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Mathematical Modeling of Age Specific Participation Rates in Bangladesh

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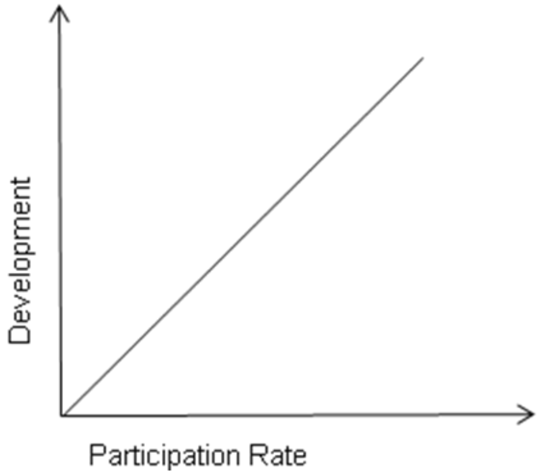
Abstract: *The main purpose of this study is to fit some mathematical models to age specific participation rates (ASPRs) for male, female and both sexes population in Bangladesh and their forward cumulative distribution. For this, the data is taken from Statistical Year Book of Bangladesh of 2011. Quasi Newton Method is employed to fit these mathematical models using the software STATISTICA. Moreover, t-test, F-test and cross validity prediction power (CVPP) are used to check the accuracy as well as validation of the model. In this study, it is found that ASPRs for male, female and both sexes population follow third degree polynomial model and their forward cumulative distribution follow cubic polynomial model. These models are well fitted in accordance with t-test, F-test and cross validity prediction power. The stability of these models are more than 95%, 84%, 96%, 99%, 99%, and 99% respectively.*

Keywords: *Age specific participation rates (ASPRs), Polynomial model, t-test, F-test, Cross validity prediction power (CVPP).*

1. INTRODUCTION

Age specific participation rate (ASPR) is the number of persons in the age specific labor force divided by the corresponding age specific population. Here, participation rate means labour force participation rates. The labour force participation of working-aged is the main indicator of the total size of labour force in Bangladesh. Distribution of income of any country depends directly on labour force participation rates. But, ASPR by sex is very helpful to view the prospective labour force changes in any country. It is observed that participation rate in Bangladesh has been increasing during the period 1961-2001 [1]. But, female participation rate in different age groups are less than male for different types of causes like lack of educational qualification of female than male, lack of safety shelter, number of labour force separation rate is more than male and so on. The proper steps may be taken to increase the participation rate for female for the overall development of a country. There is no way to avoid the interrelation between participation rate and development of a country. Here, development of any country means the development of the population in every sector such as social, economical, environmental, educational and specially health sector. But, any kind of development of a country depends directly or indirectly on economic solvency of the nation. It is observed that development will be high if participation rate is high and it will make about a upward straight line.



The age specific participation rate for female may be utilized to assess solvency of women of a society. Therefore, the objectives of this study are addressed below:
i) to study the level and pattern of age specific participation rates for male, female and both sexes population in Bangladesh,
ii) to fit some mathematical models to ASPRs and forward cumulative ASPRs for male, female and both sexes population in Bangladesh in 2010, and
iii) to check the validation of these fitted models using CVPP.

2. DATA AND DATA SOURCE

A secondary data on ASPRs for male, female and both sexes population in Bangladesh have been taken from Statistical Year Book of Bangladesh of 2011 [2] which is shown in Table 1 and their forward cumulative distribution are demonstrated in Table 2.

