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$\theta-$Filters in Almost Distributive Lattices

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Abstract: *This paper aims to introduce newly θ-filters for the purpose of deriving some fruitful properties by utilizing congruence in an Almost Distributive Lattice (ADL). A set of equivalent conditions are obtained for changing θ-filter into a θ-Prime filter.*

Keywords: *Almost Distributive Lattice (ADL), congruence, ideal, filter, Prime filter, θ-filter; θ-Prime filter*

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1. INTRODUCTION

After Booles axiomatization of two valued propositional calculus as a Boolean algebra, a number of generalizations both ring theoretically and lattice theoretically have come into being. The concept of an Almost Distributive Lattice (ADL) was introduced by Swamy and Rao [6] as a common abstraction of many existing ring theoretic generalizations of a Boolean algebra on one hand and the class of distributive lattices on the other. In that paper, the concept of an ideal in an ADL was introduced analogous to that in a distributive lattice and it was observed that the set PI(L) of all principal ideals of L forms a distributive lattice. This enables us to extend many existing concepts from the class of distributive lattices to the class of ADLs. Swamy, G.C. Rao and G.N. Rao introduced the concept of Stone ADL and characterized it in terms of its ideals. In [5], Sambasiva Rao introduced θ-ideal in a lattices and proved their properties. In this paper, the conceptof θ-filters is introduced in an ADL and then characterized in terms of ADL congruences. A set of equivalent conditions are derived for every ideal of an ADL to become a θ-filter. The concept of θ-prime filters is also introduced and established a set of equivalent conditions for every θ-filter which becomes a θ-prime filter. Some properties of θ-filters and θ-prime filter are studied. The class of all θ-filters of an ADL can be made into a bounded distributive lattice. Finally, the prime ideal theorem is generalized in the case of θ-prime filter in an ADL.

2. PRELIMINARIES

Definition 2.1.[6] An Almost Distributive Lattice with zero or simply ADL is an algebra $(L, \vee, \wedge, 0)$ of type (2, 2, 0) satisfying

1. $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
2. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

3. $(x \vee y) \wedge y = y$
4. $(x \vee y) \wedge x = x$
5. $x \vee (x \wedge y) = x$
6. $0 \wedge x = 0$
7. $x \vee 0 = x$, for any x, y, z ∈ L.

Every non-empty set X can be regarded as an ADL as follows. Let $x_0 \in X$. Define the binary operations ∨, ∧ on X by

$$x \vee y = \begin{cases} x & \text{if } x \neq x_0 \\ y & \text{if } x = x_0 \end{cases} \qquad x \wedge y = \begin{cases} y & \text{if } x \neq x_0 \\ x_0 & \text{if } x = x_0 \end{cases}$$

Then $(X, \vee, \wedge, x_0)$ is an ADL (where x_0 is the zero) and is called a discrete ADL. If (L, ∨, ∧, 0) is an ADL, for any a, b ∈ L, define $a \leq b$ if and only if $a = a \wedge b$ (or equivalently, $a \vee b = b$), then ≤ is a partial ordering on L.

Theorem 2.2: ([6]) If *(L, ∨, ∧, 0) is an ADL, for any a, b, c ∈ L, we have the following:*

(1) $a \vee b = a \Leftrightarrow a \wedge b = b$
(2) $a \vee b = b \Leftrightarrow a \wedge b = a$
(3) ∧ is associative in L
(4) $a \wedge b \wedge c = b \wedge a \wedge c$
(5) $(a \vee b) \wedge c = (b \vee a) \wedge c$
(6) $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$
(7) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
(8) $a \wedge (a \vee b) = a$, $(a \wedge b) \vee b = b$ *and* $a \vee (b \wedge a) = a$
(9) $a \leq a \vee b$ *and* $a \wedge b \leq b$
(10) $a \wedge a = a$ *and* $a \vee a = a$
(11) $0 \vee a = a$ *and* $a \wedge 0 = 0$
(12) If $a \leq c$, $b \leq c$ *then* $a \wedge b = b \wedge a$ *and* $a \vee b = b \vee a$
(13) $a \vee b = (a \vee b) \vee a$.

It can be observed that an ADL L satisfies almost all the properties of a distributive lattice except the right distributivity of ∨ over ∧, commutativity of ∨, commutativity of ∧. Any one of these properties make an ADL L a distributive lattice.

