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On the Stability of a Commensal – Host Harvested (Immigration) Species Pair with Limited Resources

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Abstract: *The present paper is devoted to an analytical investigation of a two species comensal - host model. Both the commensal and host with limited resources and are harvested (immigration) at a constant rate. The model is characterized by couple of first order non-linear ordinary differential equations. The only one equilibrium point for the model is identified and stability criteria are discussed. Also global stability is discussed by constructing suitable Liapunov's function.*

Keywords: *commensal, host, harvesting, immigration, Equilibrium points, Normal Steady state, stability, Liapunov's function.*

1. INTRODUCTION

Lotka [12] and Volterra [21] initiated mathematical studies of eco-systems in general, more particularly problems related to growth and decay of fisheries. The ecological symbiosis of living species can be broadly classified as Prey- Predation, Competition, Mutualism, Commensalism, Ammensalism, and Mutation and so on. Meyer [13], Kapoor [6,7] and several others dealt at length in their treatises ,the general concepts of mathematical modeling of ecosystems. The stability of biological communities in nature was discussed by Svirezher and D.O.Logofet [19]. Competition between two and three species with limited and unlimited resources was studied earlier by Srinivas [20] .This was followed by Lakshminarayan and PattabhiRamacharyulu [8, 9, 10] with their investigations on Prey-Predator ecological models with partial cover for the prey and alternate food for the predator and also models with harvesting. A Prey-Predator model with a variable cover for the prey and alternate food for the predator was studied by Lakshminarayan and Apparao[11].Later PattabhiRamacharyulu et.al [2], Archanareddy [3] and Rama sarma [4,5] investigated the stability of species in competition. While mutualism was considered by Ravindra Reddy [17]. Following this PattabhiRmacharyulu, Phanikumar, et.al investigating the stability of species in commensalism [14,15,16,18] while ammensalism was considered by PattabhiRamacharyulu N.Ch,and K.V.L.N Aacharyulu [1].

This paper deals with an analytical investigation of a two species comensal - host model. Both the commensal and host are with limited resources and are harvested (immigration) at a constant rate. The model is characterized by couple of first order non-linear ordinary differential equations. The only equilibrium point (state) for the model is identified based on model equations. It is noticed that the equilibrium state is co-existent and criteria for the asymptotic stability of the state have been derived and observed that it is stable.

2. NOTATION ADOPTED

N_1 and N2 are the populations of the commensal and host species with natural growth rates a_1 and a_2 respectively.

a_{11} is rate of decrease of the commensal due to insufficient food.

a_{12} is rate of increase of the commensal due to inhibition by the host.

a_{22} is rate of decrease of the host due to insufficient food.

$h_1 = a_{11} H_1$ is rate of harvest of the commensal

$h_2 = a_{22} H_2$ is rate of harvest of the host.

$K_i = \frac{a_i}{a_{ii}}$ are the carrying capacities of N_i, i = 1, 2

$C = a_{12}/a_{11}$ is the coefficient of commensalism.

t^* is the dominance reversal time

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, C, h_1, h_2 are assumed to be non-negative constants.

3. BASIC EQUATIONS

A commensal –host model with limited resources and with constant harvesting(immigration) rates is characterized by the following pair of coupled non-linear ordinary differential equations.

(I) Equation for the growth rate of commensal species (S_1)

$$\frac{dN_1}{dt} = \boldsymbol{a_{11} (K_1 N_1 - N_1^2 + C N_1 N_2 + H_1)} \quad (1)$$

(II) Equation for the growth rate of host species (S_2)

$$\frac{dN_2}{dt} = \boldsymbol{a_{22}} (\boldsymbol{K_2 N_2 - N_2^2 + H_2}) \quad (2)$$

4. EQUILIBRIUM STATES

The system under investigation has *only one* equilibrium state given by $\frac{dN_i}{dt}$=0,i=1,2.