Table 1. Age Specific Participation Rate by Sex, 2010

Age Group	Central Age	Male	Female	Both Sex
15-19	17.5	48.44	29.40	39.37
20-24	22.5	75.93	40.98	56.70
25-29	27.5	92.19	44.71	66.61
30-34	32.5	97.29	46.62	70.80
35-39	37.5	98.34	47.67	72.81
40-44	42.5	98.05	46.25	72.82
45-49	47.5	97.37	47.58	74.28
50-54	52.5	94.11	10.25	56.07
55-59	57.5	88.52	11.18	51.91
60-64	62.5	77.20	6.63	45.03
65+	67.5	57.93	8.32	34.77

Table 2. Forward Cumulative Distribution of Age Specific Participation Rates by Sex, 2010

Age Group	Central Age	FCD of Male	FCD of Female	FCD of Both Sex
15-19	17.5	48.44	29.4	39.37
20-24	22.5	124.37	70.38	96.07
25-29	27.5	216.56	115.09	162.68
30-34	32.5	313.85	161.71	233.48
35-39	37.5	412.19	209.38	306.29
40-44	42.5	510.24	255.63	379.11
45-49	47.5	607.61	303.21	453.39
50-54	52.5	701.72	313.46	509.46
55-59	57.5	790.24	324.64	561.37
60-64	62.5	867.44	331.27	606.4
65+	67.5	925.37	339.59	641.17

Note: FCD: Forward Cumulative Distribution

3. Methods and Methodological Issues

3.1 Model Fitting

A) Using the scattered plot (Fig.1-Fig.3) of ASPRs by age group in years for male, female and both sexes population in Bangladesh, it seems that ASPRs for male, female and both sexes population in Bangladesh can be fitted by polynomial model with respect to different ages in year. Therefore, an nth degree polynomial model is considered and the form of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i \ x^i + u \ ;$$

where, x is the mean value of the age group; y is ASPRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the disturbance term of the model. Here, a suitable n is found for which the error sum of square is minimum.

B) Using the dotted plot (Fig.4-Fig.6) of forward cumulative ASPRs for male, female and both sexes population in Bangladesh by age groups, it is observed that these follow nth degree polynomial model with respect to ages. Therefore, the structure of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i \ x^i + u \ ;$$

where, x is the average value of the age group; y is forward cumulative ASPRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the disturbance term of the model. A suitable n is selected such that the error sum of square is lowest.

The software STATISTICA was used to fit these mathematical models to ASPRs.