Theorem 2.3. ([6]) Let *(L, ∨, ∧, 0) be an ADL with 0. Then the following are equivalent:*

(1) (L, ∨, ∧, 0) is a distributive lattice
(2) $a \vee b = b \vee a$, *for all a, b ∈ L*
(3) $a \wedge b = b \wedge a$, *for all a, b ∈ L*
(4) $(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$, *for all a, b, c ∈ L.*

As usual, an element m ∈ L is called maximal if it is a maximal element in the partially ordered set (L, ≤). That is, for any a ∈ L, $m \leq a \Rightarrow m = a$.

Theorem 2.4: ([6]) Let *L be an ADL and m ∈ L. Then the following are equivalent:*

(1) m is maximal with respect to ≤
(2) $m \vee a = m$, *for all a ∈ L*
(3) $m \wedge a = a$, *for all a ∈ L*
(4) $a \vee m$ *is maximal, for all a ∈ L.*

As in distributive lattices [[1], [2]], a non-empty sub set I of an ADL L is called an ideal of L if $a \vee b \in I$ and $a \wedge x \in I$ for any a, b ∈ I and x ∈ L. Also, a non-empty subset F of L is said to be a filter of R if $a \wedge b \in F$ and $x \vee a \in F$ for a, b ∈ F and x ∈ L. The set I(L) of all ideals of L is a bounded distributive lattice with least element {0} and greatest element L under set inclusion in which, for any I, J ∈ I(L), $I \cap J$ is the infimum of I and J while the supremum is given by $I \vee J := \{a \vee b \mid a \in I, b \in J\}$. A proper ideal P of L is called a prime ideal if, for any x, y ∈ L, $x \wedge y \in P \Rightarrow x \in P$ or $y \in P$. A proper ideal M of L is said to be maximal if it is not properly contained in any proper ideal of L. It can be observed that every maximal ideal of L is a prime ideal. Every proper ideal of L is contained in a maximal ideal. For any subset S of L the smallest ideal containing S is given by (S] := { $(\bigvee_{i=1}^{n} s_i) \wedge x$ | $s_i \in$ S, x $\in$ L and n $\in$ N}. If S = {s}, we write (s] instead of (S]. Similarly, for any S $\subseteq$ L, [S) := { $x \vee (\bigwedge_{i=1}^{n} s_i)$ | $s_i \in$ S, x $\in$ L and n $\in$ N}.

If S = {s}, we write [s) instead of [S).

Theorem 2.5 ([6]). *For any* x, y *in* L *the following are equivalent:*

1). (x] $\subseteq$ (y]
2). y $\wedge$ x = x
3). y $\vee$ x = y
4). [y) $\subseteq$ [x).

For any x, y $\in$ L, it can be verified that (x] $\vee$ (y] = (x $\vee$ y] and (x] $\wedge$ (y] = (x $\wedge$ y]. Hence the set PI(L) of all principal ideals of L is a sublattice of the distributive lattice I(L) of ideals of L.

Definition 2.7 ([3]). An equivalence relation θ on an ADL L is called a congruence relation on L if (a $\wedge$ c, b $\wedge$ d), (a $\vee$ c, b $\vee$ d) $\in \theta$, for all (a, b), (c, d) $\in \theta$.

Definition 2.8 ([3]). For any congruence relation θ on an ADL L and a $\in$ L, we define $[a]_\theta$ = {b $\in$ L | (a, b) $\in$ θ} and it is called the congruence class containing a.

Theorem 2.9 ([3]). *An equivalence relation θ on an ADL* L *is a congruence relation if and only if for any* (a, b) $\in \theta$, x $\in$ L, (a $\vee$ x, b $\vee$ x), (x $\vee$ a, x $\vee$ b), (a $\wedge$ x, b $\wedge$ x), (x $\wedge$ a, x $\wedge$ b) *are all in* θ.

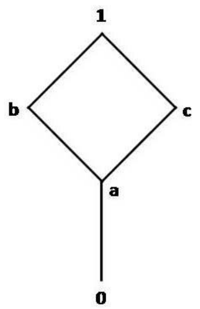
3. θ-FILTERS IN ADLS

In this section we define a θ-filter in an ADL, analogously. Though many results look similar, the proofs are not similar because of the lack of the properties like commutativity of $\vee$, commutativity of $\wedge$ and the right distributivity of $\vee$ over $\wedge$ in an ADL. Through out this paper L represents an ADL with 0.

Now we have the following definition of a θ-filter.

Definition 3.1: Let θ be a congruence relation on an ADL L. A filter F of L is called a θ-filter of L, if for any $a \in F \Rightarrow [a]_\theta \subseteq F$.

