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Generalised Lattice Betweenness (Gl-Betweenness) and Generalised Lattice Triangular Inequality (Gl-Triangular Inequality) 3-Relations on a Generalised Lattice

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Abstract: *In this paper introduced and discussed about 3-relations generalised lattice triangular inequality (gl-triangular inequality) and generalised lattice betweenness (gl-betweenness) on a generalised lattice.*

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1. Introduction

Mellacheruvu Krishna Murty and U. MadanaSwamy (Professors of Andhra University)[2] introduced the concept of generalised lattice. The author P.R.Kishore [3,4,5,6] developed the theory of generalised lattices that can play an intermediate role between the theories of lattices and posets. Later the author P.R.Kishore introduced and developed the concept Brouwerian generalised lattice in [10] and the concept generalised lattice metrized space (gl-metrized space) in [11]. In this paper section 2 contains some preliminary concepts that are from the references. In section 3 introduced and discussed about a 3-relation generalised lattice triangular inequality (gl-triangular inequality) on a generalised lattice. In section 4 discussed about the 3-relation generalised lattice betweenness (gl-betweenness) which is already introduced in [11].

2. Preliminaries

This section contains some preliminaries from the references those are useful in the next sections. The concepts of generalised lattice, subgeneralised lattice and distributive generalised lattice are known from [3,4,5].

Definition 2.1 [Kishore [11]] Let P be a generalised lattice and a,b,c$\in$ P. Then b is said to be gl-between a and c in P if ML(mu(ML{a, b}$\cup$ ML{b, c})) = {b} = mu(ML(mu{a,b} $\cup$ mu{b,c})), denoted by (a,b,c) $\in$ glb.

Kishore [11] observed that the 3-relation glb has transitivity t_1.

Definition 2.2 [Kishore [11]] Let S be a set and P be a generalised lattice having least element 0. If there exists a map d: S $\times$ S $\dashrightarrow$ P such that (i) d(a,b) $\geq$ 0 and d(a,b) = 0 $\Longrightarrow$ a= b (ii) d(a,b) = d(b,a) (iii) d(a,c) $\in$ L(mu(d(a,b), d(b,c))), then the ordered pair (S,d) is called a generalised lattice metrized space (gl-metrized space). We denote d(a,b) by a $*$ b.

Lemma 2.3 [Kishore [6]] Let P be a distributive generalised lattice. Then for any a, b, c $\in$ P we have (i) mu(ML{a,b} $\cup$ {c}) = mu(ML(mu{a, c}$\cup$ mu{b, c})) (ii) ML(mu{a,b}$\cup$ {c}) = ML(mu(ML{a, c}$\cup$ ML{b, c})).

3. GL-TRIANGULAR INEQUALITY

Definition 3.1 Let P be a generalised lattice and a, b, c∈ P. Then a, b, c are said to satisfy the triangular inequality in P if a∈ L(mu{b, c}), b∈ L(mu{c, a}), c∈ L(mu{a, b}), denoted by (a, b, c)glt.

Note: Let P be a generalised lattice. Then by definition 3.1 we have glt = {(a,b,c)∈ P ×P× P | (a, b, c)glt} = {(a,b,c)∈ P^3 | a∈ L(mu{b, c}), b∈ L(mu{c, a}), c∈ L(mu{a, b})} is a 3-relation on P.

Theorem 3.2 Let P be a distributive generalised lattice and a, b, c∈ P. Then (a, b, c)∈ glt if and only if ML(mu{a, b}) = ML(mu{a, c}) = ML(mu{b, c}) = ML(mu(ML{a, c} ∪ ML{b, c})).

Proof: Suppose (a, b, c)∈ glt. Then a∈ L(mu{b, c}), b∈ L(mu{c, a}), c∈ L(mu{a, b}). This implies L(a) ⊆ L(mu{b, c}) = L(b)∨ L(c), L(b)⊆ L(mu{c, a}) = L(c) ∨ L(a), L(c)⊆ L(mu{a, b}) = L(a) ∨ L(b). That is L(a) ∨ L(b) = L(b) ∨ L(c) = L(c) ∨ L(a) = L(a) ∨ L(b) ∨ L(c). Therefore L(mu{a, b}) = L(mu{b, c}) = L(mu{c, a}) = L(mu(ML(mu{a, b}) ∪ {c})). By lemma 2.3 we have mu(ML{a,b} ∪ {c}) = mu(ML(mu{a, c} ∪ mu{b, c})). This implies L(mu{a, b}) = L(mu{b, c}) = L(mu{c, a}) = L(mu(ML(mu{a, c} ∪ mu{b, c}))). Therefore ML(mu{a, b}) = ML(mu{b, c}) = ML(mu{c, a}) = ML(mu(ML(mu{a, c} ∪ mu{b, c}))).□

4. GL-BETWEENNESS

Definition 4.1 Let P be a poset and a, b, c ∈ P. If a ≤ b ≤ c then we say that b is poset between a and c, denoted by (a, b, c)∈ pob.

Theorem 4.2 Let P be a generalised lattice and a, b, c ∈ P with a ≤ c. Then (a, b, c) ∈ pob if and only if (a, b, c) ∈ glb.

