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[J, K]- Set vertex-edge and edge-vertex domination of path graphs

N. Murugesan, P. Elangovan*

PG and Research Department of Mathematics Government Arts College, Coimbatore

***Corresponding Author:** **P. Elangovan,** PG and Research Department of Mathematics Government Arts College, Coimbatore*

Abstract: *Domination is an Advanced Research in Graph Theory. There are various dominating sets defined by graph Theorists. In this paper, one such dominating set is known as [J, K]- setvertex edge and edge-vertexdomination of path graphs have been discussed.The generalization of the graphs, the number of dominating sets, and the number of dominating vertices and edges have been discussed.*

Keywords: *[j, k] – set vertex- edge dominating set, [j, k] – set vertex- edge domination number.*

[j, k] – set edge-vertex- dominating set, [j, k] – set edge-vertex- domination number.

1. Introduction

The basic definitions and concepts of graph theory have been learned from D.B.West [9]. Fundamental definitions of dominations and various basic theorems on this were studied by T.W.Haynes et. al[2].

[1,2] Dominations in line graphs have been introduced by N.Murugesan and Deepa s. Nair[4]. Definitions and fundamentals of graph domination were discussed by Arash Behzad et. al[1].

Mustapha Chellali et. al[5] have explained [1,2]- the domination of graphs in their paper. Various theorems have been discussed by Xiaojing Yang et. al[8]

[1. K] domination of graphs was explained by E.Sampath Kumar et. al[7]. Edge-related domination has been explained in [3,6]

2. Preliminaries

Let G(V, E) be a simply connected graph with vertex set V and edge set E. Order and size of the graph is n =|v| and m=|E| Open and closed neighbourhood of the vertex and edges N(v)= {uϵV | uvϵ E} , N[v] =N(v)u{v } and $N(e_i) = \{e_j \in E_j\}$,i,j=1,2,3,……

The number of edges incident to a vertex v is the vertex degree, deg(v) = I N(v)|. The edge degree of the edge is defined as the number of neighbors of e i.e. | N(u)uN(v)|-2.

Definition: 2.1 A subset D of the vertex set V of a graph G is a dominating set if every vertex in the complement of D in V has a neighbor in D.

Definition: 2.2 A Minimum dominating set D in a graph G is a [j, k]- dominating set if there are vertices in the complement of D in G that have at least j and atmost k number of neighbors in D for j=1 and k =2.

Definition: 2.3 A subset D of the vertex set V is said to be a vertex-edge dominating set of the graph G if for each edge uv in G, there is a vertex w in D such that w ϵ {u,v} or w dominates at least one of u,v. The vertex edge domination number γ(G) is the minimum cardinality of a vertex–edge dominating set of G.

Definition: 2.4 A subset D of E is an edge–vertex dominating set (ev – ev-dominating set) of G if every vertex of graph G is ev dominated by at least one edge of G.

Symbol : 1.$D_{(j,k)}(P_{V,E})_n$| or $|D_{(j,k)}(P_{E,V})_n|$: Number of dominating sets

2.$\gamma_{\{j,k\}}$ $(P_{V,E})_n$ or $\gamma_{\{j,k\}}$ $(P_{E,V})_n$: Number of dominating vertices or edges

3. [J, K] – SET VERTEX-EDGE DOMINATION

Theorem 3.1 the number of [1,1] – set vertex-edge domination of path graph is

$|D_{(1,1)} (P_{V,E})_k| = \{ k-2, \quad k=3,4,5\ldots.$

proof: Let v_1, v_2, v_3 ………………… v_n are the vertices and e_1, e_2………………$e_{n-1}$are the edges of the path graph. deg (v_1) = deg(v_n) = 1. The deg(v_i) = 2, i=2,3,4……. n-1. The vertices v_i, i=2,3,……… dominate the vertices v_i or v_j, i=1,2,3…n-…….. n-2 and j=3,4,5………… n.