Example 3.2: Let $D = \{0', a'\}$ be a discrete ADL and $A = \{0, a, b, c, 1\}$ is a distributive lattice as shown in the following figure:



Then $L = D \times A$ is an ADL. Define $[(0',1)]_\theta = \{(0',1)\}$, $[(0',b)]_\theta = \{(0',b)\}$, $[(0',c)]_\theta = \{(0',c)\}$, $[(0',a)]_\theta = [(0',0)]_\theta = \{(0',a),(0',0)\}$, $[(a',1)]_\theta = \{(a',1')\}$, $[(a',b)]_\theta = \{(a',b)\}$, $[(a',c)]_\theta = \{(a',c)\}$, $[(a',a)]_\theta = [(a',0)]_\theta = \{(a',a),(a',0)\}$. Clearly, θ is a congruence relation on L. Consider a filter $F = \{(a',c),(a',1)\}$. Clearly θ-filter of L. Now consider a filter $F_1 = \{(a',a),(a',b),(a',c),(a',1)\}$. Then $(a',0) \notin F_1$ and $(a',0) \in [(a',a)]_\theta$. Therefore $[(a',a)]_\theta \not\subset F_1$. Hence F_1 is not a θ-filter of L.

Now we prove the following lemma.

Lemma 3.3: Let L be an ADL with a maximal element m and θ a congruence relation on L. For any filter F of L, the following hold:

1. $\{m\}$ is a θ-filter if and only if $[m]_\theta \subseteq \{m\}$

2. If F is a θ-filter, then $[m]_\theta \subseteq F$

3. If F is a proper θ-filter, then $F \cap [0]_\theta = \phi$.

Proof: 1. It is obvious.

2. Suppose F is a θ-filter of L. We have always $m \in F.$ Then $[m]_\theta \subseteq F.$

3. Let F be a proper θ-filter of L. Suppose $F \cap [0]_\theta \neq \phi.$ Choose $x \in F \cap [0]_\theta.$ Then $x \in F$ and $(x,0) \in \theta$ and hence $0 \in [x]_\theta \subseteq F.$ This implies $0 \in F,$ which is a contradiction. Therefore $F \cap [0]_\theta = \phi.$

Theorem3.4: Let θ be a congruence relation on an ADL L. Then for any filter F of L, the following conditions are equivalent:

1. F is a θ-filter

2. For any $x, y \in L,$ $(x, y) \in \theta$ and $x \in F \Rightarrow y \in F$

3. $F = \bigcup_{x \in F} [x]_\theta.$

Proof: $(1) \Rightarrow (2):$ Assume that F is a θ-filter of an ADL. Let $(x, y) \in \theta$ and $x \in F$. Then $y \in [x]_\theta$ and $[x]_\theta \subseteq F.$ This implies that $y \in F.$

$(2) \Rightarrow (3):$ Assume that (2). Clearly, we have $F \subseteq \bigcup_{x \in F} [x]_\theta.$ Let $a \in \bigcup_{x \in F} [x]_\theta.$ Then $a \in [y]_\theta,$ for some $y \in F.$ This implies $(a, y) \in \theta.$ By our assumption we get $a \in F.$ Hence $F = \bigcup_{x \in F} [x]_\theta.$

$(3) \Rightarrow (1):$ Assume that (3). Let $a \in F.$ Then $a \in [y]_\theta,$ for some $y \in F.$ We have to prove that $[a]_\theta \subseteq F.$ Let $t \in [a]_\theta.$ Then $(a,t) \in \theta$ and hence $(t, y) \in \theta.$ That implies $t \in [y]_\theta \subseteq F.$ Therefore $[a]_\theta \subseteq F.$ Hence F is θ-filter of an ADL L.

The following result is verified easily.

Theorem 3.5: If θ is the smallest congruence relation on an ADL L, then every filter of L is a θ-filter.

Now, the concept of θ-Prime filters is introduced in an ADL.

Definition 3.6: Let θ be a congruence relation on an ADL L with any maximal element m. A proper θ-filter P of an ADL L is called a θ-prime filter of L if for any $a,b \in L$ with $a \vee b \in [m]_\theta \Rightarrow$ either $a \in P$ or $b \in P.$

We prove the following lemma.

Lemma3.7: If θ is the smallest congruence relation on an ADL L with maximal elements, then every prime filter of an ADL L is a θ-prime filter of an ADL L.

