid starts due to a sudden pressure gradient. Working with the Cartesian coordinates (x, y, z), with x-axis along the lower plate, y-axis perpendicular to the plates, and z-axis is perpendicular to the plane of motion. The flow is along x-axis then the velocity and microrotation vectors take the forms

$$\vec{q} = (u(y,t), 0, 0) \text{ and } \vec{c} = (0, 0, c(y,t)).$$

The equation of continuity (2.1) is satisfied automatically and equations (2.2)-(2.3) reduce to

$$(\mu + \kappa) \frac{\partial^2 u}{\partial y^2} + \kappa \frac{\partial c}{\partial y} - \rho \frac{\partial u}{\partial t} - \frac{\partial p}{\partial x} = 0, \tag{2.6}$$

$$\gamma \frac{\partial^2 c}{\partial^2 y} - \kappa \frac{\partial u}{\partial y} - 2\kappa c - \rho j \frac{\partial c}{\partial t} = 0, \tag{2.7}$$

The initial and slip boundary conditions applied to the problem at hand are assumed to be

$$u(y,0) = 0, \ \ c(y,0) = 0 \ \text{ for all } \ 0 \le y \le b, \tag{2.8}$$

$$\beta_1 u(0,t) = \tau_{yx}(0,t), \quad \xi_1 c(0,t) = m_{yz}(0,t), \tag{2.9}$$

$$\beta_2 u(b,t) = -\tau_{yx}(b,t), \quad \xi_2 c(b,t) = -m_{yz}(b,t), \tag{2.10}$$

where $0 \le \beta_1, \beta_2 \le \infty$ are the velocity slip parameters of the two plates. Also, $0 \le \xi_1, \xi_2 \le \infty$ are representing the microrotation slip parameters of the two plates. These parameters depend only on the fluid and the material of the plates. The no-slip case can be recovered when the slip parameters $\beta_1, \beta_2 \to \infty$ and the no-spin case is obtained when $\xi_1, \xi_2 \to \infty$.

Using (2.4) and (2.5), the non-vanishing stress and couple stress components are found to be

$$\tau_{yx} = (\mu + \kappa) \frac{\partial u}{\partial y} + \kappa c(y,t), \quad m_{yz} = \gamma \frac{\partial c(y,t)}{\partial y} \tag{2.11}$$

The following non-dimensional variables will be used

$$u^* = \frac{\rho b}{\mu + k} u, \ \ c^* = \frac{\rho \kappa b^2}{(\mu+\kappa)^2} c, \ \ x^* = \frac{x}{b}, \ \ y^* = \frac{y}{b}, \ \ t^* = \frac{(\mu+\kappa)}{\rho b^2} t, \tag{2.12}$$

$$p^* = \frac{\rho b^2}{(\mu+k)^2} p\,, \ \ t_{ij}{}^* = \frac{\rho b^2}{(\mu+\kappa)^2} t_{ij}, \ \ m_{ij}^* = \frac{\rho \kappa b^3}{\gamma(\mu+\kappa)^2} m_{ij}\,. \tag{2.13}$$

In terms of these variables, after dropping the asterisks for convenience, the differential equations (2.6) and (2.7) can be rewritten in the forms

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial c}{\partial y}-\frac{\partial u}{\partial t}-\frac{\partial p}{\partial x}=0, \tag{2.14}$$

$$\frac{\partial^2 c}{\partial y^2}-f\frac{\partial u}{\partial y}-gc-h\frac{\partial c}{\partial t}=0, \tag{2.15}$$

where $f=\frac{k^2b^2}{\gamma(\mu+\kappa)}$, $g=\frac{2\kappa b^2}{\gamma}$, $h=\frac{(\mu+\kappa)}{\gamma}j$, and $j=\frac{2\gamma}{(2\mu+\kappa)}$.

In view of (2.12)-(2.13), the non-dimensional boundary conditions (2.9) and (2.10) become

$$\alpha_1 u(0,t)=\left(1+\frac{\kappa}{\mu}\right)t_{yx}(0,t), \quad \eta_1 c(0,t)=\frac{\partial c(0,t)}{\partial y}, \tag{2.16}$$

$$\alpha_2 u(1,t)=-\left(1+\frac{\kappa}{\mu}\right)t_{yx}(1,t), \quad \eta_2 c(1,t)=-\frac{\partial c(1,t)}{\partial y}, \tag{2.17}$$

where $\alpha_1=\frac{\beta_1 b}{\mu}$, $\alpha_2=\frac{\beta_2 b}{\mu}$, $\eta_1=\frac{\xi_1 b}{\gamma}$, $\eta_2=\frac{\xi_2 b}{\gamma}$.