Co-existence state E_1:

$$E_1: \overline{N}1 = \left(K_1 + C\left(K_2 + \frac{H_2}{K_2} \right) \right) + \frac{H_1}{K_1 + C\left(K_2 + \frac{H_2}{K_2} \right)} ; \overline{N}2 = K_2 + \frac{H_2}{K_2} \quad (3)$$

5. STABILITY OF EQULIBRIUM STATES

Let $N = (N_1, N_2) = \bar{N} + U = \left(\overline{N_1} + u_1, \overline{N_2} + u_2 \right)$ (4)

where $U = (u_1, u_2)$ is a small perturbation over the equilibrium state: $\bar{N} = (\bar{N_1}, \bar{N_2})$.Substituting (3) in (1) and (2) and neglecting higher powers of the perturbations u_1, u_2 ,we get

$$\frac{dU}{dt} = AU \tag{5}$$

where

$$A = \begin{bmatrix} a_{11}(K_1 - 2\bar{N}_1 + C\bar{N}_2) & a_{11}C\bar{N}_1 \\ 0 & a_{22}(K_2 - 2\bar{N}_2) \end{bmatrix} \tag{6}$$

The characteristic equation for the system is

$$\det [A - \lambda I] = 0 \tag{7}$$

The equilibrium state is stable only when the roots of the equation (7) are negative, in case they are real or have negative real parts, in case they are complex.

5.1 Stability of the Equilibrium State E_1:

$$\bar{N}1 = \left(K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)\right) + \frac{H_1}{K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)}; \bar{N}2 = K_2 + \frac{H_2}{K_2}$$

From (6), the corresponding linearized perturbed equations are

$$\frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -a_{11}\left[\frac{2H_1}{K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)} + \left(K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)\right)\right] & Ca_{11}\left[\left(K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)\right) + \frac{H_1}{K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)}\right] \\ 0 & -a_{22}\left(K_2 + \frac{2H_2}{K_2}\right) \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{8}$$

The characteristic equation for the above system is

The $\lambda^2 + \lambda\left[a_{22}\left(K_2 + \frac{H_2}{K_2}\right) + a_{11}\left[\frac{2H_1}{K_1 + C\left(K_2 - \frac{H_2}{K_2}\right)} + \left(K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)\right)\right]\right]$

$$+a_{11}a_{22}\left(K_2 + \frac{H_2}{K_2}\right)\left(\frac{2H_1}{K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)} + \left(K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)\right)\right) = 0 \tag{9}$$

The characteristic roots of (9) are

$$\lambda_1 = -a_{11}\left(\frac{2H_1}{K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)} + \left(K_1 + C\left(K_2 + \frac{H_2}{K_2}\right)\right)\right); \lambda_2 = -a_{22}\left(K_2 + \frac{H_2}{K_2}\right)$$

both the roots λ_1 and λ_2 are negative. Hence the equilibrium State E_1 is **stable**.

The corresponding linearized perturbed equations are

By solving the system of equations in (8) we get

$$u_1 = \left[u_{10} - M\right] e^{-a_{11}\left(\frac{H_1}{K_1 + C\overline{N_2}}\right)t} + M e^{-a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)t} \tag{10}$$

where

$$M = \frac{C a_{11} u_{20} \overline{N_1}}{a_{11}\left(\frac{H_1}{K_1 + C\overline{N_2}} + \overline{N_1}\right) - a_{22}\left(\frac{H_2}{K_2} + \overline{N_2}\right)}$$

$$u_2 = u_{20}\, e^{-a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)t} \tag{11}$$

The solution curves in (10) and (11) are illustrated as follows:

There arise the following three cases.

Case A: when $u_{10}=M$ Case B: when $u_{10} > M$ and

Case C: when $u_{10} < M$

The solution cases in these three cases are illustrated below.

Case A: when $u_{10} = M$, equations (10) and (11) become

$$u_1 = u_{10} e^{-a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)t} \tag{12}$$

$$u_2 = u_{20} e^{-a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)t} \tag{13}$$

CaseA.1: When $u_{10} > u_{20}$

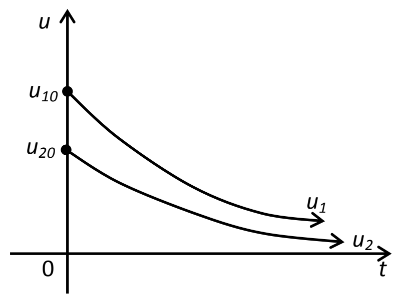

Fig. 1

The initial strength of the commensal is greater than the host. In this case the commensal outnumbers the host. It is evident that both the species converge to the equilibrium point, as shown in Fig 1.

Case A.2: When $u_{10} < u_{20}$

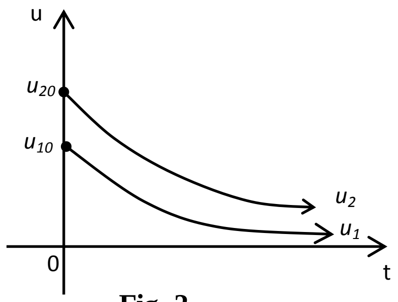

Fig. 2

The initial strength of the host is greater than the commensal. In this case the host continues to outnumber the commensal. It is evident that both the species converge to the equilibrium point as shown in Fig .2.