Proof: Let L be an ADL with maximal elements. Let θ be the smallest congruence relation on L.

Suppose that P is a prime filter of an ADL L. Then by the above result, we get P is a θ-filter of L. Let $a,b \in L$ with $a \vee b \in [m]_\theta,$ where m is any maximal element of L. Then $[a \vee b]_\theta = [m]_\theta.$ Since θ is the smallest congruence relation on an ADL L, we get $a \vee b = m.$ since P is a prime filter of L, we have $a \vee b = m \in P$. This implies that either $a \in P$ or $b \in P$. Therefore P is θ-prime filter of an ADL L.

Lemma 3.8: Let θ be a congruence relation on an ADL L with maximal elements. Then every prime θ-filter of L is a θ-prime filter of L.

Proof: Let P be a prime θ-filter of an ADL L. Let $x, y \in L$ with $x \vee y \in [m]_\theta$, where m is any maximal element of L. Since $m \in P$ and P is θ-filter of L, we get $[m]_\theta \subseteq P$ and hence $x \vee y \in P$. This implies either $x \in P$ or $y \in P$. Thus P is a θ-prime filter of L.

Theorem 3.9: Let θ be a congruence relation on an ADL L with maximal element 'm' and P, a θ-filter of L. If $[a]_\theta = [m]_\theta \Rightarrow (a] \subseteq [m]_\theta$, for all $a \in L$. Then the following conditions are equivalent:

1. P is a θ-prime filter of L

2. For any filters I, J of L with $I \cap J \subseteq [m]_\theta \Rightarrow I \subseteq P$ or $J \subseteq P$

3. For any $a, b \in L$, $[a]_\theta \vee [b]_\theta = [m]_\theta \Rightarrow$ either $a \in P$ or $b \in P$.

Proof: $(1) \Rightarrow (2)$: Assume that P is a θ-prime filter of L. Let I and J be any filters of L with $I \cap J \subseteq [m]_\theta$. Let $a \in I$ and $b \in J$. Then $a \vee b \in I \cap J \subseteq [m]_\theta$. This implies that $a \vee b \in [m]_\theta$.

By our assumption, we have either $a \in P$ or $b \in P$. Therefore either $I \subseteq P$ or $J \subseteq P$.

$(2) \Rightarrow (3)$: Assume that for any filters I, J of L with $I \cap J \subseteq [m]_\theta \Rightarrow I \subseteq P$ or $J \subseteq P$. Let $a, b \in L$, with $[a]_\theta \vee [b]_\theta = [m]_\theta$. Then $[a \vee b]_\theta = [a]_\theta \vee [b]_\theta = [m]_\theta$. This implies that $[a \vee b) \subseteq [m]_\theta$ and hence $[a) \cap [b) \subseteq [m]_\theta$. By our assumption we get either $[a) \subseteq P$ or $[b) \subseteq P$. Therefore either $a \in P$ or $b \in P$.

$(3) \Rightarrow (1)$: Assume that condition (3). Let $a \vee b \in [m]_\theta$. Then $[a]_\theta \vee [b]_\theta = [a \vee b]_\theta = [m]_\theta$. By our assumption, we have $[a]_\theta \subseteq P$ or $[b]_\theta \subseteq P$. This implies that $a \in P$ or $b \in P$. Hence P is a θ-prime filter of an ADL L.

Lemma 3.10: Let θ be a congruence relation on an ADL L. Then every minimal prime filter disjoint from $[0]_\theta$ is a θ-filter of an ADL L.

Proof: Let M be a minimal prime filter of L such that $M \cap [0]_\theta \neq \phi$. Let $x, y \in L$ with $(x, y) \in \theta$ and $x \in M$. We prove that $y \in M$. Suppose $y \notin M$. Then $M \vee [y) = L$. This implies $a \wedge y = 0$, for some $a \in M$. Since $(x, y) \in \theta$, we get that $(a \wedge x, 0) \in \theta$ and hence $a \wedge x \in [0]_\theta$. So that $a \wedge x \in M$. Therefore $M \cap [0]_\theta \neq \phi$, which is a contradiction. Hence $y \in M$. Thus M is a θ-filter of an ADL L.

Corollary 3.11: Let θ be a congruence relation on an ADL L. If $[0]_\theta = \{0\}$, then every minimal prime filter of L is a θ-filter of L.

Definition 3.12: Let θ be a congruence relation on an ADL L. For any filter F of L, define the set F^θ as given by $F^\theta = \{x \in L : (x, a) \in \theta,$ for some $a \in F\}$.