3. SOLUTION OF THE PROBLEM

Applying the Laplace transform defined by

$$L\{F(y,t)\}=\bar{F}(y,s)=\int_0^\infty e^{-st}F(y,t)dt, \tag{3.1}$$

The differential equations (2.14) and (2.15) reduce to

$$\frac{d^2\bar{u}}{dy^2}+\frac{d\bar{c}}{dy}-s\bar{u}-\frac{\partial\bar{p}}{\partial x}=0, \tag{3.2}$$

$$\frac{d^2\bar{c}}{dy^2}-f\frac{d\bar{u}}{dy}-a\bar{c}=0, \tag{3.3}$$

where $a=g+hs$.

To obtain the solution of the coupled differential equations (3.2)-(3.3) subject to the boundary conditions (2.16)-(2.17), in the Laplace domain, we employ the technique of state space approach. To do this, the two equations (3.1)-(3.2) are written in the matrix form

$$\frac{d}{dy}\bar{V}(y,s)=A(s)\bar{V}(y,s)+\bar{b}(s), \tag{3.4}$$

where

$$A(s)=\begin{pmatrix}0&0&1&0\\0&0&0&1\\s&0&0&-1\\0&a&f&0\end{pmatrix},\quad \bar{V}(y,s)=\begin{pmatrix}\bar{u}(y,s)\\\bar{c}(y,s)\\\bar{u}'(y,s)\\\bar{c}'(y,s)\end{pmatrix},\quad \bar{b}(s)=\begin{pmatrix}0\\0\\-\bar{\emptyset}(s)\\0\end{pmatrix}, \tag{3.5}$$

$$\bar{u}'=\frac{d\bar{u}}{dy},\quad \bar{c}'=\frac{d\bar{c}}{dy},\quad \bar{\emptyset}(s)=-\frac{\partial\bar{p}}{\partial x}. \tag{3.6}$$

The solution of the matrix differential equation (3.4) can be easily found to be

$$\bar{V}(y,s)=\exp[A(s)y]\left\{\bar{V}(0,s)+\int_{z=0}^{y}\exp(-A(s)z)\,\bar{b}\,(s)dz\right\}. \tag{3.7}$$

The characteristic equation of the matrix $A(s)$ is

$$k^4-(s+a-f)k^2+sa=0. \tag{3.8}$$

This characteristic equation has the roots $\pm k_1$ and $\pm$ k_2, where

$$k_1=\sqrt{\frac{(s+a-f)+\sqrt{(s+a-f)^2-4sa}}{2}},\qquad k_2=\sqrt{\frac{(s+a-f)-\sqrt{(s+a-f)^2-4sa}}{2}}. \tag{3.9}$$

The Maclaurin's series expansion of $exp[A(s)y]$ is given by

$$\exp[A(s)y]=\sum_{r=0}^{\infty}\frac{[A(s)y]^r}{r!}. \tag{3.10}$$

Using Cayley-Hamilton theorem, the infinite series (3.10) can be written in the form

$$\exp[A(s)y] = L(y,s) = a_0 I + a_1 A + a_2 A^2 + a_3 A^3, \tag{3.11}$$

where I is the fourth order unit matrix and a_0, a_1, a_2 and a_3 are parameters depending on y and s. Then, the characteristic roots $\pm k_1$ and $\pm k_2$ satisfy equation (3.11) and hence we obtain the following system of linear equations after replacing the matrix A by its characteristic roots

$$\exp[\pm k_i y] = a_0 \pm a_1 k_i + a_2 k_i^2 \pm a_3 k_i^3, \qquad i = 1, 2. \tag{3.12}$$

Solving this system of algebraic equations, we get a_0,a_1 a_2 and a_3 in the forms

$$a_0 = \frac{1}{F}[k_1^2 \cosh(k_2 y) - k_2^2 \cosh(k_1 y)], \quad a_1 = \frac{1}{F}\left[\frac{k_1^2}{k_2}\sinh(k_2 y) - \frac{k_2^2}{k_1}\sinh(k_1 y)\right], \tag{3.13}$$

$$a_2 = \frac{1}{F}[\cosh(k_1 y) - \cosh(k_2 y)], \quad a_3 = \frac{1}{F}\left[\frac{1}{k_1}\sinh(k_1 y) - \frac{1}{k_2}\sinh(k_2 y)\right], \tag{3.14}$$

where $F = k_1^2 - k_2^2$.