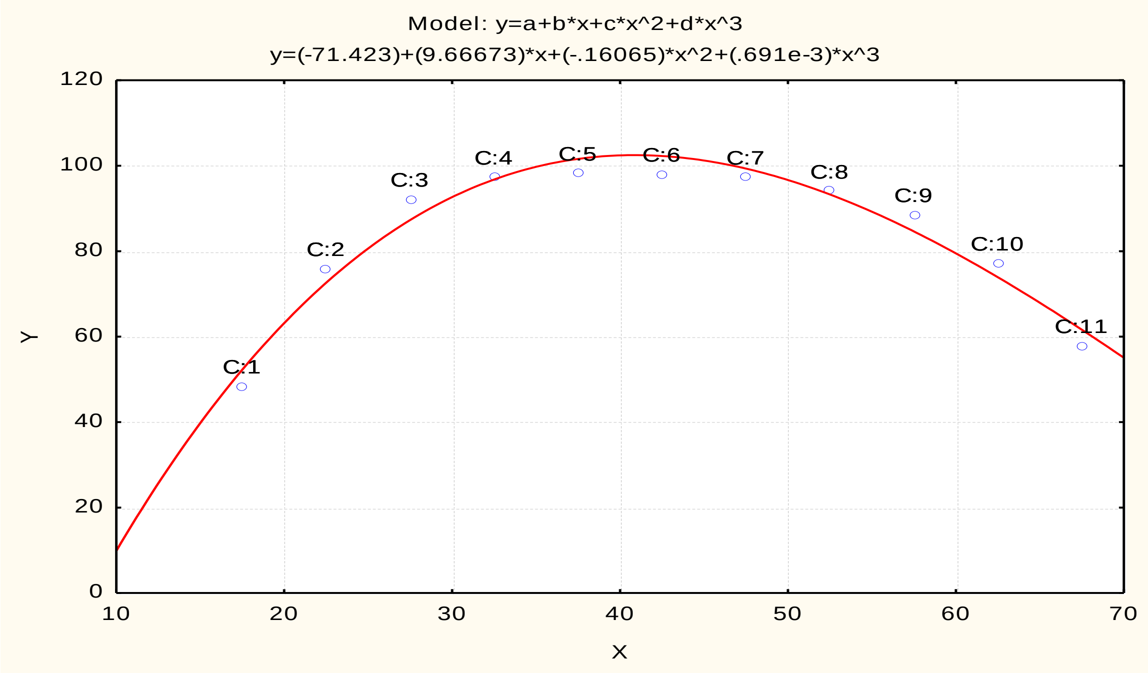


Fig.1. Observed and Fitted ASPRs for Male in 2010. X: Age in Years and Y: ASPRs for Male.

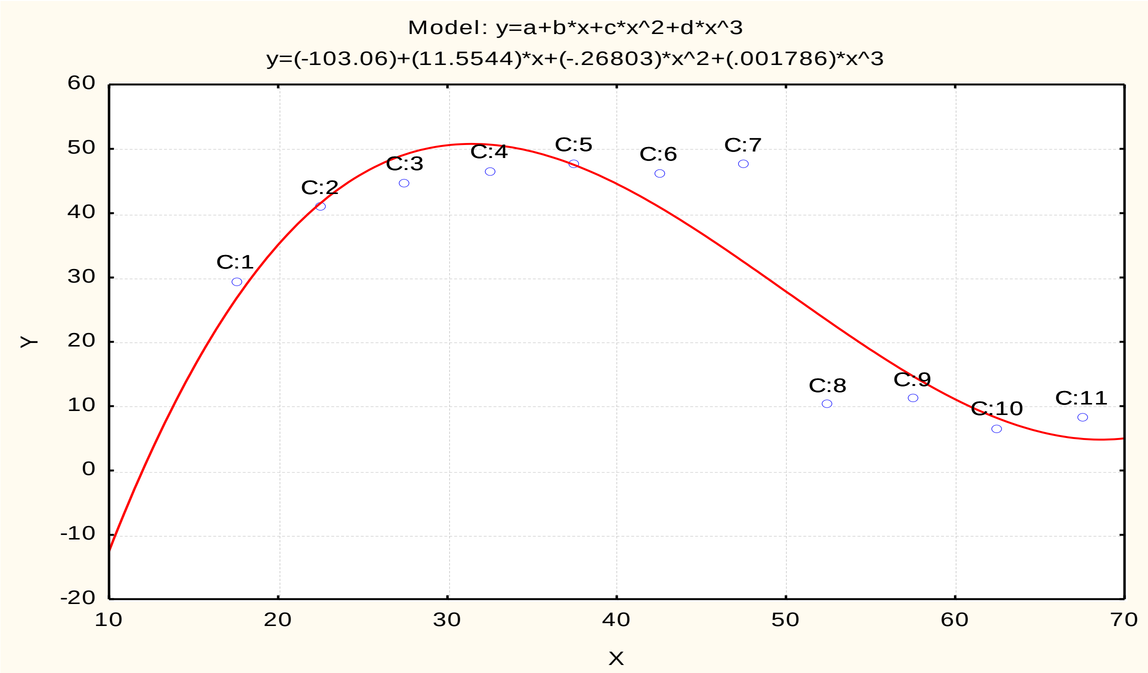


Fig.2. Observed and Fitted ASPRs for Female in 2010. X: Age in Years and Y: ASPRs for Female.

