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Comparative Performances Analysis of Exponential Decline Curve Method and Logistic Growth Model in Oil and Gas Wells Forecasting

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Abstract: *The forecasting process has been carried out for decades, using the Arps model. This model yields reliable estimates which can be used to forecast future production.Various new models and techniques have been proposed to improve the accuracy and reliability of reserve estimates; however, none have gained widespread industry acceptance. This paper compares the Arps exponential decline model to a new decline model; the logistic growth model, using numerical optimization, plotting and production curve fitting, to determine which model is more reliable. The new model incorporates known physical volumetric quantities of oil and gas into the forecast to constrain the reserve estimate to a reasonable quantity. Historical production data from an oil well and a gas well were obtained, and production rates were predicted and forecasted using the two methods under investigation. The predicted values were compared to the historical data to gauge their accuracy levels. It was found that the logistic growth model provided more accurate oil rate estimates and slightly better gas rate estimates than the exponential decline model.The resultsare limited to the conditions surrounding the wells of interest. Overall, the logistic growth model proved to be a superior forecasting tool in this study.*

Keywords: *Production forecasting, reserves, decline curve analysis, logistic growth model*

1. Introduction

Production forecasting of oil wells in the petroleum industry has been carried out for decades using the Arps equations. These models yield reliable estimates which can be used to forecast future production. Since the adoption of the Arps models, multiple decline models have been proposed by various authors, one of which is the logistic growth model. The industry has however been slow to adopt the new methods for forecasting petroleum production.

The logistic growth model equation is a newly propounded empirical model used for production forecasting. It is a population growth model from the field of biology, where its formula was likened to petroleum forecasting after empirical manipulation. It has a term referred to as the carrying capacity, this carrying capacity is the maximum size a population can grow to, at which point the size of the population will stabilize and the rate of growth will terminate (Spencer and Coulombe, 1966). This concept is applied to the field of petroleum engineering, in that, there is an early-time region which ultimately leads to a middle-time region during production from a well. This empirically imitates the logistic growth model; therefore, they can be related mathematically. This form of the model proposed is adopted from an equation found in work by Spencer and Coulombe (1966), which was used to model the regrowth of livers. They proposed a simple mathematical model to predict this growth, with the original size of the liver before reduction being used as the carrying capacity. For the purposes of forecasting production in oil and gas wells, the model has been altered through empirical analysis to concur with petroleum parameters.

Production Forecasting is an important input into the decision-making process and investment scenario evaluation, which are crucial for an upstream organization. The production forecast flows through the central nervous system of an organization and helps to identify opportunities and decide on the best way forward (Watts, 2016). There are many reasons for making production forecasts, and to make it even more complex, the different purposes quite often have several aspects. Often, the overall integrating factor is that forecasts are made to see how the maximum value of an asset can be achieved.

This paper aims to make a comparison between the forecast accuracies of the logistic growth model and the Arps exponential decline model in oil and gas wells.

1.1. Logistic Growth Model

This model was developed by mathematician Verhulst in 1838. The theoretical background of this model entails a biological population withlots of food, space to grow and no threat from predators, which tends to grow at a rate that is proportional to the population. That is, in each unit of time, a certain percentage of the individuals produce new individuals.Verhulst drew inspiration from Malthus (1872)'s theories, which proposed that the population of a specific country or region could only expand up to a certain extent before resource competition would lead to a stabilization of growth.Logistic growth models have been previously utilized in the petroleum industry, specifically through Hubbert's model. Hubbert's model (1956) was employed to predict production for entire oil fields or regions. However, the model under investigation in this study diverges from Hubbert's model as it focuses on forecasting production for an individual well.