The elements ($L_{ij}; i,j = 1,2,3,4$) of the matrix $L(y,s)$ can be obtained after substituting A, A^2, A^3 into equation (3.11) to be

$$L_{11} = a_0 + a_2 s, \; L_{12} = -a_3 a, \; L_{13} = a_1 + a_3 s - a_3 f, \; L_{14} = -a_2, \tag{3.15}$$

$$L_{21} = a_3 f s, \; L_{22} = a_0 + a_2 a, \; L_{23} = a_2 f, \; L_{24} = a_1 + a_3 a - a_3 f, \tag{3.16}$$

$$L_{31} = a_1 s + a_3 s^2 - a_3 f s, \; L_{32} = -a a_2, \; L_{33} = L_{11} - L_{23}, \quad L_{43} = -f L_{34}, \tag{3.17}$$

$$L_{34} = -a_1 - a_3 a - a_3 s + f a_3, \; L_{41} = a_2 f s, \; L_{42} = a L_{24}, \; L_{44} = L_{22} - L_{23}, \tag{3.18}$$

Using (3.11), the solution (3.7) can be rewritten in the form

$$\bar{V}(y,s) = L(y,s)\{\bar{V}(0,s) + \int_{\tau=0}^{y} \exp(-A(s)\tau)\, \bar{b}\,(s) d\tau\}, \tag{3.19}$$

$$= L(y,s)\bar{V}(0,s) + L(y,s)\begin{pmatrix} H_1(y,s) \\ H_2(y,s) \\ H_3(y,s) \\ H_4(y,s) \end{pmatrix}. \tag{3.20}$$

The explicit solution of $H_i(y,s), i = 1,2,3,4$, can be determined by using the Maclaurin series expansion.

$$\exp[-A(s)y] = \sum_{r=0}^{\infty} \frac{(-1)^r [A(s)y]^r}{r!}. \tag{3.21}$$

By using Cayley-Hamilton theorem, the infinite series (3.21) can be written as

$$\exp[-A(s)y] = a_0 I - a_1 A + a_2 A^2 - a_3 A^3 \tag{3.22}$$

Therefore

$$\begin{pmatrix} H_1(y,s) \\ H_2(y,s) \\ H_3(y,s) \\ H_4(y,s) \end{pmatrix} = \int_{\tau=0}^{y} \begin{pmatrix} L_{11} & -L_{12} & -L_{13} & L_{14} \\ -L_{21} & L_{22} & L_{23} & -L_{24} \\ -L_{31} & L_{32} & L_{33} & -L_{34} \\ L_{41} & -L_{42} & -L_{43} & L_{44} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -\bar{\phi}(s) \\ 0 \end{pmatrix} d\tau. \tag{3.23}$$

Hence, equation (3.20) can be written in the form

$$\bar{u}(y,s) = L_{11}\bar{u}(0,s) + L_{12}\bar{c}(0,s) + L_{13}\bar{u}'(0,s) + L_{14}\bar{c}'(0,s) + M_1(y,s), \tag{3.24}$$

$$\bar{c}(y,s) = L_{21}\bar{u}(0,s) + L_{22}\bar{c}(0,s) + L_{23}\bar{u}'(0,s) + L_{24}\bar{c}'(0,s) + M_2(y,s), \tag{3.25}$$

$$\bar{u}'(y,s) = L_{31}\bar{u}(0,s) + L_{32}\bar{c}(0,s) + L_{33}\bar{u}'(0,s) + L_{34}\bar{c}'(0,s) + M_3(y,s), \tag{3.26}$$

$$\bar{c}'(y,s) = L_{41}\bar{u}(0,s) + L_{42}\bar{c}(0,s) + L_{43}\bar{u}'(0,s) + L_{44}\bar{c}'(0,s) + M_4(y,s), \tag{3.27}$$

where

$$M_1(y,s) = L_{11}H_1(y,s) + L_{12}H_2(y,s) + L_{13}H_3(y,s) + L_{14}H_4(y,s), \tag{3.28}$$

$$M_2(y,s) = L_{21}H_1(y,s) + L_{22}H_2(y,s) + L_{23}H_3(y,s) + L_{24}H_4(y,s), \tag{3.29}$$

$$M_3(y,s) = L_{31}H_1(y,s) + L_{32}H_2(y,s) + L_{33}H_3(y,s) + L_{34}H_4(y,s), \tag{3.30}$$

$$M_4(y,s) = L_{41}H_1(y,s) + L_{42}H_2(y,s) + L_{43}H_3(y,s) + L_{44}H_4(y,s). \tag{3.31}$$

Applying the two boundary conditions (2.16), after taking the Laplace transform, we get

$$\bar{u}(0,s) = \left(1+\frac{\kappa}{\mu}\right)\left[\frac{\bar{u}'(0,s)}{\alpha_1} + \frac{\bar{c}'(0,s)}{\alpha_1\eta_1}\right], \quad \bar{c}(0,s) = \frac{\bar{c}'(0,s)}{\eta_1}. \tag{3.32}$$