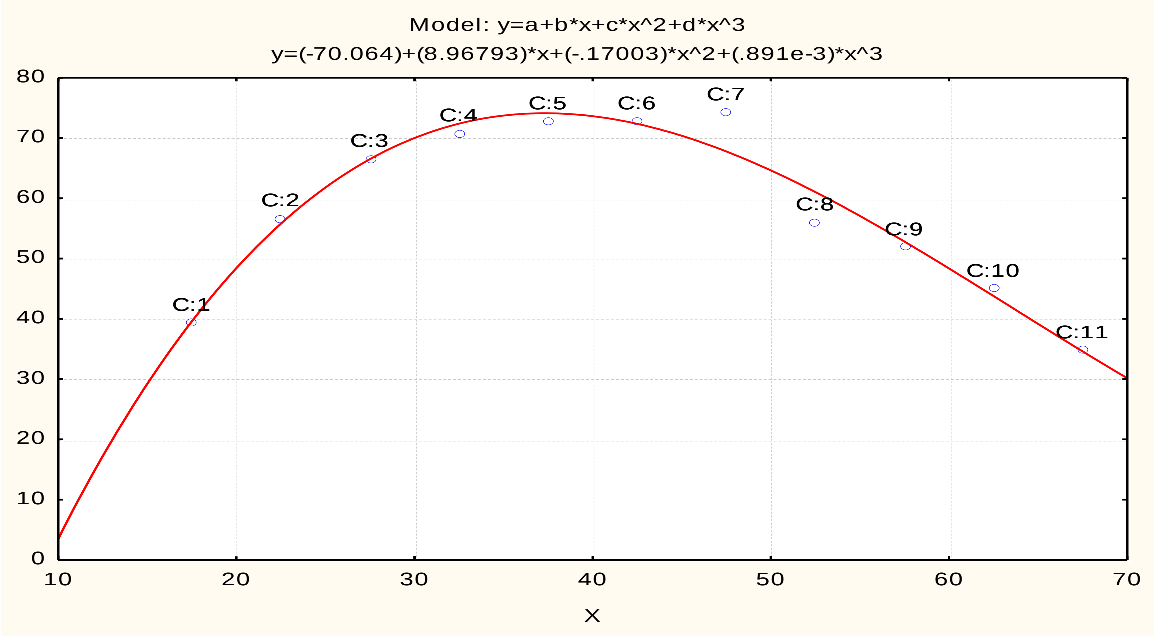


Fig.3. Observed and Fitted ASPRs for Both Sexes in 2010. X: Age in Years and Y: ASPRs for Both Sexes.