1.1.1. Applications

Equation 1 represents the generalized form of the logistic growth model, where a significant component is the concept of carrying capacity. The carrying capacity signifies the maximum size that a population can attain, leading to stabilization and the cessation of growth rate. Tsoularis and Wallace (2010) developed a comprehensive equation by combining various factors to derive this generalized form.

$$\frac{dN}{dt} = rN^{\alpha}[1 - (\frac{N}{K})^{\beta}]^{\gamma} \qquad (1)$$

Where; N = Population

r = Constant

α = Exponent

β = Exponent

γ = Exponent

K = Carrying Capacity

According to Clark (2011), the Logistic Growth Models have come to be used in different fields including the petroleum industry. The generalised Equation 1 above has been altered through empirical analyses to yield Equation 2:

$$\mathrm{Q(t)} = \frac{\mathrm{Kt^n}}{\mathrm{a + t^n}} \qquad (2)$$

Where; Q = Cumulative Production

K = Carrying Capacity

a = Constant

n = Hyperbolic Exponent

t = Time

The logistic growth model is a growth equation. In this case, the growth is cumulative oil or gas production. The derivative with respect to time can be taken to obtain the rate form which is penultimate to this work. We arrive at Equation 3:

$$\mathrm{q(t)} = \frac{\mathrm{dQ}}{\mathrm{dt}} = \frac{\mathrm{Knbt^{n-1}}}{\mathrm{(a + t^n)^2}} \qquad (3)$$

Where; q = Production Rate.

While it does not appear to be so in this form, this is a specific case of the generalized logistic growth model (eq. 1) shown above where 'r' is equal to n(K/a)1/n, 'α' is equal to 1-1/n, 'β' is equal to 1 and 'γ' equals 1+1/n.

1.1.2. Method

In practical applications, the logistic growth model requires the determination of either two or three unknown parameters to achieve a proper fit with production data. These parameters include the carrying capacity (K), the hyperbolic exponent (n), and the constant (a). The carrying capacity represents the maximum recoverable amount of oil or gas from primary depletion in the well, disregarding time or economic limitations. Essentially, K serves as an estimate of the well's Estimated Ultimate Recovery (EUR) without economic constraints. It functions as a constraint on cumulative production, eventually leading to a decline in the production rate to zero. As cumulative production approaches the carrying capacity, the rate gradually diminishes until reaching termination. The determination of the number of unknowns in the equation also hinges on the value of this parameter.

The Estimated Ultimate Recovery (EUR) can be derived through volumetric calculations and a recovery factor. In cases where the initial EUR is unknown before well production, it can be employed as an adjustable parameter. By optimizing the fit to the data, the value of K can be determined. The logistic model demonstrates high flexibility and allows for multiple satisfactory fits to the data when the carrying capacity is not known in advance. Figure 1 provides an illustration of a shale gas well where different carrying capacities were utilized, resulting in reasonable fits for all three scenarios.

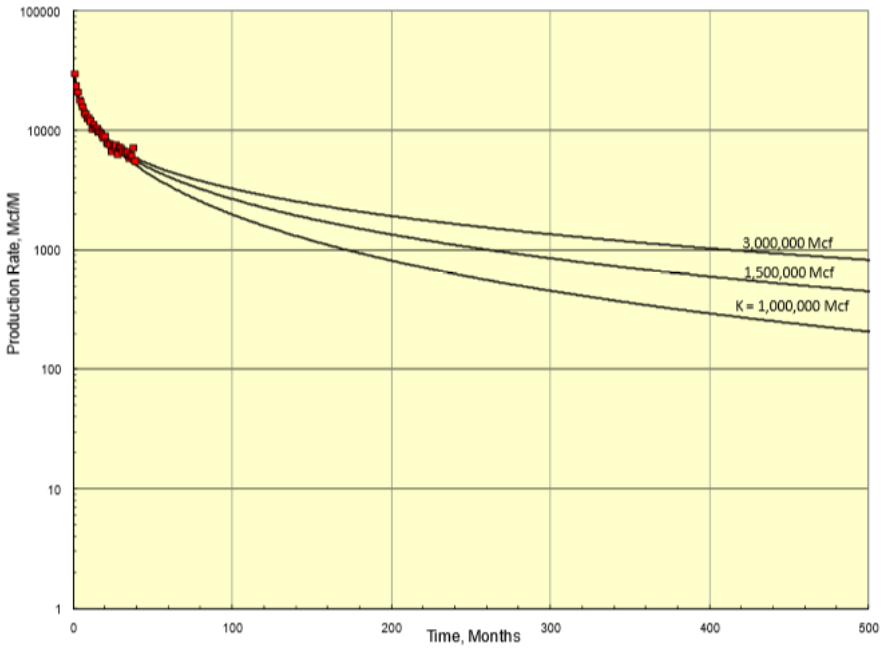

Figure1. *Rate vs. Time Data Fit with the Logistic Model using varying 'K' Values (Clark, 2011)*

Once the status of the carrying capacity being known in advance is established, the next step involves determining the hyperbolic decline exponent. Regardless of whether or not the carrying capacity (K) is known beforehand, the constant (n) must be determined. The value of n plays a crucial role in governing the decline behavior of the model and enables it to exhibit greater flexibility for a more accurate fit to the production data.