Case B: When u_{10} > M

Case B.1: When $u_{10} > u_{20}$

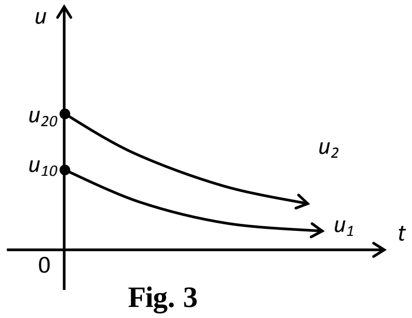

Fig. 3

The initial strength of the host is greater than that of the commensal i.e., $u_{20}>u_{10}$. In this case the host dominates the commensal as shown in Fig.3.

Case B.2: When $u_{10} < u_{20}$

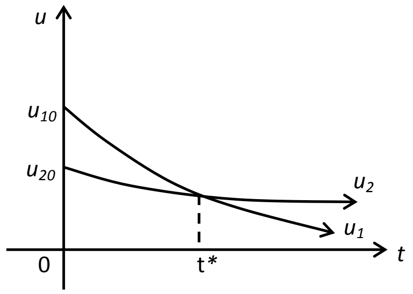

Fig.4

The initial strength of the commensal is greater than that of the host i.e., $u_{10}<u_{20}$. In this case the commensal dominates the host till the time

$$t = t^* = \frac{1}{a_{11}\left(\frac{H_1}{K_1 + C\overline{N}2} + \overline{N}_1\right) - a_{22}\left(\frac{H_2}{K_2} + \overline{N}_2\right)} \log\left(\frac{u_{10} - M}{u_{20} - M}\right)$$

This is the dominance reversal time in this case. This is shown in Fig .4.

Case C: When u_{10} < M

Case C.1: When $u_{10} > u_{20}$

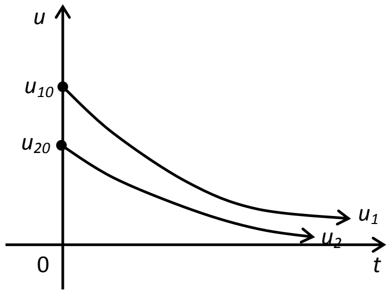

Fig .5

The initial strength of the commensal is greater than that of the host i.e.,
$u_{10} > u_{20}$. In this case the commensal dominates the host as shown in Fig.5.

Case C.2: When $u_{10} < u_{20}$

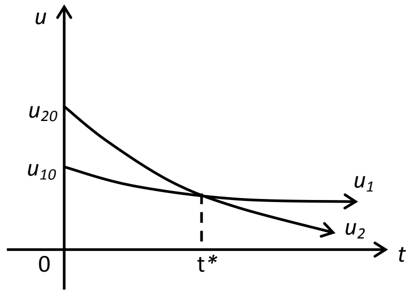

Fig.6.

The initial strength of the host is greater than that of the commensal i.e. $u_{20}>u_{10}$. In this case the host dominates the commensal till the time

$$t = t^* = \frac{1}{a_{11}\left(\frac{H_1}{K_1+C\overline{N}_2}+\overline{N}_1\right)-a_{22}\left(\frac{H_2}{K_2}+\overline{N}_2\right)}\log\left(\frac{u_{10}-M}{u_{20}-M}\right)$$

after which the commensal dominates. The dominance reversal time t^* is shown in Fig.6.

5.1. (a) Trajectories of perturbed species

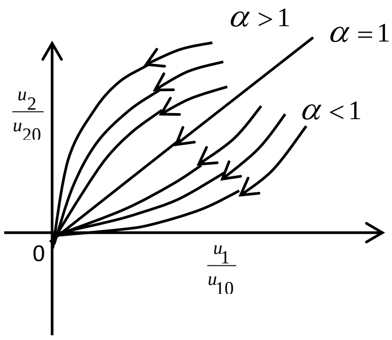

Fig 7

Eliminating't' between the equations (10) and (11)

we obtain $\frac{u_1}{u_{10}} = \frac{M}{u_{10}}\left(\frac{u_2}{u_{20}}\right) + \left(1-\frac{M}{u_{10}}\right)\left(\frac{u_2}{u_{20}}\right)^{\alpha}$

where
$$\alpha = \frac{a_{11}\left(\frac{H_1}{K_1+C\overline{N}_2}+\overline{N}_1\right)}{a_{22}\left(\overline{N}_2+\frac{H_2}{K_2}\right)} \qquad (14)$$

and these curves are seen in Fig .7 represents the stability of the equilibrium point.