Vertices v_i, I = 2,3………. (n-1) dominates v_j, j=1,3,2,4,3,5……

Thereforevertex- edge dominating vertices are { v_2, v_3, v_4 ……… v_{k+1}}

The Generalized form of [1,1] – dominating sets of the vertex edge are

$D_{(1,1)}(p_{V,E})_k \quad = \quad \{ v_{k-1}, k=3,4,5,\ldots\ldots.$

and the number of dominating vertices are

$\gamma_{[1,1]} [(P_{V,E})_k] = k-2$

Theorem 3,2. The number of [1, 2] – set vertex–edge domination of path graph p_n is

$| D_{(1,2)} [(P_{V,E})_K]| = \{ k-2, k=4,5,6$ …………

Proof: The vertex–edge dominating sets of the path P_4 are { v_1, v_3 } and { v_2, v_4 }. The vertex – edge dominating sets of the path P_5 is { v_1, v_3}, {v_2, v_4} and {v_3, v_5}

Proceeding like this we could find the vertex–edge dominating sets of the path

$P_n$is { v_1, v_3}, {v_2, v_4}, {v_3, v_5}……………………..{v_{n-2}, v_n}

The Generalized form of [1,2] – dominating sets of the vertex-edge of the path graph P_n is

When n= k+3 is, k=1,2,3…….

$D_{(1,2)} [(P_{v,e})_n] = \{v_k, v_{k+2}$, k=1,2,3……. and the number of dominating vertices in each set is

$\gamma_{[1,2]}[(P_{v,e})_n]= k$

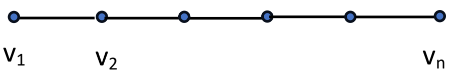

Fig (1)

4. [J, K] – EDGE- VERTEX DOMINATION

Theorem: 4.1. The number of [1,1] – set edge – vertex domination of path graph P_n when n=k+1, k=1,2,3, etc is

$|D_{[1,1]} (P_{E,v})_n | = \{ k, \ k=1,2,3\ldots\ldots\ldots$

Proof: Let v_1, v_2, v_3 ……………….v_n are the vertices and e_1, e_2, e_3 ……………e_n are the edges of the path graph P_n. For the path graph P_2, edge $e_1$dominates the vertex v_1 or v_2. For the graph P_3 edge e_1 dominates the vertex v_1 or v_2, and edge e_2 dominates the vertex v_2 or v_3. Proceeding like this we get for the path graph P_n, e_{n-1}, when n=2,3,4,………. Dominates the vertices v_k or v_{k+1}, k=1,2,3…………..

The number of [1,1] domination of the edge - vertex when n=k+1 is

$|D_{[1,1]} (P_{e,v})_n|=k$, k=1,2,3……………

The generalized form of [1,1]- set edge -vertex when n=k+1 is

$D_{[1,1]} (P_{e,v})_n = \{ e_k$, when k=1,2,3 …………………

Therefore, the number of dominating edges in each dominating set is

$\gamma_{[1,1]} (P_{E,V})_n = k$

Theorem 4.2: The number of [1,2]- set edge – vertex domination of path graph when n=k+1,k=1,2,3 ……… is

$D_{[1,2]} (P_{E, V})_n = \{ k$, when k=1,2,3…………

Proof: Let v_1, v_2, v_3 ……………….v_n are the vertices and e_1, e_2, e_3 ……………e_{n-1} are the edges of the path graph P_n. For the path graph P_3 edges e_1 and e_2 dominates the vertex v_2 and e_1 dominates v_1 or v_2 and edge e_2 dominates v_2 or v_3. For the graph p_4, edges e_1 and $e_2$dominate the vertex v_2, and edges e_2 and e_3 dominate the vertex v_3. Proceeding like this up to the path graph P_n, e_{n-1}, when n=2,3, 4,………. Dominates the vertices v_k, k=1,2,3…………..n-1.

The number of [1,2] – set edge – vertex domination of the path graph P_n when n=k+1 is

$|D_{[1,2]} (P_{e,v})_n| = \{ k+1$, k=1,2,3 ………………………

The generalized form of [1,2] – set edge – vertex when n=k+1 is

$$D_{[1,2]} (P_{e,v})_n = \{ e_k, e_{k+1}, k=1,2,3\ldots$$

Therefore the number of dominating edges in each dominating set when n=k+2, k=1,2,3………………… is $\gamma_{[1,1]} (P_{E, V})_n = k+1$, k=1,2,3………………………

5. Conclusion

In this paper [J,K]- set vertex-edge and edge-vertex domination of the path graph has been discussed in detail.

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