In order to illustrate the impact of the parameter "n" on forecasts, dimensionless terms were introduced to the analysis. These include the dimensionless rate (qD), which represents the production rate relative to the peak production rate, and the dimensionless cumulative (QD), which normalizes the cumulative production based on the carrying capacity. In this form, the dimensionless cumulative reflects the fraction of the EUR that has been recovered. As the oil/gas well approaches the EUR, the QD value tends to approach 1. Figure 2 presents the relationship between the dimensionless rate and dimensionless cumulative, showcasing different values of the decline exponent.

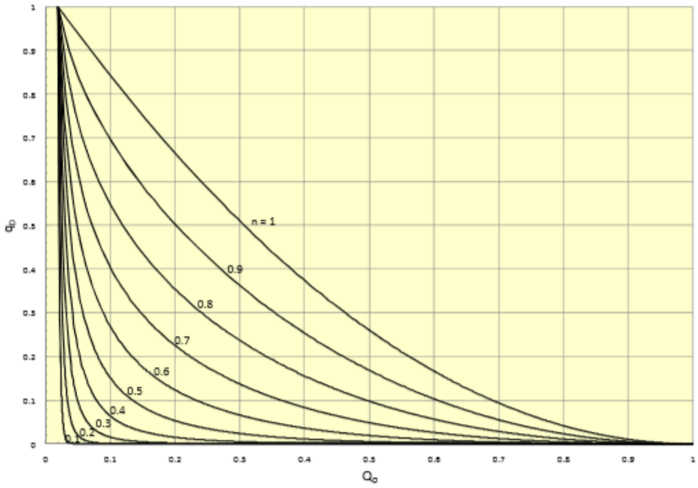

Figure2. *Dimensionless Curve for Varying 'n' Values (Clark, 2011)*

Figure 2 illustrates the behaviour of the model within the range of n values from 0 to 1. The specific K and a values used in this example are arbitrary and do not impact the general understanding of the model's behaviour. The parameter 'n' plays a crucial role in determining the steepness of the decline curve. For smaller values of 'n,' the well experiences a rapid initial decline, followed by a stabilization at a lower production rate with a slower decline over time. Conversely, higher 'n' values result in a more gradual decline throughout the lifespan of the well. When 'n' exceeds 1, the model exhibits an inflection point where the production rate briefly increases before declining. This behaviour does not indicate a forecasting error but can be utilized to match data for wells that do not initially reach their peak rate.

The third parameter, 'a,' represents the time to the power of 'n' at which half of the carrying capacity has been reached. It is important to note that this is not equivalent to half of the time it will take for the well to reach its carrying capacity. Equation 4 demonstrates that as time to the power of 'n' approaches 'a,' the logistic model approaches half of the carrying capacity

$$\frac{\lim}{t^n \rightarrow a}\left[\frac{Kt^n}{a+t^n}\right] = \frac{K}{2} \tag{4}$$

This results in a similar behaviour for 'a' as seen with the initial decline parameter ('di') in the Arps equation. When 'a' has a lower value, the production rate experiences a rapid decline before stabilizing. On the other hand, a higher value of 'a' leads to a more consistent and stable production throughout the lifespan of the well. In simpler terms, a very low 'a' value indicates a high initial production rate, followed by a rapid recovery of half of the EUR, and subsequently, a gradual decline with a low production rate for an extended period. Figures 3 and 4 depict the relationship between dimensionless rate versus time and dimensionless cumulative versus time, respectively, for different values of 'a'.