6. Liapunov's Function For Global Stability

In section 5.1 we have discussed the local stability of the state of co-existence. We now examine the global stability of the dynamical system (1) and (2).We have already noted that this system has a unique, stable non-trivial co-existent equilibrium state at

$$\overline{N}1 = \left(K_1 + C\left(K_2+\frac{H_2}{K_2}\right)\right) + \frac{H_1}{K_1+C\left(K_2+\frac{H_2}{K_2}\right)};\overline{N}2 = K_2+\frac{H_2}{K_2}$$

Basic Equations:

$$\frac{dN_1}{dt} = a_1N_1 - a_{11}N_1^{\,2} + a_{12}N_1N_2 + a_{11}H_1 \qquad (15)$$

$$\frac{dN_2}{dt} = a_2N_2 - a_{22}N_2^{\,2} + a_{22}H_2 \qquad (16)$$

The lineraized basic equations are

$$\frac{du_1}{dt} = -a_{11}\left(\overline{N}_1 + \frac{H_1}{K_1+C\overline{N}_2}\right)u_1 + Ca_{11}\overline{N}_1u_2 \qquad (17)$$

$$\frac{du_2}{dt} = -a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)u_2 \tag{18}$$

The characteristic equation is

$$\left(\lambda + a_{11}\left(\overline{N_1} + \frac{H_1}{K_1 + C\overline{N_2}}\right)\right)\left(\lambda + a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)\right) = 0$$

i.e., $\lambda^2 + \left[a_{22}\left(\overline{N}2 + \frac{H_2}{K_2}\right) + a_{11}\left(\overline{N}1 + \frac{H_1}{K_1 + C\overline{N_2}}\right)\right]\lambda + a_{22}a_{11}\left(\overline{N}1 + \frac{H_1}{K_1 + C\overline{N_2}}\right)\left(\overline{N}2 + \frac{H_2}{K_2}\right) = 0$

This is in the form of $\lambda^2 + p\lambda + q = 0$

$$\text{where } p = a_{22}\left(\overline{N}2 + \frac{H_2}{K_2}\right) + a_{11}\left(\overline{N}1 + \frac{H_1}{K_1 + C\overline{N_2}}\right) > 0 \tag{19}$$

$$q = a_{11}a_{22}\left(\overline{N}1 + \frac{H_1}{K_1 + C\overline{N_2}}\right)\left(\overline{N}2 + \frac{H_2}{K_2}\right) > 0 \tag{20}$$

Therefore the conditions for Liapunov’s function are satisfied.

Now define

$$E(u_1, u_2) = \tfrac{1}{2}(au_1^2 + 2b\, u_1u_2 + cu_2^2) \tag{21}$$

Where

$$a = \frac{a_{22}^2\left(\overline{N}2 + \frac{H_2}{K_2}\right)^2 + \left[a_{11}a_{22}\left(\overline{N}2 + \frac{H_2}{K_2}\right)\left(\overline{N}1 + \frac{H_1}{K_1 + C\overline{N_2}}\right)\right]}{D} \tag{22}$$

$$b = \frac{Ca_{11}a_{22}\overline{N}1\left(\overline{N}2 + \frac{H_2}{K_2}\right)}{D} \tag{23}$$

$$c = \frac{a_{11}^2\left(\overline{N}1 + \frac{H_1}{K_1 + C\overline{N_2}}\right)^2 + C^2a_{11}^2\overline{N_1}^2 + \left[a_{11}a_{22}\left(\overline{N}2 + \frac{H_2}{K_2}\right)\left(\overline{N}1 + \frac{H_1}{K_1 + C\overline{N_2}}\right)\right]}{D} \tag{24}$$

$$D = pq = \left[a_{22}\left(\overline{N}2 + \frac{H_2}{K_2}\right) + a_{11}\left(\overline{N}1 + \frac{H_1}{K_1 + C\overline{N_2}}\right)\right]\left[a_{22}a_{11}\left(\overline{N}1 + \frac{H_1}{K_1 + C\overline{N_2}}\right)\left(\overline{N}2 + \frac{H_2}{K_2}\right)\right] \tag{25}$$

From (19) and (20) it is clear that D > 0 and a > 0.