Substituting from (3.32) into equations (3.24)-(3.27), we get the solution in terms of the two unknowns $\bar{u}'(0,s)$ and $\bar{c}'(0,s)$as

$$\bar{u}(y,s) = L_{11}\left(1+\frac{\kappa}{\mu}\right)\left(\frac{\bar{u}'(0,s)}{\alpha_1} + \frac{\bar{c}'(0,s)}{\alpha_1\eta_1}\right) + L_{12}\frac{\bar{c}'(0,s)}{\eta_1} + L_{13}\bar{u}'(0,s) + L_{14}\bar{c}'(0,s) + M_1(y,s) \tag{3.33}$$

$$\bar{c}(y,s) = L_{21}\left(1+\frac{\kappa}{\mu}\right)\left(\frac{\bar{u}'(0,s)}{\alpha_1} + \frac{\bar{c}'(0,s)}{\alpha_1\eta_1}\right) + L_{22}\frac{\bar{c}'(0,s)}{\eta_1} + L_{23}\bar{u}'(0,s) + L_{24}\bar{c}'(0,s) + M_2(y,s) \tag{3.34}$$

$$\bar{u}'(y,s) = L_{31}\left(1+\frac{\kappa}{\mu}\right)\left(\frac{\bar{u}'(0,s)}{\alpha_1} + \frac{\bar{c}'(0,s)}{\alpha_1\eta_1}\right) + L_{32}\frac{\bar{c}'(0,s)}{\eta_1} + L_{33}\bar{u}'(0,s) + L_{34}\bar{c}'(0,s) + M_3(y,s), \tag{3.35}$$

$$\bar{c}'(y,s) = L_{41}\left(1+\frac{\kappa}{\mu}\right)\left(\frac{\bar{u}'(0,s)}{\alpha_1} + \frac{\bar{c}'(0,s)}{\alpha_1\eta_1}\right) + L_{42}\frac{\bar{c}'(0,s)}{\eta_1} + L_{43}\bar{u}'(0,s) + L_{44}\bar{c}'(0,s) + M_4(y,s) \tag{3.36}$$

The unknowns functions $\bar{u}'(0,s)$ and $\bar{c}'(0,s)$ can be determined by satisfying the boundary conditions (2.17), after applying the Laplace transform, to be

$$\bar{c}'(0,s) = (\alpha_2\zeta_{21}H_1(1,s) - \eta_2\zeta_{11}H_2(1,s) + \frac{\kappa}{\mu}\zeta_{21}(H_2(1,s)+H_3(1,s)) - \zeta_{11}H_4(1,s) + \zeta_{21}(H_2(1,s)+H_3(1,s))/(\zeta_{12}\zeta_{21} - \zeta_{11}\zeta_{22}), \tag{3.37}$$

$$\bar{u}'(0,s) = (\alpha_2\zeta_{22}H_1(1,s) - \eta_2\zeta_{12}H_2(1,s) + \frac{\kappa}{\mu}\zeta_{22}(H_2(1,s)+H_3(1,s)) - \zeta_{12}H_4(1,s) + \zeta_{22}(H_2(1,s)+H_3(1,s))/(\zeta_{11}\zeta_{22} - \zeta_{12}\zeta_{21}), \tag{3.38}$$

$$\zeta_{11} = -\left(1+\frac{\kappa}{\mu}\right)\left(\frac{\alpha_2}{\alpha_1}L_{11}^1 + L_{23}^1 + L_{33}^1\right) - \left(1+\frac{\kappa}{\mu}\right)^2\left(\frac{L_{31}^1}{\alpha_1} + \frac{L_{21}^1}{\alpha_1}\right) - \alpha_2 L_{13}^1, \tag{3.39}$$

$$\zeta_{12} = -\left(1+\frac{\kappa}{\mu}\right)\left(\frac{\alpha_2}{\alpha_1\eta_1}L_{11}^1 + \frac{L_{22}^1}{\eta_1} + L_{24}^1 + \frac{L_{32}^1}{\eta_1} + L_{34}^1 + \frac{\alpha_2}{\eta_1}L_{12}^1\right) - \left(1+\frac{\kappa}{\mu}\right)^2\left(\frac{L_{31}^1}{\alpha_1\eta_1} + \frac{L_{21}^1}{\alpha_1\eta_1}\right) - \alpha_2 L_{14}^1, \tag{3.40}$$

$$\zeta_{21} = \frac{-\eta_2}{\alpha_1}\left(1+\frac{\kappa}{\mu}\right)L_{21}^1 - \eta_2 L_{23}^1 - \left(1+\frac{\kappa}{\mu}\right)\frac{L_{41}^1}{\alpha_1} - \left(1+\frac{\kappa}{\mu}\right)L_{34}^1 - L_{43}^1, \tag{3.41}$$

$$\zeta_{22} = \frac{-\eta_2}{\alpha_1\eta_1}\left(1+\frac{\kappa}{\mu}\right)L_{21}^1 - \frac{\eta_2}{\eta_1}L_{22}^1 - \eta_2 L_{24}^1 - \left(1+\frac{\kappa}{\mu}\right)\frac{L_{41}^1}{\alpha_1\eta_1} - \frac{L_{42}^1}{\eta_1} - L_{44}^1, \tag{3.42}$$

where L_{ij}^1 are the values of L_{ij} evaluated at $y = 1$.