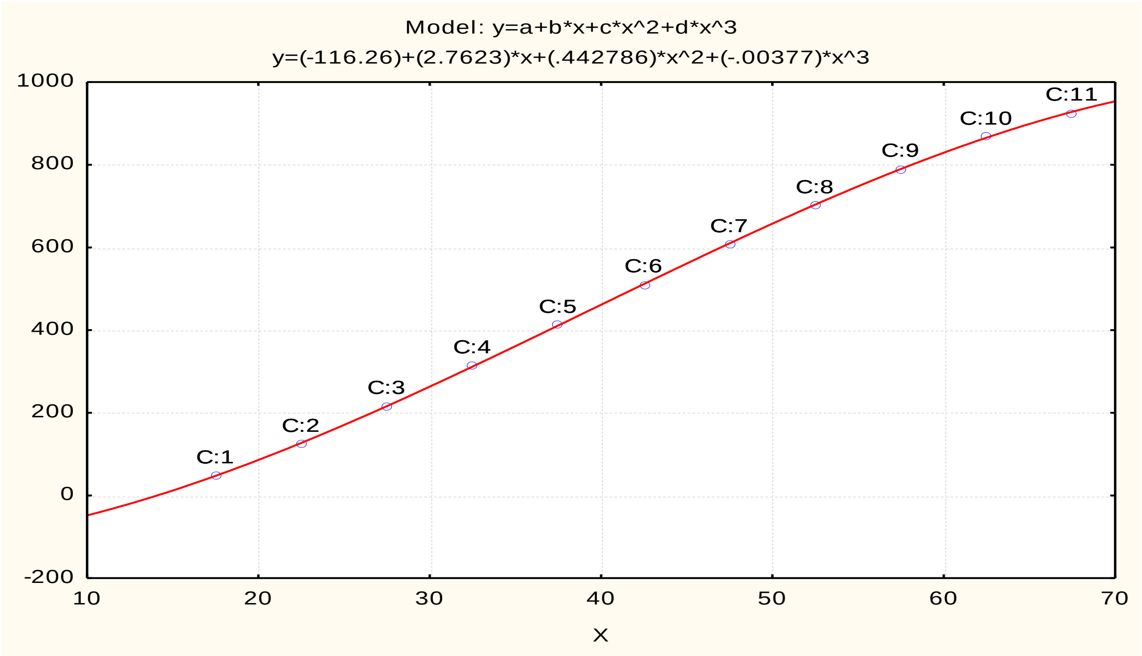


Fig.4. Observed and Fitted FCD of ASPRs for Male in 2010. X: Age in Years and Y: FCD of ASPRs for Male.

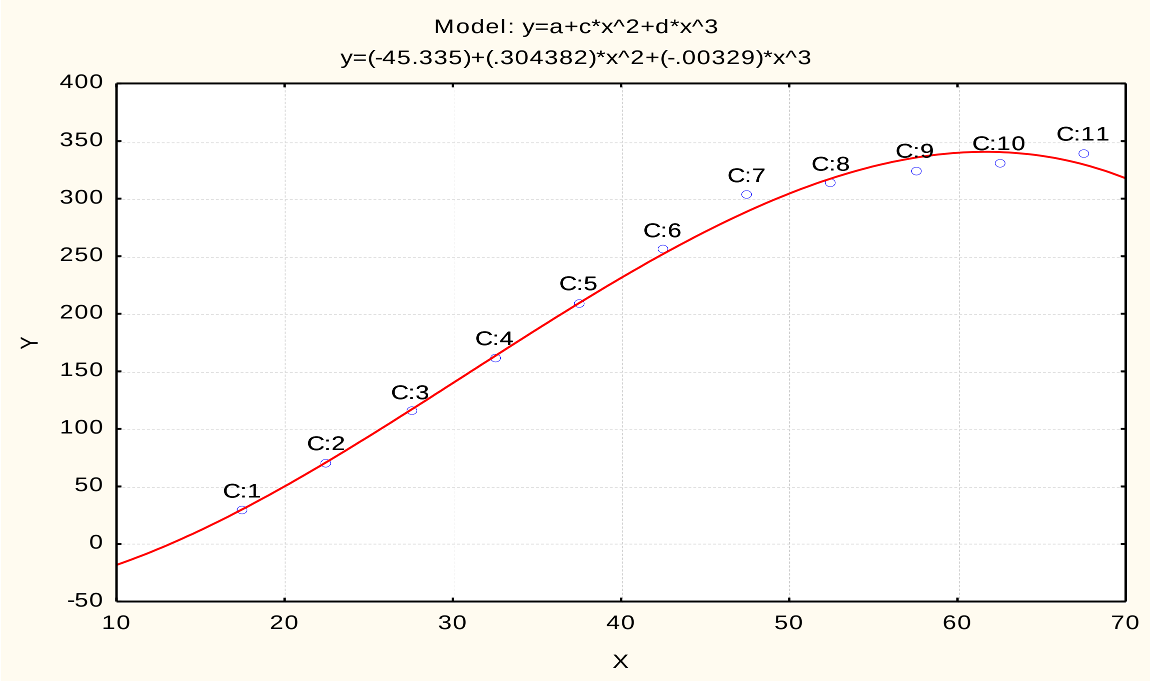


Fig.5. Observed and Fitted FCD of ASPRs for Female in 2010. X: Age in Years and Y: FCD of ASPRs for Female.