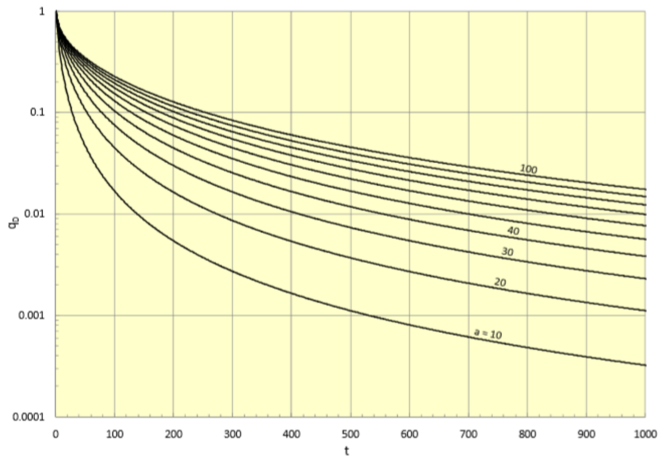

Figure3. *Dimensionless Rate vs. Time for Varying 'a' Values (Clark, 2011)*

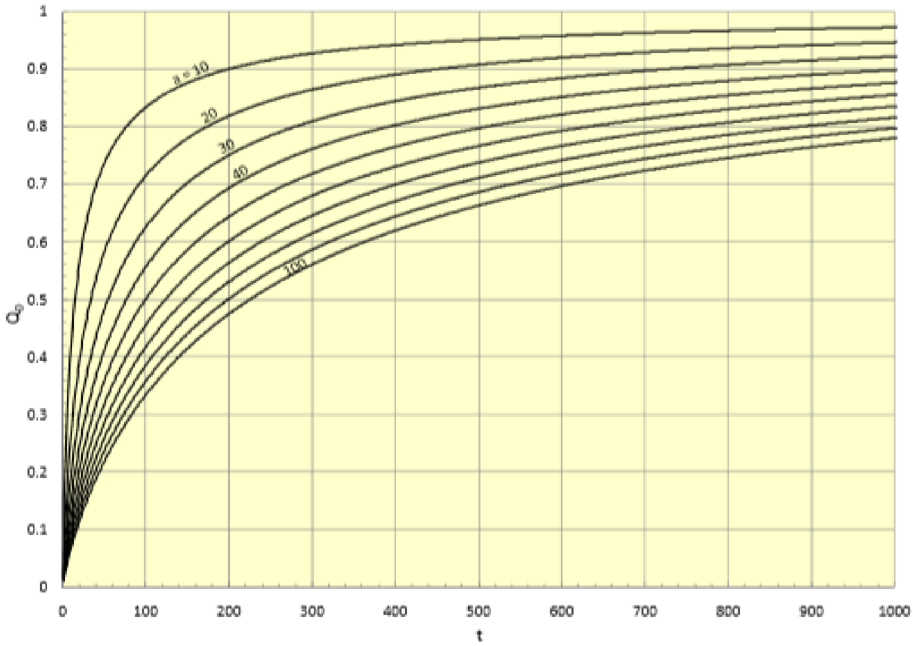

Figure4. *Cumulative Production vs. Time for Varying 'a' Values (Clark, 2011)*

In Figures 3 and 4, the range of 'a' values varies from 10 to 100. It is noticeable that lower 'a' values result in an initial steep decline, followed by a stabilization at a significantly lower decline rate. Conversely, higher 'a' values exhibit a more gradual decline over time.

1.2. Arps Exponential Decline Model

For the exponential decline model, the hyperbolic decline constant, b, is assumed to be zero since this is an exponential function. The exponential decline model is derived from the hyperbolic decline model. The generalized form of the exponential equation proposed by Arps (1944) is shown in Equation 5:

$$q = qie^{-dt} \tag{5}$$

Where, q = current production rate

qi = initial production rate

t = cumulative time since the start of production

d = dt = nominal decline rate (a constant).

2. METHODOLOGY

Data from a gas producer and an oil producer were obtained and used to compare the goodness of fit of both forecasting methods. The process involved predicting production rates for the analysis periods, calculating errors to determine accuracylevels and forecasting production 8 years into the future to see the nature of the generated curves.

The gas well produces entirely under depletion, with support only from an underlying aquifer. It produces mainly gas and some condensate. The oil well is supported by gas and water injection. There is no support derived from an aquifer or gas cap. It produces oil and gas only. Declining production rates in the gas well are mainly due to declining reservoir pressure, while declining rates in the oil producer are due to a combination of gas breakthrough, formation damage and declining reservoir pressure.