Lemma 3.13: Let θ be a congruence relation on an ADL L. For any filter F of L, the set F^θ is a filter of L.

Proof: Clearly $F^\theta \neq \phi$, since $F \neq \phi$. Let $x, y \in F^\theta$. Then $(x, a) \in \theta$ and $(y, b) \in \theta$, for some $a, b \in F$. This implies that $(x \wedge y, a \wedge b) \in \theta$ and $a \wedge b \in F$. Therefore $x \wedge y \in F^\theta$. Let $x \in F^\theta$ and $r \in L$. Then $(x, a) \in \theta$, for some $a \in F$. Then $(r \vee x, r \vee a) \in \theta$ and $r \vee a \in F$. Hence $r \vee x \in F^\theta$. Thus F^θ is a filter of L.

Lemma 3.14: Let θ be a congruence relation on an ADL L. For any two filters I, J of L, we have the following:

1. $I \subseteq I^\theta$
2. If $I \subseteq J$ then $I^\theta \subseteq J^\theta$
3. $(I \cap J)^\theta = I^\theta \cap J^\theta$
4. $(I^\theta)^\theta = I^\theta$.

Proof: 1. It is obvious.

2. Suppose that $I \subseteq J$. Let $x \in I^\theta$. Then $(x,a) \in \theta$, for some $a \in I$. Since $I \subseteq J$, we get $(x,a) \in \theta$ and $a \in J$. Therefore $x \in J^\theta$. Hence $I^\theta \subseteq J^\theta$.

3. Let $x \in (I \cap J)^\theta$. Then $(x,a) \in \theta$, for some $a \in I \cap J$. That implies $(x,a) \in \theta$ and $a \in I,\ a \in J$. Therefore $x \in I^\theta \cap J^\theta$. Hence $(I \cap J)^\theta \subseteq I^\theta \cap J^\theta$. Let $x \in I^\theta \cap J^\theta$. This implies $(x,a),(x,b) \in \theta$, for some $a \in I$ and $b \in J$. So that $(x, a \vee b) \in \theta$ and $a \vee b \in I \cap J$. Implies that $x \in (I \cap J)^\theta$. Therefore $I^\theta \cap J^\theta \subseteq (I \cap J)^\theta$. $(I \cap J)^\theta \subseteq I^\theta \cap J^\theta$. Hence $(I \cap J)^\theta = I^\theta \cap J^\theta$.

4. Let $x \in (I^\theta)^\theta$. Then $(x,a) \in \theta$, for some $a \in I^\theta$. Since $a \in I^\theta$, we have $(a,b) \in \theta$, for some $b \in I$. This implies $(x,b) \in \theta$ and $b \in I$ and hence $x \in I^\theta$. Therefore $(I^\theta)^\theta \subseteq I^\theta$. Let $x \in I^\theta$. Then $(x,a) \in \theta$, for some $a \in I$. Since $a \in I$, we have $a \in I^\theta$. That implies $(x,a) \in \theta$, for some $a \in I^\theta$. Therefore $x \in (I^\theta)^\theta$ and hence $I^\theta \subseteq (I^\theta)^\theta$. Thus $(I^\theta)^\theta = I^\theta$.

Proposition 3.15: Let θ be congruence relation on an ADL L. For any filter F of L , F^θ is the smallest θ -filter of L such that $F \subseteq F^\theta$. $x \in (I \cap J)^\theta$.

Proof: Clearly, F^θ is a filter of L and $F \subseteq F^\theta$. Let $x \in F^\theta$. Then $(x,a) \in \theta$, for some $a \in F$. We have prove that $[x]_\theta \subseteq F^\theta$. Let $t \in [x]_\theta$. Then $(t,x) \in \theta$. Since $(x,a) \in \theta$ and $a \in F$, we get $(t,a) \in \theta$ and hence $t \in F^\theta$. Therefore F^θ is a θ a-filter of L containing F. Let K be any θ -filter of L containing F. Now we prove that $F^\theta \subseteq K$. Let $x \in F^\theta$. Then $(x,a) \in \theta$, for some $a \in F$. Since $F \subseteq K$, we have $a \in K$. Since K is a θ -filter of L, we get $x \in K$. Therefore F^θ is the smallest θ -filter of L such that $F \subseteq F^\theta$.

Theorem 3.16: Let θ be a congruence relation on an ADL L with maximal elements. For any proper θ -filter F of L we have $F = \bigcap\{P \mid P$ is a θ -prime filter and $F \subseteq P\}$.

