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Prime Spectrum of a Ternary Semiring

N. Koteswaramma
Assistant Professor of Mathematics,
VVIT,Nambur, Guntur, A.P,
keerthimedha@gmail.com

Dr. J. Venkateswara Rao
Professor of Mathematics, Mekelle
University, Mekelle, Ethiopia,
venkatjonnalagadda@yahoo.co.in

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Abstract: *This paper is a study about prime ideals and spectrum of a ternary semiring. In this manuscript the results are obtained by characterizing prime ideals and spectrum of a ternary semiring. Also here regular ternary semiring is described in terms of prime ideals, semiprime ideals , irreducible ideals and an m-system.*

Keywords: *Ternary semiring, left ideal, right ideal, lateral ideal, prime ideal, semiprime ideal, irreducible ideal, m-system, regular ternary semiring.*

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1. Introduction

The theory of ternary algebraic system was introduced by D.H.Lehmer [1] in 1932. Ternary rings and their structures were investigated by Lister [9] in 1971 by characterizing some additive subgroups of rings which are closed under the triple ring product. He also studies about imbedding of ternary rings in rings.

In 2002 S.Kar and T.K.Dutta introduced notion of ternary semiring which was the generalization of ternary ring .The same authors in 2003[5] introduced regular ternary semiring and studied some of its properties and in 2005[6,7] they studied prime ideals ,semiprime ideals , irreducible ideals and prime radical of a ternary semiring. In [3, 4] we study about Quasi-ideals, Bi-ideals and its properties in regular ternary semiring.

The notion of semiring was introduced in 1934 [8] by Van diver. In [2] we see the studies of various types of ideals in semirings and its characterizations. In the semiring theory ideals, prime ideals, semiprime ideals and irreducible ideals play an important role. So in this paper we proved some results on these ideals in regular ternary semirings.

2. Preliminaries

Definition 2.1[5]. A nonempty set S together with a binary addition and a ternary multiplication, denoted by juxtaposition, is said to be a ternary semiring if S is an additive commutative semigroup satisfying the following conditions:

(i) $(abc)\,de = a(bcd)e = ab(cde)$

(ii) $(a+b)cd = acd + bcd$

(iii) $a(b+c)d = abd + acd$

(iv) $ab(c+d) = abc + abd$, for all $a, b, c, d, e \in S$

Example2.1. [5] Let X be a topological space and R^{-} the set of all negative real numbers. Suppose that $S = \{f: X \to R^{-} / f \text{ is a continuous map}\}$. We define addition and multiplication on S by (f + g)(x) = f(x) + g(x) and (fgh)(x) = f(x) g(x) h(x) for all $x \in X$ and f, g, h $\in$ S. Then we can easily check that S forms a ternary semiring.

Definition2.2. [5]A ternary semiring S is called a commutative ternary semiring

if abc=bac=bca for all a,b,c Є S.

Definition2.3. [5] An additive sub semigroup T of S is called a ternary sub semiring of S if $t_1t_2t_3 \in T$, for all t_1, t2, t3 $\in$ T.

If A, B, C are three subsets of S, then ABC = $\{\Sigma a_ib_ic_i / a_i \in A, b_i \in B, c_i \in C\}$.

Definition2.4. [5] An additive sub semigroup I of S is called a left (resp., right, lateral) ideal of S if s_1s_2i (resp. is_1s_2, s_1is_2) $\in$ I, for all s_1, $s_2 \in$ S and i $\in$ I. If I is both left and right ideal of S, then I is called a two-sided ideal of S. If I is a left, a right, a lateral ideal of S, then I is called an ideal of S.

An ideal I of S is called a proper ideal if $I \neq S$.

Definition 2.5. [6] A proper ideal P of a ternary semiring S is called a prime ideal of S if $ABC \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$, for any three ideals A,B,C of S**.**

Definition 2.6. [7] A proper ideal Q of a ternary semiring S is called a semiprime ideal of S if $I^3 \subseteq Q$ implies $I \subseteq Q$ for any ideal I of S.