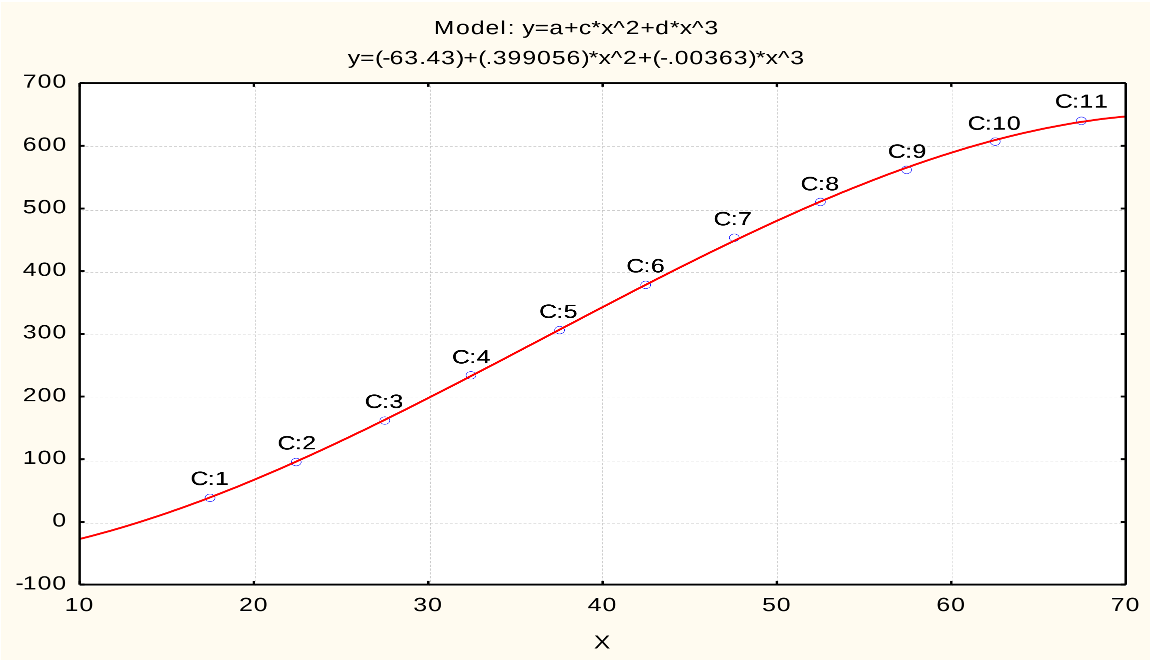


Fig.6. Observed and Fitted FCD of ASPRs for Both Sexes in 2010. X: Age in Years and Y: FCD of ASPRs for Both Sexes.

3.2 Model Validation Technique

To check how much these models are stable over the population, the cross validity prediction power (CVPP), ρ_{cv}^2, is applied. Here

$$\rho_{cv}^2 = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$$; where, n is the number of cases, k is the number of predictors in the model and the cross validated R is the correlation between observed and predicted values of the dependent variable [3] (Stevens, 1996). The shrinkage

coefficient of the model is the positive value of (ρ^2_{cv} - R^2); where ρ^2_{cv} is CVPP and R^2 is the coefficient of determination of the model. 1-shrinkage is the stability of R^2 of the model. The information on model fitting and estimated CVPP has been demonstrated in Table 3. Note that CVPP was also employed as model validation or accuracy test [4, 5, 6, 7, 8].

3.3 F-test

To verify the measure of the overall significance of the model as well as the significance of R^2, the F-test is employed here. The formula for F-test is given below:

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

where k = the number of parameters is to be estimated, n = the number of cases and $\mathbf{R^2}$ is the coefficient of determination in the model [9]. These estimates are shown in Table 4.