2.1. Exponential Decline Analysis

Equation 5 was used to estimate gas and oil production rates, assuming the initial rates and time variations of 1 day. The decline rate was solved for numerically using the Solver feature in Microsoft Excel. This permitted the use of the decline rate that gave the best production decline fit.

2.2. Logistic Growth Production Curve Fitting

There are two preferred methods for fitting the logistic growth model to the data. The first method involves optimizing the parameters with a numerical scheme, and the second method involves linearizing the equation and plotting it on a Cartesian grid to obtain the parameters. This paper applies only the numerical optimization method. The numerical optimization was performed using the Microsoft Excel Solver. This allowed the most accurate forecasts to be estimated. Equation 2 was used to generate production rates by first calculating cumulative production and finding the difference between consecutive values. The parameter that was known to a degree in both cases was the carrying capacity, K. K was assumed to be the contacted reservoir volume for both wells. These figures were obtained from rate transient analysis using available production data. The K values placed a limit on the possible cumulative production from the well and allowed for more reasonable values of the other unknown parameters. Cases where K was not fixed were ran, also numerically. The resulting K values were unreasonably large, with corresponding unreasonable values for a and n.

For both forecasting methods, the degree of accuracy was determined using the Mean Average Percentage Error (MAPE) KPI.

3. Results

3.1. Gas Well

Figure 5 illustrates the forecasted gas production rates against time using the Arps exponential decline method. Also shown are approximately two years' worth of the historical production rates from the well.The initial production rate was about 60 MMscf/d and the well produced at a fairly constant choke size throughout the two-year period. The Arps method can be used in this case because the circumstances affecting production did not change. Any forecasts into the future will therefore be bound by the conditions of the past, as stated by the golden rule. As can be seen, a fairly goodmatchbetween predicted and historical rates was obtained, with a MAPE of 2.51%. The decline rate that gave the best fit was 15.76% per year. Production from the well does not decline quickly, and this behaviour was modelled well by the Arps equation.

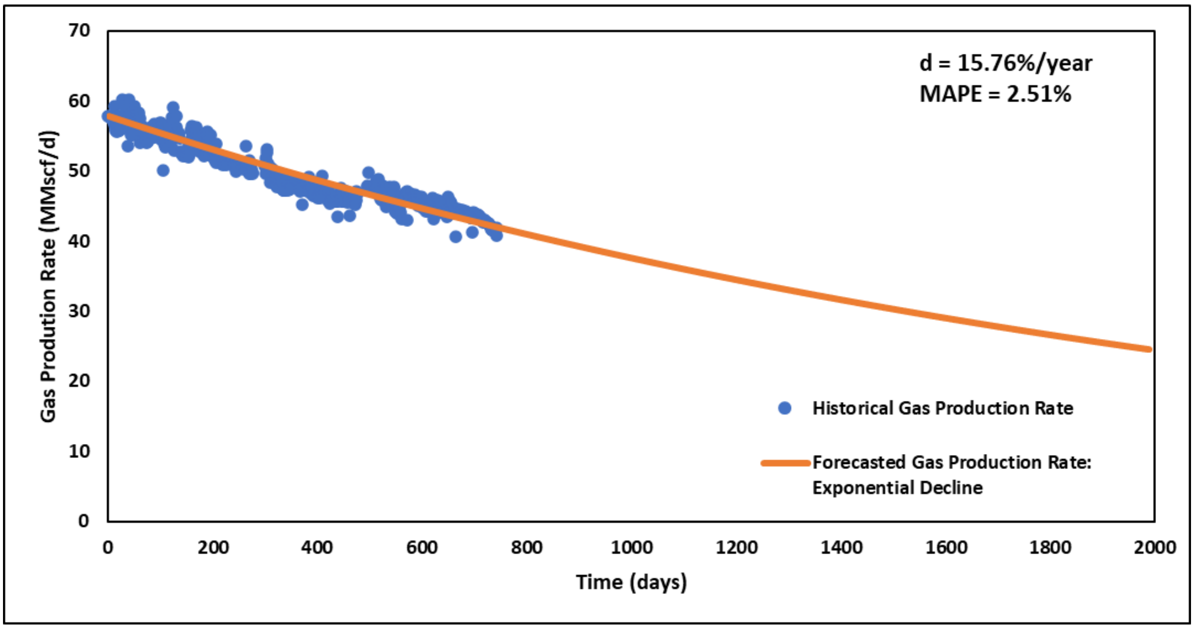

Figure5. *Resulting Fit of Exponential Decline Model with Historical Production Data*