Definition 2.7[7].A proper ideal I of a ternary semiring S is called a strongly irreducible ideal of S if for ideals H and K of S, $H \cap K \subseteq I$ implies $H \subseteq I$ or $K \subseteq I$.

Definition 2.8[7].A proper ideal I of a ternary semiring S is called an irreducible ideal of S if for ideals H and K of S, $H \cap K = I$ implies H=I or K=I.

Remark 2.1. A strongly irreducible ideal is surely irreducible.

Definition2.9.[6]A non empty subset A of a ternary semiring S is called an m-system if for each a,b,c $\in$ A there exists elements x_1, x_2, x_3, x_4 of S such that $ax_1bx_2c \in A$ or $ax_1x_2bx_3\,x_4c \in A$ or $ax_1x_2bx_3cx_4 \in A$ or $x_1ax_2bx_3x_4c \in A$.

Definition2.10.[7]A non empty subset B of a ternary semiring S is called a

P-system if for b $\in$ B there exists elements x_1, x_2, x_3, x_4 of S such that $bx_1bx_2b \in B$ or $bx_1x_2bx_3\,x_4b \in B$ or $bx_1x_2bx_3bx_4 \in B$ or $x_1bx_2bx_3x_4b \in B$.

Remark 2.2. In a ternary semiring S every m-system is a p-system.

Definition2.11. [5] An element a in a ternary semiring S is called regular if there exists an element x in S such that axa = a. A ternary semiring is called regular if all of its elements are regular.

Definition 2.12. The spectrum of a ternary semiring S is defined as the set of all prime ideals of S and is denoted by spec(S), read as spectrum of S.

Definition 2.13. Let I be an ideal of a ternary semiring S then

V(I) = $\{H \in Spec(s) / I \subset H\}$.

Definition 2.15. Let I be an ideal of a ternary semiring S then $\sqrt{I} = \cap V(I)$.

Remark 2.3. $\sqrt{I}$ is a semiprime ideal .

3. RESULTS

Proposition 3.1: If I and H are proper ideals of a ternary semiring S then

(i) $I \subseteq H \Longrightarrow \sqrt{I} \subseteq \sqrt{H}$

(ii) $\sqrt{\sqrt{I}} = \sqrt{I}$

(iii) $\sqrt{I+H} = \sqrt{\sqrt{I}+\sqrt{H}}$

Proof: (i) Suppose $I \subseteq H$. We know that $\sqrt{I} = \cap V(I)$

$\sqrt{I} = \cap V(I)$

$= \cap \{K \in Spec(S)/I \subseteq K\}$

Since $I \subseteq H \Longrightarrow \{K/I \subseteq K\} \supseteq \{K/H \subseteq K\}$

$\Longrightarrow \cap \{K/I \subseteq K\} \subseteq \cap \{K/H \subseteq K\}$

So, $\cap \{K \in Spec(S)/I \subseteq K\} \subseteq \cap \{K \in Spec(S)/H \subseteq K\}$

$= \cap V(H) = \sqrt{H}$

Thus $\sqrt{I} \subseteq \sqrt{H}$.

(ii) Consider $\sqrt{\sqrt{I}} = \cap V(\sqrt{I})$

$= \cap \{K \in Spec(S)/\sqrt{I} \subseteq K\}$

$= \cap \{K \in Spec(S)/\cap \{H \in Spec(S)/I \subseteq H\} \subseteq K\}$

$= \cap \{K \in Spec(S)/I \subseteq K\} = \sqrt{I}$

Thus $\sqrt{\sqrt{I}} = \sqrt{I}$.

(iii) Clearly we have $I \subseteq \sqrt{I}$ and $H \subseteq \sqrt{H}$ implies $I + H \subseteq \sqrt{I} + \sqrt{H}$.

By (i), $\sqrt{I+H} \subseteq \sqrt{\sqrt{I}+\sqrt{H}}$

Now it is enough if we show $\sqrt{\sqrt{I}+\sqrt{H}} \subseteq \sqrt{I+H}$

For this first we show that $\sqrt{I}+\sqrt{H} \subseteq \sqrt{I+H}$

We know that $I \subseteq I + H \subseteq \sqrt{I+H} \Longrightarrow I \subseteq \sqrt{I+H}$

$\Longrightarrow \sqrt{I} \subseteq \sqrt{\sqrt{I+H}} = \sqrt{I+H}$ (by (ii))

Therefore $\sqrt{I} \subseteq \sqrt{I+H}$. Similarly $\sqrt{H} \subseteq \sqrt{I+H}$.