Proof: Take $F_0 = \bigcap\{P \mid P$ is a θ -prime filter and $F \subseteq P\}$. Clearly $F \in F_0$. Let $a \notin F$. Consider $\mathfrak{F} = \{J \mid J$ is a θ -filter, $F \subseteq J$ and $a \notin F\}$. Clearly $F \in \mathfrak{F}$. Let $\{J_\alpha\}_{\alpha\in\Delta}$ be a chain of θ -filters in $\mathfrak{F}$. Clearly, $\bigcup_{\alpha\in\Delta} J_\alpha$ is a θ -filter of L such that $F \subseteq \bigcup_{\alpha\in\Delta} J_\alpha$ and $a \notin \bigcup_{\alpha\in\Delta} J_\alpha$. Hence by the Zorn's lemma, $\mathfrak{F}$ has a maximal element M, say. That is M is a θ -filter, $F \subseteq M$ and $a \notin M$. Let $x, y \in L$ with $x \vee y \in [m]_\theta$, where m is any maximal element of an ADL L. Suppose $x \notin M$ and $y \notin M$. Then $M \subset M \vee [x) \subseteq (M \vee [x))^\theta$ and $M \subset M \vee [y) \subseteq (M \vee [y))^\theta$. By the maximality of M, we get that $a \in (M \vee [x))^\theta \cap (M \vee [y))^\theta = (M \vee [x \vee y))^\theta$. Since $x \vee y \in [m]_\theta$, we get that $a \notin M$, which is a contradiction. Hence M is θ -prime filter of L. Therefore for any $a \notin F$, there exists a θ -prime filter M of an ADL L such that $F \subseteq M$ and $a \notin M$. Thus $a \notin F_0$. Hence $F_0 \subseteq F$. Therefore $F_0 = F$.

Corollary 3.17: Let L be an ADL with maximal element m. Then $[m]_\theta = \bigcap\{P \mid P$ is a θ-prime filter of L $\}$.

Corollary 3.18: Let L be an ADL with maximal element m and θ be a congruence relation on L. If $a \notin [m]_\theta$ then there exists a θ-prime filter P of L such that $a \notin P$.

Theorem 3.19: Let L be an ADL with maximal element m and θ be a congruence on L. Suppose F is a θ-filter and I is an ideal of L such that $F \cap I = \phi$. Then there exists a θ-prime filter P of L such that $F \subseteq P$ and $I \cap P = \phi$.

Proof: Let F be a θ-filter and I, an ideal of L with $F \cap I = \phi$. Consider $\mathfrak{F} = \{J \mid J$ is a θ-filter, $F \subseteq J$ and $J \cap I = \phi\}$. Clearly $F \in \mathfrak{F}$. Let $\{J_\alpha\}_{\alpha\in\Delta}$ be a chain of θ-filters in $\mathfrak{F}$. Clearly, $\bigcup_{\alpha\in\Delta} J_\alpha$ is a θ-filter of L such that $F \subseteq \bigcup_{\alpha\in\Delta} J_\alpha$ and $(\bigcup_{\alpha\in\Delta} J_\alpha) \cap I = \phi$. Hence by the Zorn's lemma $\mathfrak{F}$ has a maximal element M, say. Let $x, y \in L$ with $x \vee y \in [m]_\theta$. We prove that $x \in M$ or $y \in M$. Suppose that $x \notin M$ and $y \notin M$. Then $M \subset M \vee [x) \subseteq (M \vee [x))^\theta$ and $M \subset M \vee [y) \subseteq (M \vee [y))^\theta$. By the maximality of M, we get that $(M \vee [x))^\theta \cap F \neq \phi$ and $(M \vee [y))^\theta \cap F \neq \phi$. Choose $a \in (M \vee [x))^\theta \cap F$ and $b \in (M \vee [x))^\theta \cap F$. Then $a \vee b \in (M \vee [x))^\theta \cap (M \vee [y))^\theta = (M \vee [x \vee y))^\theta$ and $a \vee b \in F$. Since $x \vee y \in [m]_\theta$, we get that $x \vee y \in M$. Since $x \vee y \in F$, we have $x \vee y \in M \cap F$, which is a contradiction. Therefore M is a θ-prime filter of an ADL L.

4. Conclusions

Some remarkable results have been established on θ-filters by using congruence in an Almost Distributive Lattice(ADL).The change of θ-filter into a θ-Prime filter is achieved with the help of a set of equivalent conditions.

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