In Figure 6, the forecasted gas production rates using the logistic growth model are shown. A carrying capacity of 200 Bscf of gas was used. There is little uncertainty surrounding this figure, and that gave way for fairly accurate values of a and n to be obtained. After numerical optimization, optimum a and n values of 48,828 and 1.408 were obtained. The combination of these values led to a good fit of the predicted production rates, with a MAPE of 2.47%. The model proved slightly more accurate than the Arps equation, though, forecasts may not differ much.

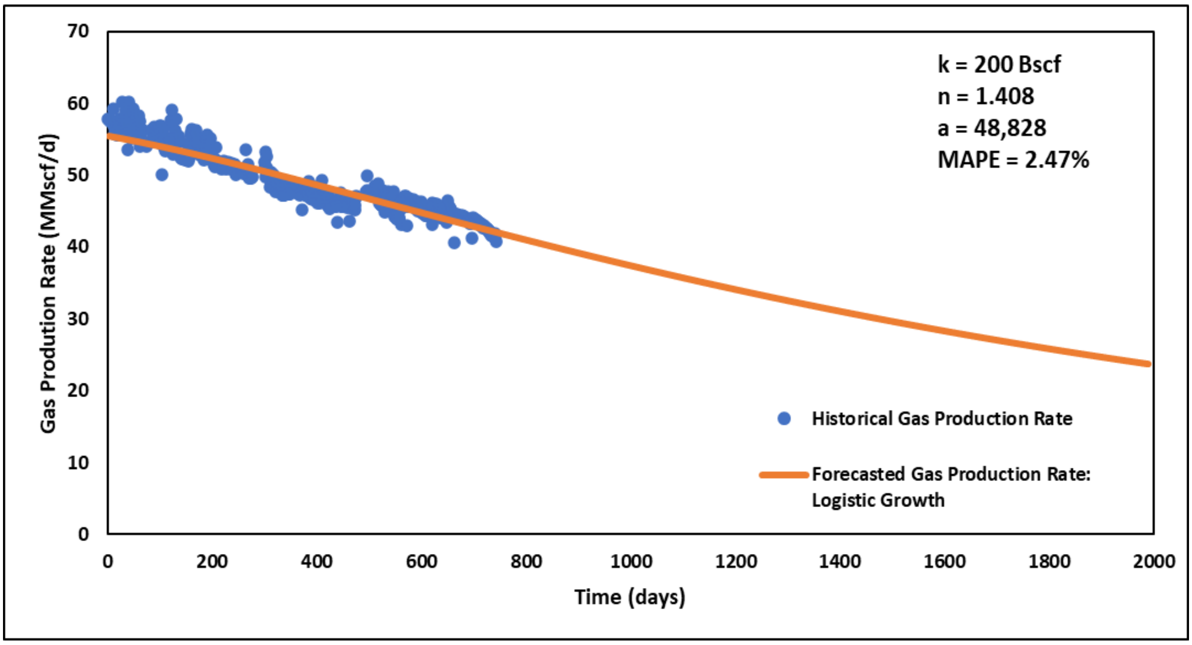

Figure6. *Resulting Fit of LGM with Historical Production Data*

The forecasts for both methods were then compared to each other in Figure 7. As can be seen, and as expected, forecasted values 8 years into the future did not differ much between the two methods. The predicted LGM values were marginally less than those of the Arps equation, leading to a slightly lower EUR in the long run.