Proof: Suppose (a, b, c) ∈ pob, that is a≤ b≤ c. To show that (a, b, c)∈ glb: Since a≤ b≤ c, we have ML{a, b} = {a}, ML{b, c} = {b}, mu{a, b} = {b} and mu{b, c} = {c}. This implies mu(ML{a, b} ∪ ML{b, c}) = mu{a, b} = {b} and ML(mu{a, b} ∪ mu{b, c}) = ML{b, c} = {b}. Therefore mu(ML{a, b} ∪ ML{b, c}) = {b} = ML(mu{a, b}∪ mu{b, c}), that is (a, b, c)∈ glb. Conversely suppose (a, b, c)∈ glb. To show that (a, b, c)∈ pob: Consider L(mu{a, b}) ∩ L(c) = (L(a) ∨ L(b)) ∧ L(c) ≥ L(a) ∨ (L(b) ∧ L(c)) = L(a) ∨ (L(a) ∧ L(b)) ∨ (L(b) ∧ L(c)) = L(a) ∨ (L({a, b}) ∨ L({b, c})) = L(a) ∨ L(mu(ML{a, b} ∪ ML{b, c})) = L(a) ∨ L(b). Therefore L(b) ⊆ L(a) ∨ L(b) = L(mu{a, b}) ∩ L(c) ⊆ L(c), that is b ≤ c. Similarly we can prove L(a) ⊆ L(a) ∨ (L(b)∧ L(c)) ⊆ (L(a) ∨ L(b)) ∧ L(c) = L(b) ∧ L(c) ⊆ L(b), that is a ≤ b. Therefore a ≤ b ≤ c, that is (a, b, c)∈ pob.□

Theorem 4.3 Let P be a generalised lattice. Then (a, b, c) ∈glb⟺ (c, b, a) ∈ glb.

Proof: Suppose (a, b, c) ∈ glb. Then by definition 2.1, we have ML(mu(ML{a, b}∪ ML{b, c})) = {b} = mu(ML(mu{a,b}∪ mu{b,c})). This implies ML(mu(ML{b, a}∪ ML{c, b})) = {b} = mu(ML(mu{b,a}∪ mu{c,b})). Therefore (c, b, a) ∈ glb. Therefore we proved that (a, b, c) ∈ glb ⟹ (c, b, a) ∈ glb. Similarly we can prove that (c, b, a) ∈ glb ⟹ (a, b, c) ∈ glb.□

Theorem 4.4 Let P be a generalised lattice and a, b, c ∈ P. Then (a, b, c)∈ glb implies (s, b, t)∈ pob for all s∈ML{a, c} and t∈ mu{a, c}.

Proof: Suppose (a, b, c)∈ glb. Then by definition 2.1 we have ML(mu(ML{a, b} ∪ ML{b, c})) = {b} = mu(ML(mu{a,b}∪ mu{b,c})). This implies L({a, c}) = L(mu{a,b}) ∩ L(mu{b,c}) ∩ L({a, c}) = L(mu{a,b} ∪ mu{b,c})∩ L({a, c}) = L(b) ∩ L({a, c}). This implies $\bigcup_{s \in ML\{a,c\}}$ L(s) = L({a, c}) ⊆ L(b). This implies L(s) ⊆L(b) for all s ∈ ML{a, c}, that is s ≤ b for all s ∈ ML{a, c}. Similarly we can prove $\bigcup_{t \in mu\{a,c\}}$ U(t) = U({a, c}) = U(b) ∩ U({a, c}) ⊆ U(b). This implies U(t) ⊆ U(b) for all t ∈ mu{a, c}, that is b ≤ t for all t ∈ mu{a, c}.Therefore s ≤ b ≤ t for all s ∈ ML{a, c} and t ∈ mu{a, c}, that is (s, b, t) ∈ pob for all s ∈ ML{a, c} and t ∈ mu{a, c}.□

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REFERENCES

[1]G.Birkhoff, Lattice theory, Amer.Math.Soc.Colloq.Publ. XXV, Providence, R.I, 1967.

[2]M.K.Murty and U.M.Swamy, Distributive partially ordered sets, The Aligarh Bull.of Math, 8 (1978) 1-16.

[3]P.R.Kishore, M.K.Murty, V.S.Ramani and M.D.P.Patnaik, On generalised lattices, Southeast Asian Bulletin of Mathematics, 33 (2009) 1091-1104.

[4]P.R.Kishore, The lattice of convex subgeneralised lattices of a generalised lattice. International Journal of Algebra, 3(17) (2009) 815-821.

[5]P.R.Kishore, Distributive generalised lattices, International Journal of Computational Cognition, 7(3) (2009) 23-26.

[6] P.R.Kishore, One-one correspondence between a class of ideals and a class of congruences in a generalised lattice. Asian European Journal of Mathematics, 3(4) (2010) 623-629.

[7] Leo Lapidus, Lattice metrized spaces, Doctoral dissertation, Department of Mathematics, Michigan State University, 1956.

[8]E.A.Nordhaus, Leo Lapidus, Brouwerian geometry, Canadian Journal of Mathematics, 6 (1954) 217-229.

[9] Everett Pitcher and M.F.Smiley, Transitivities of betweenness, Transactions of American Mathematical Society, 52 (1942) 95-114.

[10] P.R.Kishore and Demelash K.M., Brouwerian generalised lattices, Advances and Applictions in Discrete Mathematics, 29(2) (2022) 205 - 222.

[11] P.R.Kishore, Generalised Lattice Metrized Spaces (gl-metrized spaces), International Journal of Scientific and Innovative Mathematical Research (IJSIMR), 11(3) (2023) 1-4.

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