4. Results and Discussion

Table 1 represents the ASPRs for male, female and both sexes population of Bangladesh in 2010. To see the level and pattern of ASPRs for male and female population of Bangladesh, the data have been presented in graph paper shown in Fig. 7. It is found that the pattern of ASPRs for male is showing reciprocal of U-shape pattern. It is seen that highest and lowest ASPRs for male in the age group 35-39 and 65+ years are 98.34 and 57.93 respectively. It is also observed that highest and lowest ASPRs for female in the age group 35-39 and 60-64 years are only 46.67 and 6.63 respectively. From the above observation and Fig. 7 it is found that participation rate for female is very low in all age group compared to male in Bangladesh. Table 2 is prepared for forward cumulative distribution of male, female and both sex population of Bangladesh and which shows the total ASPRs in every age group for clear understanding.

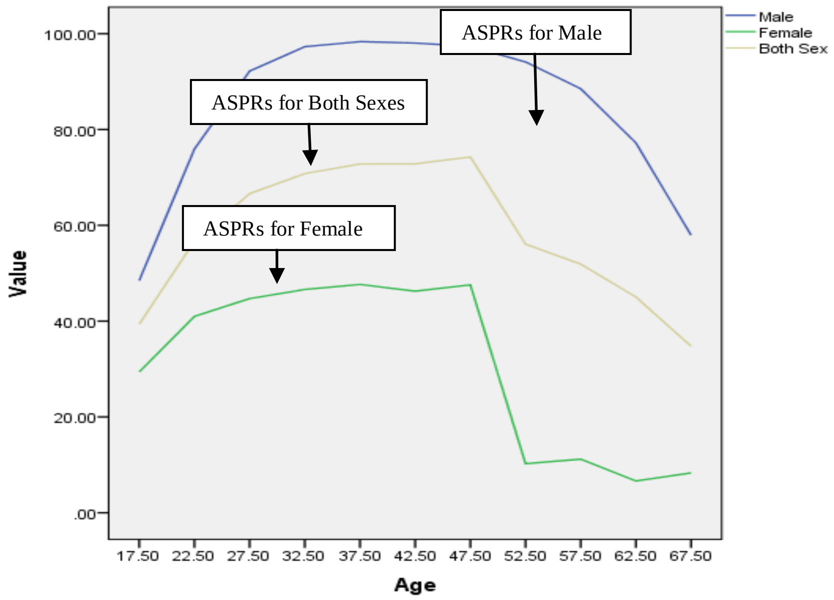


Fig.7. ASPRs for male, female and both sexes population of Bangladesh in 2010.

The polynomial model is constructed for ASPRs for male population of Bangladesh in 2010 and the fitted equation is as follows:

$$y = -71.423 + 9.66673x - 0.16065x^2 + 0.000691x^3 \tag{1}$$

t-stat (-2.7012) (4.4455) (-2.9425) (1.6222)

P-value (0.0305) (0.0029) (0.0216) (0.1487)

Again, another polynomial model is constructed for ASPRs for female population of Bangladesh in 2010 and the fitted equation is

$$y = -103.06 + 11.5544x - 0.26803x^2 + 0.001786x^3 \quad (2)$$

t-stat (-1.9386) (2.6427) (-2.4416) (2.0855)

P-value (0.0937) (0.0332) (0.0446) (0.0754)

Another polynomial model is fitted for ASPRs for both sexes population of Bangladesh in 2010 and the fitted equation is given below:

$$y = -70.0639 + 8.967925x - 0.170033x^2 + 0.000891x^3 \quad (3)$$

t-stat (-3.3931) (5.2809) (-3.9879) (2.6804)

P-value (0.0115) (0.0011) (0.0052) (0.0315)

Moreover, another polynomial model is constructed for forward cumulative distribution of ASPRs for male population of Bangladesh in 2010 and the fitted equation is as follows:

$$y = -116.262 + 2.7623x + 0.442786x^2 - 0.003774x^3 \quad (4)$$

t-stat (-8.4760) (2.4487) (15.6338) (-17.0820)

P-value (0.0000) (0.0441) (0.0000) (0.0000)

Again, another polynomial model is fitted for forward cumulative distribution of ASPRs for female population of Bangladesh in 2010 and the fitted equation is

$$y = -45.3354 + 0.304382x^2 - 0.003290x^3 \quad (5)$$

t-stat (-5.4938) (22.4983) (-17.3108)

P-value (0.0005) (0.0000) (0.0000)

In which the term containing x is excluded from the model because of the insignificant of the parameter.