4. THE NUMERICAL INVERSION OF LAPLACE TRANSFORM

To get the inverse Laplace transform of the velocity and microrotation components, we employ a numerical inversion technique developed by Honig and Hirdes [16]. In this method, the inverse Laplace transform of a function $\bar{g}(s)$ is approximated by

$$g(t) = \frac{e^{ht}}{T}\left[\frac{1}{2}\bar{g}(h) + \sum_{k=1}^{N} Re\left\{\bar{g}\left(h + \frac{ik\pi}{T}\right)\exp\left(\frac{ik\pi t}{T}\right)\right\}\right], \qquad 0 < t < 2T, \tag{4.1}$$

where N is sufficiently large integer chosen such that,

$$e^{ht} Re\left\{\bar{g}\left(\mathrm{h} + \frac{iN\pi}{T}\right)\exp\left(\frac{iN\pi t}{T}\right)\right\} < \varepsilon, \tag{4.2}$$

where ε is a small positive number that corresponds to the degree of accuracy required. The parameter h is a positive free parameter that must be greater than real parts of all singularities of $\bar{g}$(s).

5. NUMERICAL RESULTS

Now we represent the obtained results graphically. Two different cases are considered; flow due to a constant pressure gradient and flow due to an oscillatory pressure gradient. In the numerical calculations, we have taken $\alpha_1 = \alpha_2 = \alpha$ and $\eta_1 = \eta_2 = \eta$.

CASE 1

In this case, we assume that the dimensionless pressure gradient is given by

$$\frac{-\partial p}{\partial x} = \varphi(t) = H(t), \tag{5.1}$$

where the Heaviside unit step function $H(t)$ is defined by

$$H(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

CASE 2

Here, the pressure gradient is assumed to be dependent of the time and is given by

$$\frac{-\partial p}{\partial x} = \varphi(t) = \sin t\ . \tag{5.2}$$

Figs. 1 and 2 show the distribution of the velocity and microrotation versus the distance between the two plates for different times when the constant pressure gradient is assumed. It can be concluded that the values of the velocity and microrotation increase with the increase of the time and the steady state case is recovered when the time approaches infinity. The velocity and microrotation distributions for different values of the velocity slip parameter α are shown in Figs. 3 and 4, respectively. It can be observed that the velocity values decrease with the increase of this parameter while the microrotation increases. Also, the case of no-slip is obtained as the slip parameter goes to infinity as shown in the figures. The velocity and microrotation profiles for different microrotation slip parameter η, keeping α fixed, are represented in Figs. 5 and 6. It is noticed from Fig.5 that this parameter does not affect on the velocity while Fig.6 shows that this parameter has a considerable effect on the microrotation. Finally, the distributions of the velocity and microrotation for different values of the micropolarity parameter are represented in Figs. 7 and 8. It is observed that the increase in this parameter results in an increase of the values of both velocity and microrotation. If the micropolarity ratio κ/μ becomes zero, we return to the classical case of viscous fluid. The variation of the velocity and microrotation for the case of sine oscillation are represented in Figs. 9 and 10, respectively.

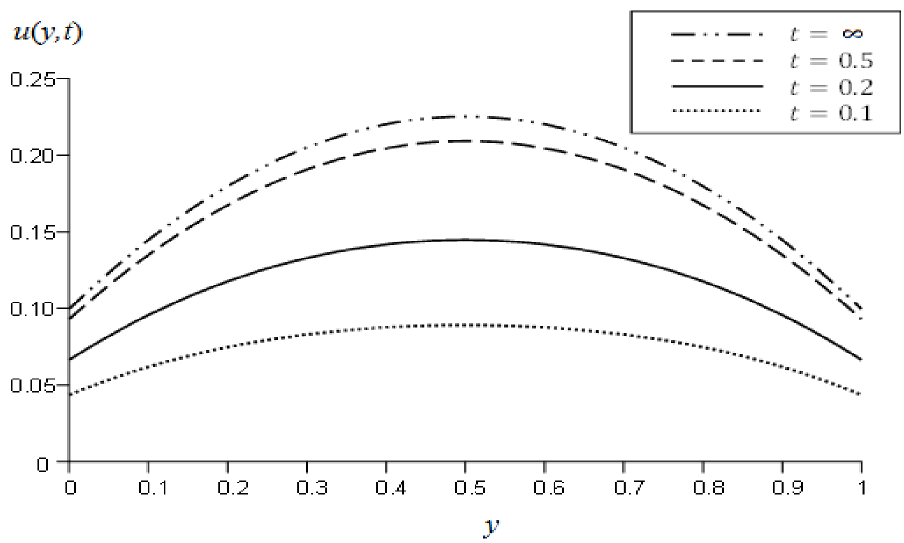

Fig.1. *Velocity variation versus distance at* $\alpha = \eta = 10$ *and* $\kappa/\mu = 1$ *for case 1*