Also,

$D^2(ac-b^2) =$

$$D^2\left\{\begin{array}{l}\left(\dfrac{a_{22}{}^2\left(\overline{N}2+\dfrac{H_2}{K_2}\right)^2+a_{11}a_{22}\left(\overline{N}2+\dfrac{H_2}{K_2}\right)\left(\overline{N}1+\dfrac{H_1}{K_1+C\overline{N_2}}\right)}{D}\right)\\ \left(\dfrac{a_{11}{}^2\left(\overline{N}1+\dfrac{H_1}{K_1+C\overline{N_2}}\right)^2+C^2a_{11}{}^2\overline{N_1}{}^2+a_{11}a_{22}\left(\overline{N}2+\dfrac{H_2}{K_2}\right)\left(\overline{N}1+\dfrac{H_1}{K_1+C\overline{N_2}}\right)}{D}\right)\\ -\left(\dfrac{Ca_{11}a_{22}\overline{N_1}\left(\overline{N}2+\dfrac{H_2}{K_2}\right)}{D}\right)^2\end{array}\right\}$$

$$\Rightarrow D^2(ac-b^2)>0$$

$$\Rightarrow b^2-ac<0 \tag{26}$$

$\therefore$ The function $E(u_1, u_2)$ at (21) is positive definite.

Further

$$\frac{\partial E}{\partial u_1}\frac{du_1}{dt}+\frac{\partial E}{\partial u_2}\frac{du_2}{dt}=$$

$$\left(au_1+bu_2\right)\left(-a_{11}\left(\overline{N_1}+\frac{H_1}{K_1+C\overline{N_2}}\right)u_1+Ca_{11}\overline{N_1}u_2\right)+\left(bu_1+cu_2\right)\left(-a_{22}\left(\overline{N_2}+\frac{H_2}{K_2}\right)u_2\right)$$

$$=-a\,a_{11}\left(\overline{N_1}+\frac{H_1}{K_1+C\overline{N_2}}\right)u_1{}^2+\left(a\,Ca_{11}\overline{N_1}-ba_{11}\left(\overline{N_1}+\frac{H_1}{K_1+C\overline{N_2}}\right)-ba_{22}\left(\overline{N_2}+\frac{H_2}{K_2}\right)\right)u_1u_2$$

$$+\left(bCa_{11}\overline{N_1}-ca_{22}\left(\overline{N_2}+\frac{H_2}{K_2}\right)\right)u_2{}^2 \tag{27}$$

Substituting the values of a, b and c from (22), (23) and (24) in (27) we get

$$\frac{\partial E}{\partial u_1}\frac{du_1}{dt}+\frac{\partial E}{\partial u_2}\frac{du_2}{dt}=-\left(\frac{a_{11}a_{22}\left(\overline{N_1}+\dfrac{H_1}{K_1+C\overline{N_2}}\right)\left(\overline{N_2}+\dfrac{H_2}{K_2}\right)\left[a_{22}\left(\overline{N_2}+\dfrac{H_2}{K_2}\right)+a_{11}\left(\overline{N_1}+\dfrac{H_1}{K_1+C\overline{N_2}}\right)+\right]}{D}\right)u_1{}^2-$$

$$\left(\frac{a_{11}a_{22}\left(\overline{N_1}+\dfrac{H_1}{K_1+C\overline{N_2}}\right)\left(\overline{N_2}+\dfrac{H_2}{K_2}\right)\left[a_{22}\left(\overline{N_2}+\dfrac{H_2}{K_2}\right)+a_{11}\left(\overline{N_1}+\dfrac{H_1}{K_1+C\overline{N_2}}\right)\right]}{D}\right)u_2{}^2$$

$$=-\left[\frac{D}{D}\right]u_1{}^2-\left[\frac{D}{D}\right]u_2{}^2 \tag{28}$$

$$=-\left(u_1{}^2+u_2{}^2\right) \tag{29}$$

$$\therefore \frac{\partial E}{\partial u_1}\frac{du_1}{dt} + \frac{\partial E}{\partial u_2}\frac{du_2}{dt} = -\left(u_1^2 + u_2^2\right)$$

which is clearly negative definite

So, E (u_1, u_2) is a Liapunov function for the Linear system.