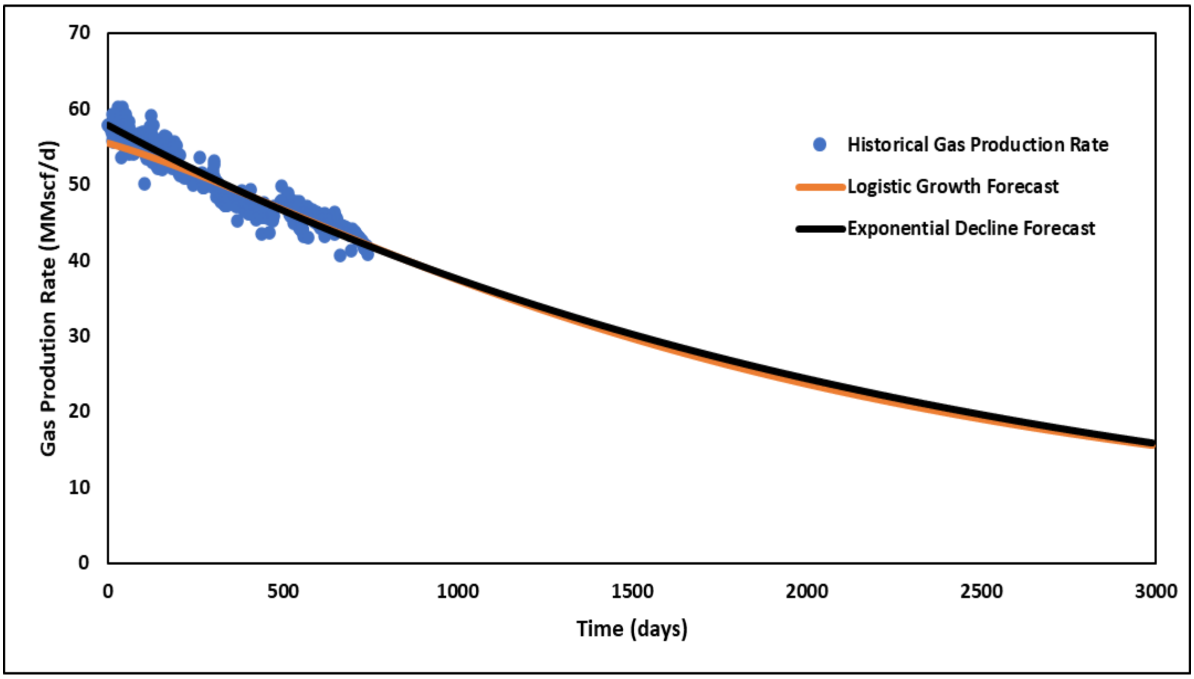

Figure7. *Arps vs. LGM Forecasts*

3.2. Oil Well

In Figure 8, the Arps exponential decline method is utilized to showcase the expected oil production rates over time. The graph also includes historical production rates from the well spanning approximately three years. Throughout this period, as was done with the gas well, the oil well maintained a relatively consistent choke size, while the initial production rate was around 9,300 STB/d. The conditions affecting production remained constant, allowing the application of the Arps method in this case.

Notably, the graph demonstrates a relatively poor match between the projected and historical rates, with a MAPE of 5.58%. The most suitable decline rate for achieving the best fit was determined to be 29.27% per year. Production declines more rapidly than in the gas well. As can be seen, good matches were obtained early in the life of the well. After about 200 days, the model was unable to replicate the well's behaviour, leading to oil rate overestimations. After about 700 days, the model started to underpredict production rates. It is safe to assume that per the historical production trend, future rates will be higher than those predicted by the model. The Arps equation in this case may therefore underestimate the well's EUR.