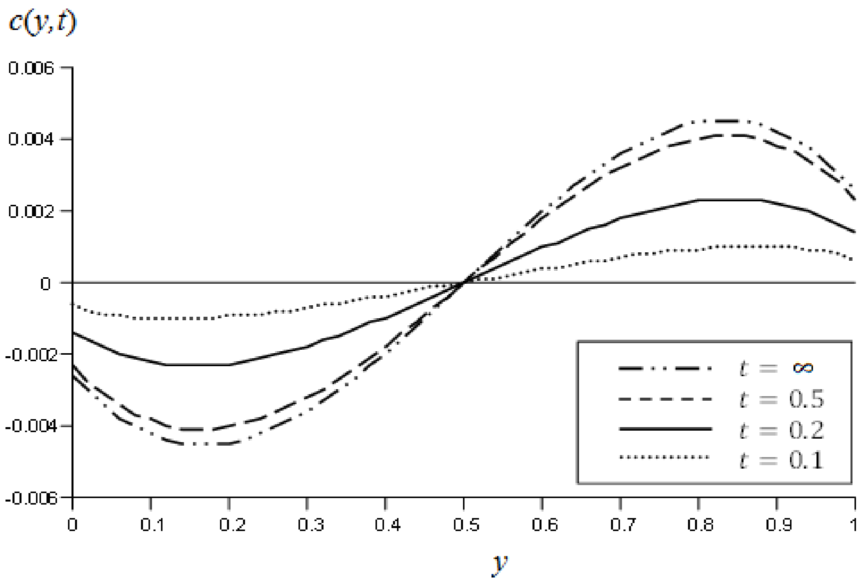

Fig.2. *Microrotation variation versus distance at* $\alpha = \eta = 10$ *and* $\kappa/\mu = 1$ *for case 1*

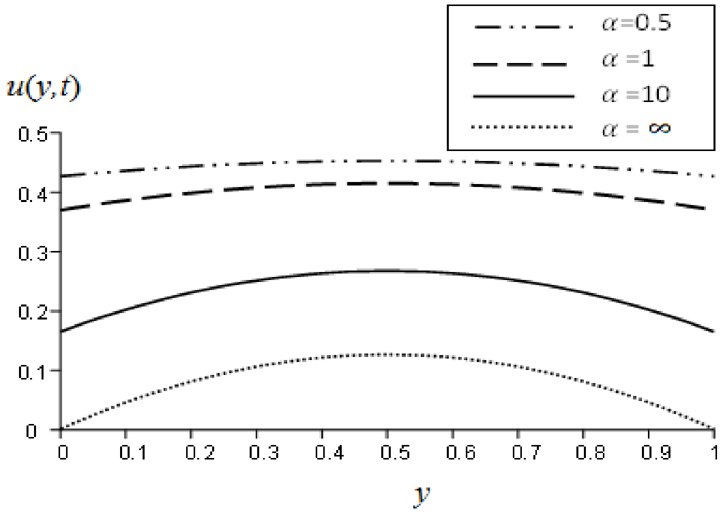

Fig. 3. *Velocity variation versus distance at* $t = 0.5, \kappa/\mu = 1$ *and* $\eta = 10$ *for case 1*

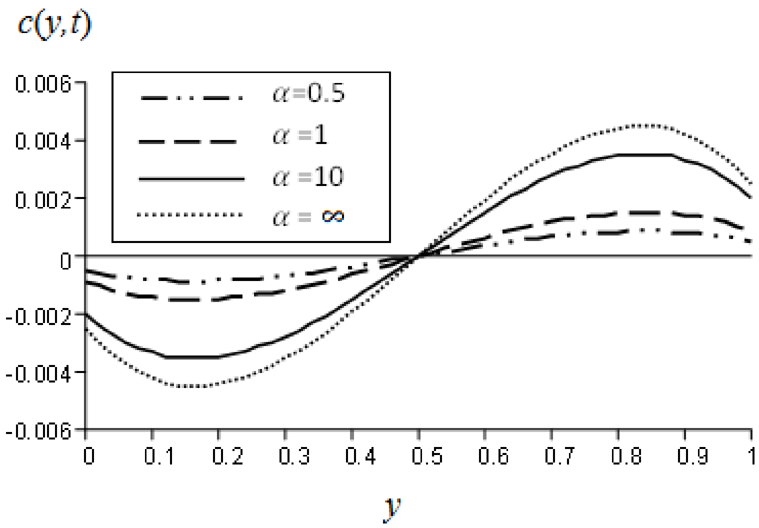

Fig. 4. *Microrotation variation versus distance at* $t = 0.5, \kappa/\mu = 1$ *and* $\eta = 10$ *for case 1*