Thus $\sqrt{I}+\sqrt{H} \subseteq \sqrt{I+H}$

So that $\sqrt{\sqrt{I}+\sqrt{H}} \subseteq \sqrt{I+H}$

Hence $\sqrt{I+H} = \sqrt{\sqrt{I}+\sqrt{H}}$.

Proposition 3.2. Let S be a ternary semiring and I be proper ideal of S. If $x \in \sqrt{I}$ then $(xs)^{n-1}x \in I$,for all $s \in S$ and for some positive integer n.

Proof: Let $x \in \sqrt{I}$. If possible, suppose for some $s \in S$, $(xs)^{n-1}x \notin I$ for all positive integers n. Then the m-system $H = \{x = (xs)^0 x, (xs)^1 x, (xs)^2 x, \ldots \ldots (xs)^n x, \ldots\}$ is disjoint from I. So, there exists a prime ideal $P \supseteq I$ such that $P \cap H = \emptyset$. Then $x \notin P$. So $x \notin \sqrt{I}$, a contradiction. So $(xs)^{n-1}x \in I$ for all $x \in S$ and for some positive integer n.

Proposition 3.3. Let S be a ternary semiring. If A is an m-system of elements of S and if I is an ideal of S maximal among all those ideals of S disjoint from A then I is prime.

Proof: Let H, K and L be three ideals of S such that HKL ⊆ I. Suppose that $H \nsubseteq I$, $K \nsubseteq I$ and $L \nsubseteq I$. Then $I \subseteq I + H, I \subseteq I + K,\ I \subseteq I + L$ and I is maximal disjoint from A. It follows that$(I + H) \cap A \neq \emptyset$, $(I + K) \cap A \neq \emptyset$ and $(I + L) \cap A \neq \emptyset$

$\Longrightarrow \exists\ a \in (I + H) \cap A,\ \ b \in (I + K) \cap A$ and $c \in (I + L) \cap A$

$\Longrightarrow a \in I + H$ and $a \in A, b \in I + K$ and $b \in A, c \in (I + L)$

$\Longrightarrow a \in I + H, b \in I + K, c \in (I + L)$ and $a, b, c \in A$

$\Longrightarrow a = i_1 + h, b = i_2 + k, c = i_3 + l$, for some $i_1, i_2, i_3 \in I,\ h\epsilon H, k\epsilon K, l\epsilon L\ and\ a, b, c\epsilon A$. Since A is m-system we have for $a, b, c\epsilon A\ there\ exists\ \ s\epsilon S\ such that\ asbsc\epsilon A$. Consider asbsc = $(i_1 + h)$s $(i_2 + k)$s $(i_3 + l)$ = $i_1si_2si_3 + i_1si_2sl + i_1sksi_3 + i_1sksl + \ hsi_2si_3 + hsi_2sl + hsksi_3 + hsksl \in I + HKL \subseteq I$.

Therefore$asbsc \in I \cap A$, a contradiction. Therefore either $H \subseteq I$ or $K \subseteq I$ or $L \subseteq I$.

Proposition 3.4. Every prime ideal P of a ternary semiring S contains a minimal prime ideal.

Proof: Let S be a ternary semiring and P be a prime ideal of S.