Furthermore, another polynomial model is constructed for forward cumulative distribution of ASPRs for both sexes population of Bangladesh in 2010 and the fitted equation is given by

$$y = -63.4298 + 0.399056x^2 - 0.003631x^3 \quad (6)$$

t-stat (-21.4841) (82.4435) (-53.3971)

P-value (0.0000) (0.0000) (0.0000)

In (6), the term containing x is kept out from the model because of the insignificant of the coefficient of x.

The estimated CVPP, ρ_{cv}^2 corresponding to their R^2 are shown in Table 3. The observed and fitted values are depicted in Figure 1 to Figure 6. In this table, all fitted models from equation (1) to equation (6) are highly cross validated and their shrinkage's are small. Moreover, it is observed that all the parameters of the fitted models are statistically significant with large proportion of variation explained excepting only one model and whose proportion of variation is explained more than 84%. The stability of R^2 of these models are more than 94%, 79%, 95%, 99%, 99%, and 99% respectively.

Table 3. Information on Model Fittings and Estimated CVPP of the Predicted Equations of ASPRs and its Forward Cumulative Distribution of Bangladesh in 2010.

Models	n	k	R^2	ρ^2_{cv}	Shrinkage	Variance explained (%)
Equation 1	11	3	0.95920	0.904623377	0.054576623	95.920
Equation 2	11	3	0.84762	0.643787013	0.203832987	84.762
Equation 3	11	3	0.96334	0.914301299	0.049038701	96.334
Equation 4	11	3	0.99996	0.999906494	0.000053506	99.996
Equation 5	11	2	0.99573	0.992513636	0.003216364	99.573
Equation 6	11	2	0.99984	0.999719481	0.000120519	99.984

The calculated values of F-statistic of the models (1) to (6) are 1152.947 with (3, 7) degrees of freedom (d.f.), 46.77013 with (3, 7) degrees of freedom (d.f.), 921.1711 with (3, 7) degrees of freedom (d.f.), 195734.8 with (3, 7) degrees of freedom (d.f.), 3302.224 with (2, 8) degrees of freedom (d.f.), 72223.45 with (2, 8) degrees of freedom (d.f.) respectively where as the corresponding tabulated values are only 8.45, 8.45, 8.45, 8.45, 8.65 and 8.65 at 1% level of significance, respectively. Therefore, from these statistics it is also concluded that all these constructed models are highly statistically significant. Hence, the fits of all these models are well.

Table 4. Information on Model Fittings and Calculated Tab. F of the Predicted Equations of ASPRs and its Forward Cumulative Distribution of Bangladesh in 2010.

Models	n	k	Cal. F	Tab.F (at 1% level)
Equation 1	11	4	1152.947	8.45 with (3,7) d.f.
Equation 2	11	4	46.77013	8.45 with (3,7) d.f.
Equation 3	11	4	921.1711	8.45 with (3,7) d.f.
Equation 4	11	4	195734.8	8.45 with (3,7) d.f.
Equation 5	11	3	3302.224	8.65 with (2,8) d.f.
Equation 6	11	3	72223.45	8.65 with (2,8) d.f.

5. CONCLUSIONS

In this study, ASPRs for female population of Bangladesh shows a very poor shape comparing to the ASPRs for male population with reciprocal of U-shape pattern. The ASPRs and its forward cumulative distribution follow cubic polynomial models with explaining large proportion of variance. The stability of these models are more than 95%, 84%, 96%, 99%, 99%, and 99% respectively. Moreover, the stability of R^2 of these models are more than 94%, 79%, 95%, 99%, 99%, and 99% respectively. It is found that the overall significance of these models is very high due to F-statistics.

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