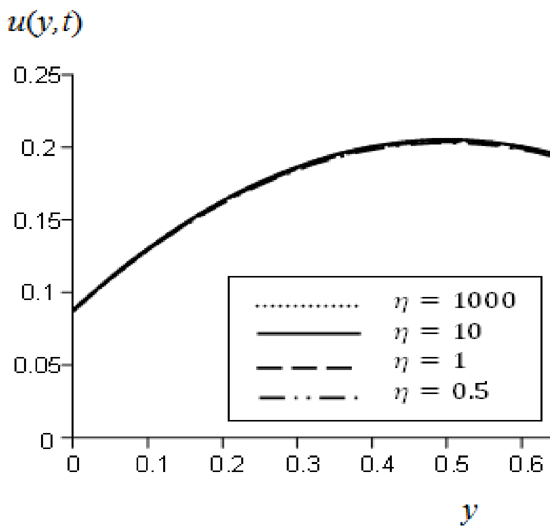

Fig. 5. *Velocity variation versus distance at* $t = 0.5, \kappa/\mu = 1$ *and* $\alpha = 10$ *for case 1*

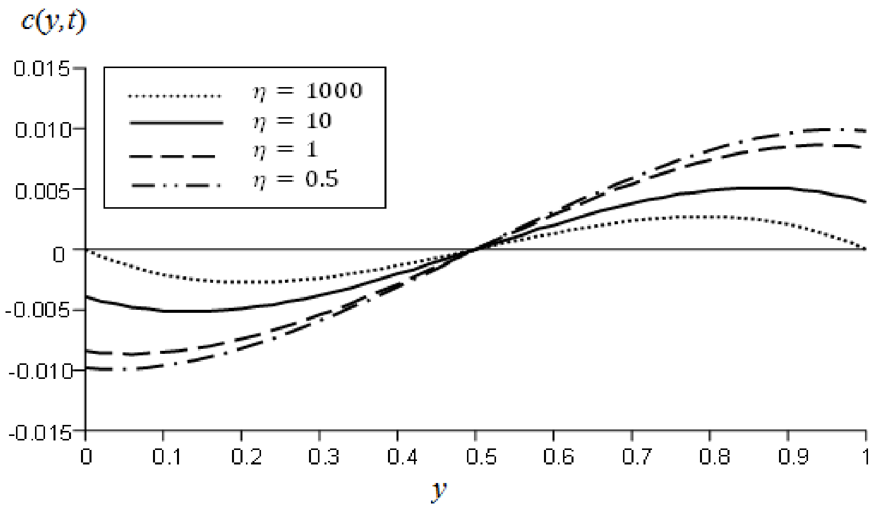

Fig. 6. *Microrotation variation versus distance at* $t = 0.5, \kappa/\mu = 1$ *and* $\alpha = 10$ *for case 1*

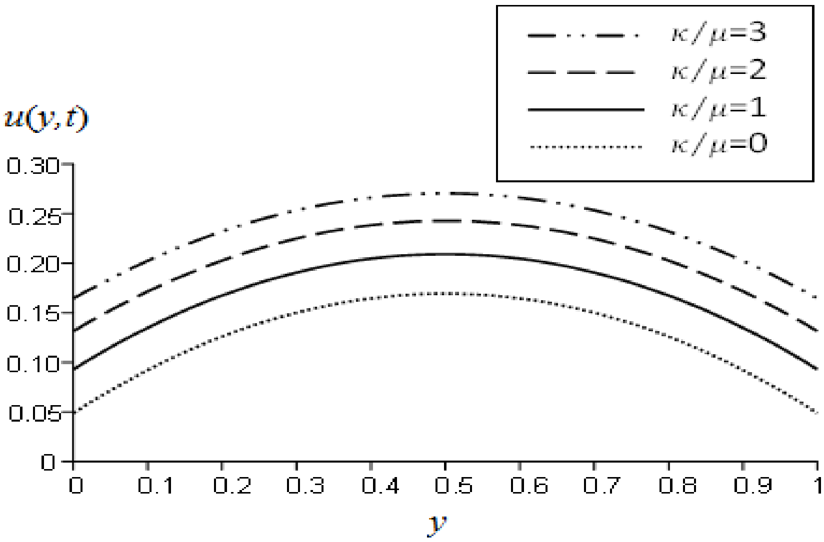

Fig.7. *Velocity variation versus distance at* α*=*η*=10 and* $t = 0.5$ *for case 1*

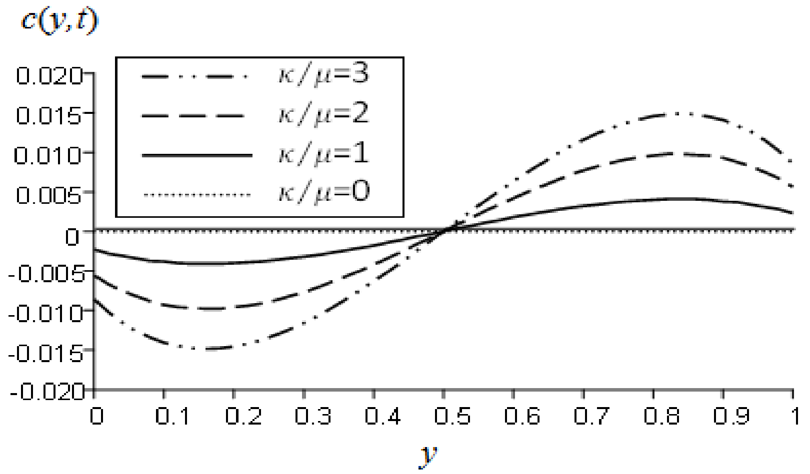

Fig.8. *Microrotation variation versus distance at* $\alpha = \eta = 10$ *and* $t = 0.5$ *for case 1*