Construct $\mathcal{A} = \{H{:}\, H \subseteq P, H \text{ is prime}\}$. Clearly $\mathcal{A} \neq \emptyset$. Since $P \in \mathcal{A}$. Let $\{H_i{:}\, i \in\Delta\}$ be a descending chain of prime ideals of S. That is $i \geq j$ in Δ if and only if $H_i \subseteq H_j$. Take $H = \bigcap_{i\in\Delta} H_i$, Clearly H is an ideal of a ternary semi ring S. Now to prove that H is prime we use the theorem3.2[6]. Let a, b, c ∈ S ∋ {asbsc : s ∈ S} ⊆ H. We prove that a∈H or b∈H or c∈H. Suppose a ∉ H, b ∉ H that is there exists $k \in\Delta$ such that $a, b \notin H_k \Rightarrow c \in H_k$, since $\{asbsc/s \in S\} \subseteq H \subseteq H_i$ for every $i \in\Delta$ and H_i is prime. $\Longrightarrow c \in H_i\ for\ all\ i \leq k$ (since we take a chain). If $i > k$ then $H_i \subseteq H_k$ and $a, b \notin H_k \Longrightarrow a, b \notin H_i$. Also by theorem (3.2,[6]) $c \in H_i$ and H_i is prime, forevery $i \in\Delta \Longrightarrow c \in \bigcap_{i\in\Delta} H_i = H$. Therefore H is a prime ideal and H is a lower bound of the descending chain of prime ideals, then by Zorn's lemma $\mathcal{A}$ has a minimal element. Therefore P contains a minimal prime ideal.

The following gives the characterization of prime spectrum of an ideal of a ternary semiring.

Proposition 3.5. If I is a proper ideal of a ternary semiring S then $\sqrt{I}$ = {s∈S : every m-system in S which contains s has a non-empty intersection with I}.

Proof: Let P^c = {s∈S : every m-system in S which contain s has a nonempty intersection with I}. Now let s ∉ p^c. Then there exists an m-system H in S such that s∈H and $H \cap I = \emptyset$. Then by theorem 3.10[6] there exists a prime ideal P of S with P ⊇ I and $P \cap H = \emptyset$.So s $\notin P$ and hence $s \notin \sqrt{I}$.Consequently$\sqrt{I} \subseteq P^c$ …(i)

Now let $s \notin \sqrt{I}$. Then there exists a prime ideal p of S such that s ∉ p that is

$s \in p^c$.Again $p^c\ \ is\ an\ \ m - system\ of\ S(by\ theorem\ 3.12[6])Since\ p \supseteq I$,

$p^c\ \cap I = \emptyset$. Thus m-system p^c in S contains s but has an empty intersection with I. So $s \notin p^c$ and hence $P^c \subseteq \sqrt{I}$……(ii).From (i) and (ii), $\sqrt{I} = P^c$. This completes the proof.

Proposition 3.6. If I is an ideal of a commutative ternary semiring S then $\sqrt{I}$ = {a∈S/a^n∈I for some positive integer n}.

Proof: Let k = {a∈S/a^n∈I for some positive integer n}. Now it is enough if we prove$\sqrt{I} = K$. Clearly K is an ideal of ternary semiring S. By a result "An ideal I of a ternary semiring S is semiprime if and only if R/I is p-system" so, to prove K is semiprime, we prove that R/K is p-system. Let c ∈ R/K. Now we prove that c^3 ∈ R/K. Contrarily suppose that c^3 ∈ K implies there exists a positive integer n such that $(c^3)^n$ ∈ I ⟹ c ∈ K, a contradiction so c^3 ∉ K⟹ c^3 ∈ R/K, therefore K is a semiprime ideal. Next we prove that V(I) = V(K). Let H be a prime ideal containing I. Let a ∈ K⟹ there exists n ∈ N such that a^n ∈ I ⊆ $H \Longrightarrow a^n \in H$ and H is prime

$\Rightarrow a \in H$ for every a $\in$ K. Therefore K $\subseteq$ H for every prime ideal H containing I $\Rightarrow$ K $\subseteq$ {H $\in$ Spec(S) : I $\subseteq$ H} $\Rightarrow$ K $\subseteq \cap V(I) = \sqrt{I}$.

Therefore K $\subseteq \sqrt{I}$. Clearly by definition of k, we have I $\subseteq K \Rightarrow \sqrt{I} \subseteq \sqrt{K} = K$ (by Proposition 3.1). Therefore $\sqrt{I} \subseteq K$. Hence $\sqrt{I} = K$.

