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Gl-Betweenness Relation And Autometrized Generalised Lattices

Parimi Radha krishna Kishore[1*], GebrieYeshiwasTilahun[2]

[1]*Associate Professor, Department of Mathematics, Arba Minch University, Arba Minch, Ethiopia and also Guest Faculty, Department of Mathematics, SRM University-AP, Andhra Pradesh, Neerukonda(Village), Mangalagiri(Mandalam), Guntur (District), Andhra Pradesh (State), PIN Code: 522240, INDIA*

[2]*Ph.D-Research Scholar, Department of Mathematics, Arba Minch University, Arba Minch, ETHIOPIA*

Corresponding Author: *Parimi Radha krishna Kishore, *Associate Professor, Department of Mathematics, Arba Minch University, Arba Minch, Ethiopia and also Guest Faculty, Department of Mathematics, SRM University-AP, Andhra Pradesh, Neerukonda(Village), Mangalagiri(Mandalam), Guntur (District), Andhra Pradesh (State), PIN Code: 522240, INDIA*

Abstract: *In this paper discussed about the 3-relatiom gl-betweenness in a generalised lattice and introduced the concept autometrized generalised lattice.*

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1. Introduction

Mellacheruvu Krishna Murty and U. MadanaSwamy (Professors of Andhra University)[2] introduced the concept of generalised lattice. The author P.R.Kishore [3,4] developed the theory of generalised lattices that can play an intermediate role between the theories of lattices and posets. The concepts metrized lattice and autometrized lattice is known from Leo Lapidus [5,6]. Later the author P.R.Kishore introduced and developed the concepts Brouwerian generalised lattice in [8] and generalised lattice metrized space (gl-metrized space) in [9, 10]. In this paper section 2 contains some preliminary concepts that are from the references. In section 3 discussed about the 3-relation gl-betweenness on a generalised lattice which is already introduced in [9]. In section 4 introduced and discussed the concept autometrized generalised lattice.

2. PRELIMINARIES

This section contains some preliminaries from the references those are useful in the next sections. The concepts of generalised lattice and distributive generalised lattice are known from [3,4].

Definition 2.1 [Kishore [9]] Let P be a generalised lattice and $a,b,c \in P$. Then b is said to be gl-between a and c in P if $ML(mu(ML\{a, b\} \cup ML\{b, c\})) = \{b\} = mu(ML(mu\{a,b\} \cup mu\{b,c\}))$, denoted by $(a,b,c) \in glb$.

Definition 2.2 [Kishore [9]] Let S be a set and P be a generalised lattice having least element 0. If there exists a map $d: S\times S \to P$ such that (i) $d(a,b) \geq 0$ and $d(a,b) = 0 \Leftrightarrow a=b$ (ii) $d(a,b) = d(b,a)$ (iii) $d(a,c) \in L(mu(d(a,b), d(b,c)))$, then the ordered pair (S,d) is called a generalised lattice metrized space (gl-metrized space). We denote d(a,b) by $a * b$.

Theorem 2.3 [Kishore [10]] Let P be a generalised lattice and $a, b, c \in P$. Then $(a, b, c) \in glb$ implies $(s, b, t) \in pob$ for all $s \in ML\{a, c\}$ and $t \in mu\{a, c\}$.

Definition 2.4 [Kishore [10]] Let P be a generalised lattice and $a, b, c \in P$. Then a, b, c are said to satisfy the triangular inequality in P if $a \in L(mu\{b, c\})$, $b \in L(mu\{c, a\})$, $c \in L(mu\{a, b\})$, denoted by $(a, b, c) \in glt$.

3. GL-BETWEENNESS

Theorem 3.1 Let P be a generalised lattice and $a, b, c \in P$. Then $b = c$ implies $(a, b, c) \in glb$ and $(a, c, b) \in glb$.

Proof: Suppose b = c. Consider mu(ML{a, b} ∪ ML{b, c}) = mu(ML{a, b} ∪ ML{b, b}) = mu(ML{a, b} ∪ {b}) = {b}. Consider ML(mu{a, b}∪ mu{b, c}) = ML(mu{a, b}∪ mu{b, b}) = ML(mu{a, b} ∪ {b}) = {b}. Therefore (a, b, c)∈ glb and (a, c, b)∈ glb.

Theorem 3.2 Let P be a generalised lattice and a, b, c, d∈ P. Then (a, b, c)∈ glb and (a, d, b)∈ glb implies (d, b, c)∈ glb.