Next we prove that E (u_1, u_2) is also a Liapunov function for the non-linear system

If F and G are two functions in N_1 and N_2 defined by

$$F(N_1, N_2) = a_1N_1 - a_{11}N_1^2 + a_{12}N_1N_2 + a_{11}H_1 \tag{30}$$

$$G(N_1, N_2) = a_2N_2 - a_{22}N_2^2 + a_{22}H_2 \tag{31}$$

Now we have to show that $\frac{\partial E}{\partial u_1}F + \frac{\partial E}{\partial u_2}G$ is negative definite

By putting $N_1 = \overline{N}_1 + u_1$ and $N_2 = \overline{N}_2 + u_2$ in (30) and (31) we get

$$\frac{du_1}{dt} = a_{11}(\overline{N}_1 + u_1) - a_{11}(\overline{N}_1 + u_1)^2 + a_{12}(\overline{N}_1 + u_1)(\overline{N}_2 + u_2) + a_{11}H_1$$

$$= -a_{11}\left[\frac{H_1}{K_1 + C\overline{N}2} + \overline{N}_1\right]u_1 + Ca_{11}\overline{N}_1u_2 + f(u_1, u_2)$$

Where $f(u_1, u_2) = -a_{11}u_1^2 + a_{12}u_1u_2$

$$\Rightarrow F(u_1, u_2) = \frac{du_1}{dt} = -a_{11}\left[\frac{H_1}{K_1 + CN_2} + \overline{N}1\right]u_1 + Ca_{11}\overline{N}1u_2 + f(u_1, u_2) \tag{32}$$

similarly

$$\frac{du_2}{dt} = a_2(\overline{N}2 + u_2) - a_{22}(\overline{N}2 + u_2)^2 - a_{22}u_2$$

$$= -a_{22}\left(\overline{N}2 + \frac{H_2}{K_2}\right)u_2 + g(u_1, u_2)$$

where $g(u_1, u_2) = -a_{22}u_{22}$

$$\Rightarrow G(u_1, u_2) = \frac{du_2}{dt} = -a_{22}\left(\overline{N}2 - \frac{H_2}{K_2}\right)u_2 + g(u_1, u_2) \tag{33}$$

From (21)

$$\frac{\partial E}{\partial u_1} = au_1 + bu_2 \tag{34}$$

$$\frac{\partial E}{\partial u_2} = bu_1 + cu_2 \tag{35}$$

Now $\frac{\partial E}{\partial u_1}F + \frac{\partial E}{\partial u_2}G = (au_1 + bu_2)\left\{-a_{11}\left[\overline{N}1 + \frac{H_1}{K_1 + C\overline{\overline{N_2}}}\right]u_1 + Ca_{11}\overline{N}1u_2 + f(u_1, u_2)\right\}$

$$+ (bu_1 + cu_2)\left\{-a_{22}\left(\overline{N}2 + \frac{H_2}{K_2}\right)u_2 + g(u_1, u_2)\right\}$$

$$= (au_1 + bu_2)\left[-a_{11}\left[\overline{N}1 + \frac{H_1}{K_1 + C\overline{\overline{N_2}}}\right]u_1 + Ca_{11}\overline{N}1u_2\right] + (bu_1 + cu_2)\left(-a_{22}\left(\overline{N}2 + \frac{H_2}{K_2}\right)\right)u_2 +$$

$$(au_1 + bu_2)f(u_1, u_2) + (bu_1 + cu_2)g(u_1, u_2) \tag{36}$$

$$\frac{\partial E}{du_2}F + \frac{\partial E}{du_1}G = -(u_1{}^2 + u_2{}^2) + (au_1 + bu_2)f(u_1, u_2) + (bu_1 + cu_2)g(u_1, u_2) \tag{37}$$

By introducing polar coordinates (37) becomes

$$\frac{\partial E}{du_2}F + \frac{\partial E}{du_1}G = -r^2 + r[(a\cos\theta + b\sin\theta)f(u_1, u_2) + (b\cos\theta + c\sin\theta)g(u_1, u_2) \tag{38}$$

Let us denote the largest of the numbers | a|, |b|, |c| by K.

Our assumptions imply that | f (u_1, u_2)| < $\frac{r}{6K}$ and | g (u_1, u_2)| < $\frac{r}{6K}$, for all sufficiently small r > 0.

So

$$\frac{\partial E}{\partial u_1}\text{F} + \frac{\partial E}{\partial u_2}\text{G} < -r^2 + \frac{4Kr^2}{6K} = -\frac{r^2}{3} < 0 \tag{39}$$

Thus E (u_1, u_2) is a positive definite function with the property that

$$\frac{\partial E}{\partial u_1}F + \frac{\partial E}{\partial u_2}G \quad \text{is negative definite.} \tag{40}$$

$\therefore$ The equilibrium state E_1 is asymptotically stable.

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