Corollary 3.1. If I is a primary ideal of a commutative ternary semiring S then $\sqrt{I}$ is a prime ideal of S.

Proof: Clearly $\sqrt{I}$ is an ideal of a ternary semiring S. Now we prove that $\sqrt{I}$ is prime ideal. Let a, b, c $\in$ S such that abc $\in \sqrt{I}$. We prove that a $\in \sqrt{I}$ or b $\in \sqrt{I}$ or c $\in \sqrt{I}$. Suppose a $\notin \sqrt{I}$, c $\notin \sqrt{I}$. We have abc $\in \sqrt{I}$ = {a$\in$S/a^n $\in$ I for some n$\in$N}

(by proposition 3.6) $\Rightarrow$ there exists a positive integer n such that $(abc)^n$ $\in$ I. Since R is commutative we have $a^n b^n c^n$ $\in$ I , also I is primary $\Rightarrow$ there exists k $\in$ N such that $(b^n)^k \in I \Rightarrow b^{nk} \in I \Rightarrow$ there exists nk $\in$ N such that b^{nk} $\in$ I $\Rightarrow$ b $\in \sqrt{I}$. This completes the proof.

Proposition 3.7. If I, H and K are ideals of a ternary semiring S then $\sqrt{IHK} = \sqrt{I \cap H \cap K} = \sqrt{I} \cap \sqrt{H} \cap \sqrt{K}$.

Proof: Clearly IHK $\subseteq I \cap H \cap K \subseteq I, H$, K

$\Rightarrow \sqrt{IHK} \subseteq \sqrt{I \cap H \cap K} \subseteq \sqrt{I}, \sqrt{H}, \sqrt{K}$ (Proposition 3.1)

$\Rightarrow \sqrt{IHK} \subseteq \sqrt{I \cap H \cap K} \subseteq \sqrt{I} \cap \sqrt{H} \cap \sqrt{K}$

Now we prove that $\sqrt{I} \cap \sqrt{H} \cap \sqrt{K} \subseteq \sqrt{IHK}$

Let $a \in \sqrt{I} \cap \sqrt{H} \cap \sqrt{K} \Rightarrow a \in \sqrt{I}, a \in \sqrt{H}$ and $a \in \sqrt{K}$

$\Rightarrow \quad a \in \sqrt{I} = \{a \in S/a^l \in I$ for some $l \in N\}$

$a \in \sqrt{H} = \{a \in S/a^m \in H$ for some $m \in N\}$

$a \in \sqrt{K} = \{a \in S/a^n \in K$ for some $n \in N\}$

$\Rightarrow$ there exists l, m, n such that a^l $\in$ I, a^m $\in$ H, a^n $\in$ K

$\Rightarrow$ there exists l + m + n such that a^{l+m+n} $\in$ IHK $\Rightarrow a \in \sqrt{IHK}$

Therefore $\sqrt{I} \cap \sqrt{H} \cap \sqrt{K} \subseteq \sqrt{IHK}$.

Hence$\sqrt{IHK} = \sqrt{I} \cap \sqrt{H} \cap \sqrt{K}$.

Finally we prove that $\sqrt{I} \cap \sqrt{H} \cap \sqrt{K} \subseteq \sqrt{I \cap H \cap K}$

Let $a \in \sqrt{I} \cap \sqrt{H} \cap \sqrt{K} \Rightarrow a \in \sqrt{I}, a \in \sqrt{H}, a \in \sqrt{K}$

$\Rightarrow$ There exists l, m, n such that $a^l \in I, a^m \in H$ and $a^n \in K$.

Take k = L.C.M (l, m, n) $\in$ N.

Now $a^k \in I$ and $a^k \in H, a^k \in K \Rightarrow a^k \in I \cap H \cap K$

$\Rightarrow a \in \sqrt{I \cap H \cap K}$. Therefore $\sqrt{I} \cap \sqrt{H} \cap \sqrt{K} \subseteq \sqrt{I \cap H \cap K}$.

Hence $\sqrt{I \cap H \cap K} = \sqrt{I} \cap \sqrt{H} \cap \sqrt{K}$.