Proof: Suppose (a, b, c)∈ glb and (a, d, b)∈ glb. Then mu(ML{a, b}∪ ML{b, c}) = {b} = ML(mu{a, b}∪ mu{b, c}) and by theorem 2.3 we have (s, d, t)∈ pob that is s≤ d ≤ t for all s∈ ML{a, b} and t∈ mu{a, b}. To show that (d, b, c)∈ glb: Consider L(mu(ML{d, b}∪ ML{b, c})) = L({d, b}) ∨ L({b, c}) = (L(d)∧ (L({a, b})∨ L({b, c}))) ∨ L({b, c}) ≥ (L(d)∧ L({a, b}))∨ L(d)∧ L({b, c}) ∨ L({b, c}) ≥ (L(d)∧ L(s))∨ (L(d)∧ L({b, c}))∨ L({b, c}) = L(s)∨ L({b, c}) for all s∈ ML{a, b}. This implies L(mu(ML{d, b}∪ ML{b, c})) ≥ $\bigvee_{s\in ML\{a,b\}}$(L(s) ∨ L({b, c})). This implies L(mu(ML{d, b}∪ ML{b, c})) ≥ L({a, b})∨ L({b, c}) = L(mu(ML{a, b}∪ ML{b, c})) = L(b). We know that L(mu(ML{d, b}∪ ML{b, c})) = L({d, b}) ∨ L({b, c}) ≤ L(b). That is L(mu(ML{d, b}∪ ML{b, c})) = L(b). Then mu(ML{d, b}∪ ML{b, c}) = {b}. Similarly we can prove ML(mu{d, b}∪ mu{b, c}) = {b}. Therefore (d, b, c)∈ glb.⊡

Theorem 3.3 Let P be a generalised lattice and a, b, c, d, x∈ P. Then (a, b, c)∈ glb, (a, d, b)∈ glb and (a, c, x)∈ glb implies (d, c, x)∈ glb.

Proof: Suppose (a, b, c)∈ glb, (a, d, b)∈ glb and (a, c, x)∈ glb. Since (a, c, x)∈ glb, we have mu(ML{a, c}∪ ML{c, x}) = {c} = ML(mu{a, c}∪ mu{c, x}). Since (a, b, c)∈ glb, (a, d, b)∈ glb; by theorem 2.3 we have s≤ b for all s∈ML{a, c} and p≤ d for all p∈ ML{a, b}. To show that L({a, c})⊆ L(d): Let s∈ ML{a, c}. Then s≤ a and s≤ b, that is s∈L({a, b}). This implies there exists p∈ML{a, b} such that s≤p. Then s≤ p ≤ d for all s∈ML{a, c}. This implies L(s)⊆L(d) for all s∈ ML{a, c}. Therefore L({a, c})⊆ L(d). To show that (d, c, x)∈ glb: Consider L(mu(ML{d, c}∪ ML{c, x})) = L({d, c})∨ L({c, x}) = (L(d)∧ (L({a, c})∨ L({c, x})))∨ L({c, x}) ≥ (L(d)∧ L({a, c}))∨ (L(d)∧ L({c, x}))∨ L({c, x}) = L({a, c})∨ L({c, x}) = L(mu(ML{a, c} ∪ ML{c, x})) = L(c)≥ (L(d)∧ L(c))∨ (L(c)∧ L(x)) = L(mu(ML{d, c}∪ ML{c, x})). Therefore L(mu(ML{d, c}∪ ML{c, x})) = L(c). This implies mu(ML{d, c}∪ ML{c, x}) = {c}. Therefore (d, c, x)∈ glb.⊡

4. Autometrized Generalised Lattices

Definition 4.1 Let P be a generalised lattice with least element 0. If there is a map d: P×P → P such that (i) a∗ b ≥ 0 and a∗ b = 0⇔ a = b (ii) a∗ b = b∗ a (iii) a∗ c ∈ L(mu{a∗ b, b∗ c}), then P is called an autometrized generalised lattice.

Note: Every autometrized generalised lattice is a gl-metrized space.

Definition 4.2 Let P be an autometrized generalised lattice with least element 0. Then P is called regular if a∗ 0 = a for all a∈ P.

Definition 4.3Anautometrizedgenerlised lattice P is said to be distributive if P is a distributive generalised lattice.

Theorem 4.4 Let P be a regular distributive autometrized generalised lattice. Then for any a,b∈ P we have ML(mu{a, b}) = ML(mu(ML{a, b}∪{a∗ b})).

Proof: Consider the triangle Δ_P(a, 0, b). By definitions 4.1 and 4.2, we have a∗ b ∈ L(mu{a∗ 0, 0∗ b}) = L(mu{a, b}), a = a∗ 0 ∈ L(mu{a∗ b, b∗ 0}) = L(mu{a∗ b, b}) and b = b∗ 0 ∈ L(mu{b∗ a, a∗ 0}) = L(mu{b∗ a, a}) = L(mu{a∗ b, a}). Then by definition 2.4 we have (a∗ b, a, b)∈ glt. This implies we get L(a)∨ L(b) = L(a)∨ L(a∗ b) = L(b)∨ L(a∗ b) = L(a)∨ L(b)∨ L(a∗ b). Consider L(mu{a, b}) = L(a)∨ L(b) = (L(a)∨ L(a∗ b))∧ (L(b)∨ L(a∗ b)) = (L(a)∧ L(b))∨ L(a∗ b) = L({a, b})∨ L(a∗ b) = L(mu(ML{a, b}∪{a∗ b})). Therefore ML(mu{a, b}) = ML(mu(ML{a, b}∪{a∗ b})).⊡

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