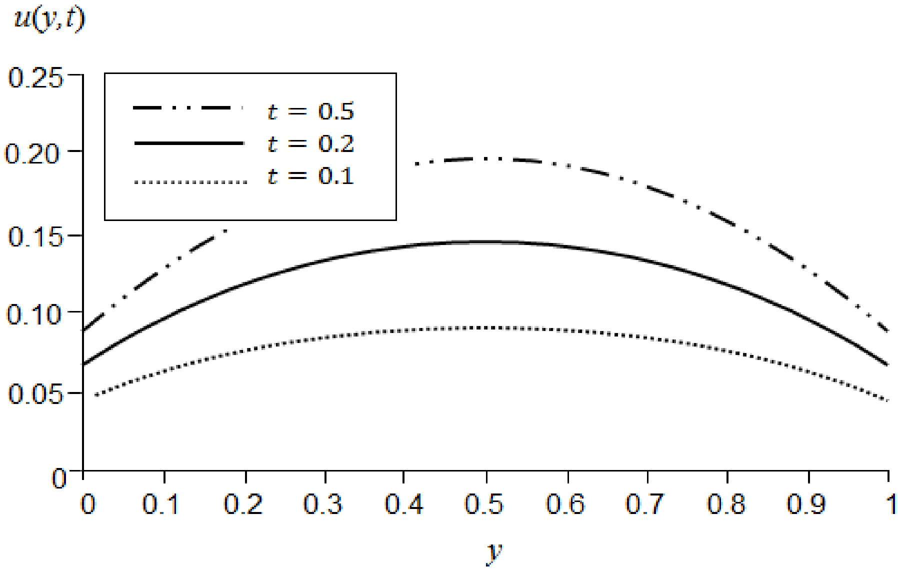

Fig.9. *Velocity variation versus distance at* $\alpha = \eta = 10$ *and* $\kappa/\mu = 1$ *for case 2*

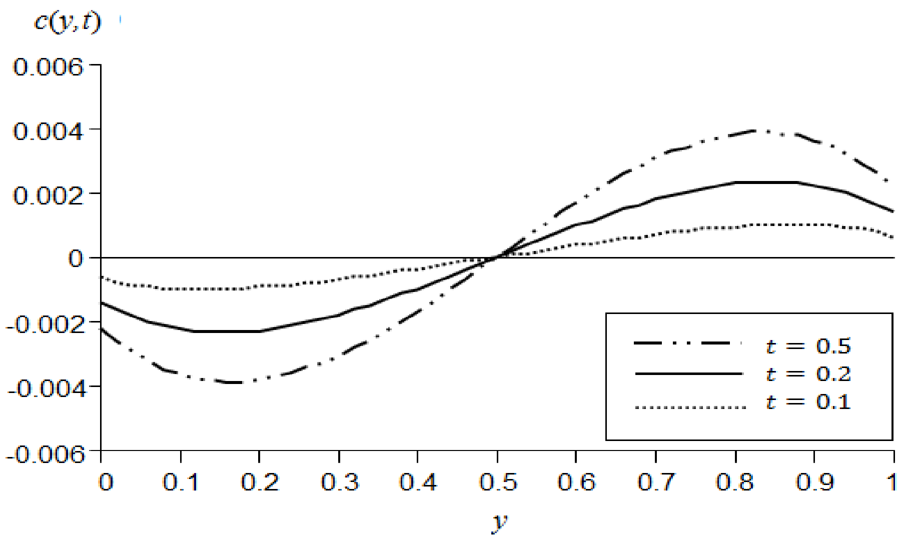

Fig.10. *Microrotation variation versus distance at* $\alpha = \eta = 10$ *and* $\kappa/\mu = 1$ *for case 2*

6. Conclusion

The State Space approach is utilized to study the unsteady micropolar fluid flow between two parallel plates. The slip boundary condition is applied for both velocity and microrotation. It is concluded that the slip for velocity has a considerable effect on the flow field. Also, it is observed that the microrotation slip has no effect on the velocity vector while it has a remarkable effect on the microrotation. The no-slip case is recovered when the slip parameters approaches infinity. It is also concluded that the time has a considerable influence on the flow field especially at small times and that the steady state is obtained at large time when the constant pressure gradient is considered. Finally, it is noticed that the micropolarity parameter increases the values of the velocity and microrotation.

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