Proposition 3.8. [7] A proper ideal I of a ternary semiring S is prime if and only if it is semiprime and strongly irreducible.

Proposition 3.9. If I is an ideal of a multiplicatively regular ternary semiring S then the following conditions are equivalent.

(i) I is prime

(ii) I is irreducible

Proof: (i) ⟹ (ii). Suppose I is prime then by proposition (3.8) I is strongly irreducible. Let H, K be ideals of S such that H ∩ K = I ⟹ H ∩ K ⊆ I and H ∩ K ⊇ I. Now H ∩ K ⊆ I and I is strongly irreducible ⟹ H ⊆ I and K ⊆ I. Also H∩K ⊇I ⟹ H⊇H∩K ⊇ I or K = I. Therefore I is irreducible.

(ii) ⟹ (i) Suppose I is irreducible. Let H, K and L be ideals of ternary semiring S such that HKL ⊆ S. L be ideals of ternary semiring S such that HKL⊆S. Since S is multiplicatively regular we have H∩K∩L = HKL. Clearly we have H + I, K + I and

L + I are ideals of S. Consider (H+I) ∩(K+I) ∩ (L+I) = (H+I) (K+I) (L+I) ⊆ I and I ⊆ (H+I) ∩ (K+I) ∩ (L+I). Therefore (H+I) ∩ (K+I) ∩ (L+I) = I and I is irreducible ⟹ (H+I) ∩ (K+I) = I or L + I = I ⟹ H + I = I or K+I = I or L+I = I ⟹ H ⊆ I or K ⊆ I or L ⊆ I. Therefore I is prime.

The following two results are analogous to irreducible ideals of semiring.

Proposition 3.10. Let a be a non zero element of a ternary semiring S and let I be an ideal of S not containing a then there exists an irreducible ideal H of S containing I and not containing a.

Proof: Let $\mathcal{A} = \{H : H \text{ is an ideal}, I \subseteq H,\ a \notin H\}$. Clearly $I \in \mathcal{A}$ so that $\mathcal{A}$ is non empty. Let $\{H_i : i \in I\}$be a chain of ideals in $\mathcal{A}$. Take $K = \bigcup_{i \in I} H_i$, Clearly K is an ideal, $I \subseteq K$ and $a \notin K$ so that K is an upper bound for $\{H_i : i \in I\}$.Then by Zorn's lemma $\mathcal{A}$ has a maximal element , let it be H that is H is an ideal of S containing I, not containing a. Now to prove H is an irreducible ideal. Let H', H'' be two ideals of S such that H=H'∩H''⊆ H' , H''. To prove H=H' or H=H'', assume that this is not the case so H⊂H' and H⊂H'' also H is maximal not containing a implies that a∈H' and a∈H''implies a∈H'∩H''=H implies a∈H, a contradiction. Therefore H=H' or H=H''. Hence the proof.

Proposition 3.11.**[7]**Any proper ideal I of a ternary semiring S is the intersection of all irreducible ideals containing it.

Proposition 3.12. A commutative ternary semiring S is multiplicatively regular if and only if every irreducible ideal of S is prime ideal.

Proof: Suppose S is multiplicatively regular then by proposition (3.9) every irreducible ideal is prime ideal conversely suppose that every irreducible ideal of S is prime. Let I be an ideal of S. Then by Proposition (3.11) I is the intersection of all irreducible ideals containing I. That is by supposition I is the intersection of all prime ideals containing I ⟹ I = $\sqrt{I}$.

Let H, K and L be ideals of S. Then HKL is an ideal of S. That implies HKL = $\sqrt{HKL}$ = $\sqrt{H \cap K \cap L} = \sqrt{H} \cap \sqrt{K} \cap \sqrt{L} = H \cap K \cap L$.Then by proposition (6.35,[2]) S is multiplicatively regular.

Proposition 3.13. A ternary semiring S is multiplicative regular if and only if RML = R∩M∩L for every left ideal L, right ideal R and lateral ideal M.

Proof: Suppose that S is multiplicatively regular ternary semiring. Let $x \in RML \subseteq R, M, L$ ⟹ x ∈ R∩M∩L ⟹ RML ⊆ R∩M∩L. Now let $a \in$ R ∩ M ∩ L

⟹ $a \in R, a \in M$ and $a \in L$. Now $a \in S$ and S is multiplicatively regular so there exists $h \in S$. Such that aba = a. therefore a = aba ∈ RML.

$\Rightarrow R\cap M\cap L \subseteq RML$ Thus $RML = R\cap M\cap L$

Conversely suppose that RML = R∩M∩L. To prove S is multiplicatively regular. Let $a \in S \Rightarrow a \in aSS$ $a \in SSa$, $a \in SaS + SSaSS$

$\Rightarrow a \in (aSS) \cap (SaS + SSaSS) \cap (SSa)$

$\Rightarrow a \in (aSS)(SaS + SSaSS)(SSa)$

$\Rightarrow a \in aSa + aSSSa$

$\Rightarrow a \in aSa + aSa$

$\Rightarrow a \in aSa$

$\Rightarrow a = axa$ for some $x \in S$.

Therefore 'a' is regular. Hence S is regular ternary semiring.

Proposition 3.14. A commutative ternary semiring S is regular if and only if every ideal of S is idempotent.

Proof: Let S be a regular ternary semiring and I be an ideal of S.

Then III ⊆ IIS ⊆ I. Let a ∈I. Now a ∈ aISIa ⊆ IISII ⊆ III.

So I ⊆ III. Hence I = III ⟹ ideal I is idempotent conversely suppose that every ideal of S is idempotent. Let R, M, L be three ideal of S. Them RML ⊆ R∩M∩L. Also (R∩M∩L)S (R∩M∩L)S (R∩M∩L) ⊆ RSMSL. Since R∩M∩L is an ideal of S,

(R∩M∩L) (R∩M∩L) (R∩M∩L) = R∩M∩L. So R∩M∩L ⊆ RML.

Hence RML = R∩M∩L and so by proposition (3.13) S is regular ternary semiring.

Proposition 3.15. A commutative ternary semiring S is regular if and only if every proper ideal of S is semiprime.

Proof: Let S be regular ternary semiring and P be any proper ideal of S with AAA ⊆P, where A is any ideal of S. By Proposition (3.14) AAA = A. So A ⊆ P. Hence P is semi prime in S. Conversely suppose that every proper ideal of S is semiprime. Let a ∈S. Then aSa is an ideal of S. If aSa = a then we are done. So, suppose asa ≠ S. Then aSa is a semirpime ideal of S. Now $\langle a\rangle\langle a\rangle\langle a\rangle \in aSa$ that is $\langle a\rangle \subseteq aSa$, since aSa is a semirpime ideal of S. consequently a = axa for some x ∈ a and hence S is a regular ternary semiring.

4. Conclusion

This manuscript determines the set of all prime ideals of ternary semiring S as spectrum of S. It is proved that if I is a primary ideal of a commutative ternary semiring S then $\sqrt{I}$ is a prime ideal of S. Also this manuscript establishes various results on prime ideals, semiprime ideals, irreducible ideals and on regular ternary semirings which are analogous to that of Jonathan S. Golan semiring theory.

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AUTHOR'S BIOGRAPHY

Prof. J. Venkateswara Rao: He has been working as a Professor of Mathematics at Mekelle University, Mekelle, Ethiopia since 2009. His total teaching experience is 21 years. He rendered his services in various positions like Professor & Principal, Reader & Head, Lecturer and Teaching Assistant. His articles more than 80 were published in International Journals of repute. He produced 8 Ph.Ds' and 19 M.Phils'. He was appointed as editorial board member and reviewer for a good number of esteemed International Journals. He evaluated Ph.D. theses from various Indian Universities as an external examiner.

N.koteswaramma: She has been working as Assistant professor of Mathematics at Vasireddy Venkatadri Institute of Technology,Nambur,Guntur since 2007.Her total teaching experience is 7 years.She awarded M.phil degree in 2007 from Nagarjuna University. At present pursuing Ph.D in the concept of ternary semirings under the guidence of Dr.J.venkateswara Rao.