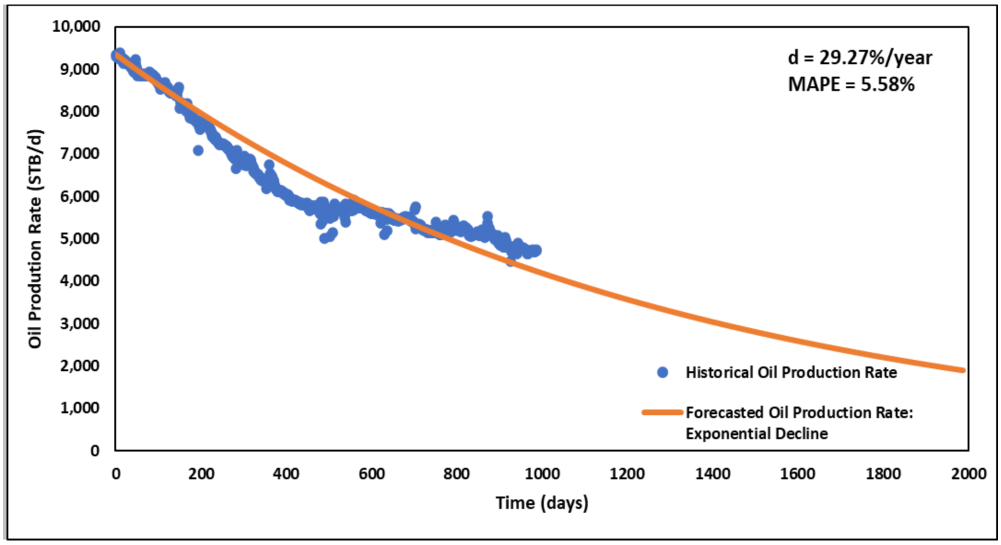

Figure8. *Resulting Fit of Exponential Decline Model with Historical Production Data*

Figure 9 illustrates the projected oil production rates based on the logistic growth model. A carrying capacity of 220 MMSTB of gas was utilized, and there is a high level of confidence associated with this value. Consequently, accurate estimates for the parameters 'a' and 'n' were obtained. Through numerical optimization, the optimal values for 'a' and 'n' were determined to be 206.28 and 0.4045, respectively. These values, in combination, resulted in a relatively well-fitting prediction of production rates, with a mean absolute percentage error of 2.85%. The logistic growth model exhibited much better accuracy compared to the Arps equation. Unlike the Arps model, it was able to predict the latter production rates and better predicted the production rates between days 200 and 600. The EUR with this model is expected to be higher than that of the Arps model. The two methods are compared in Figure 10.

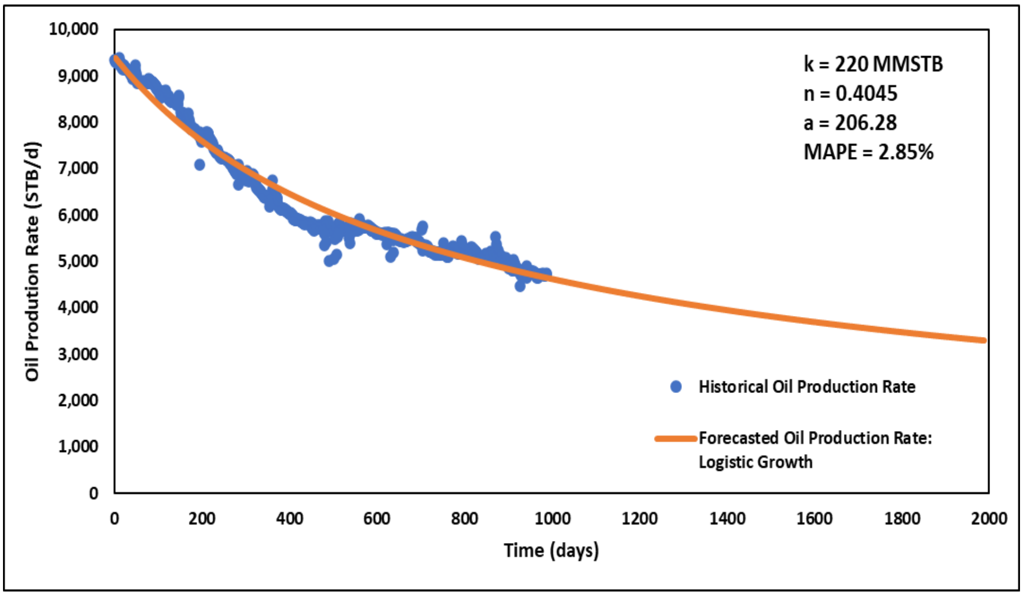

Figure9. *Resulting Fit of LGM with Historical Production Data*

Production rates from the logistic growth model were consistently higher than those of the Arps model for the forecasted period and provided, most likely, better production forecasts. Production rates were projected 8 years into the future. The LGM was able to do this by better matching the tail end of the production data. With the Arps model, the well reaches its economic limit faster and leads to a lower EUR.

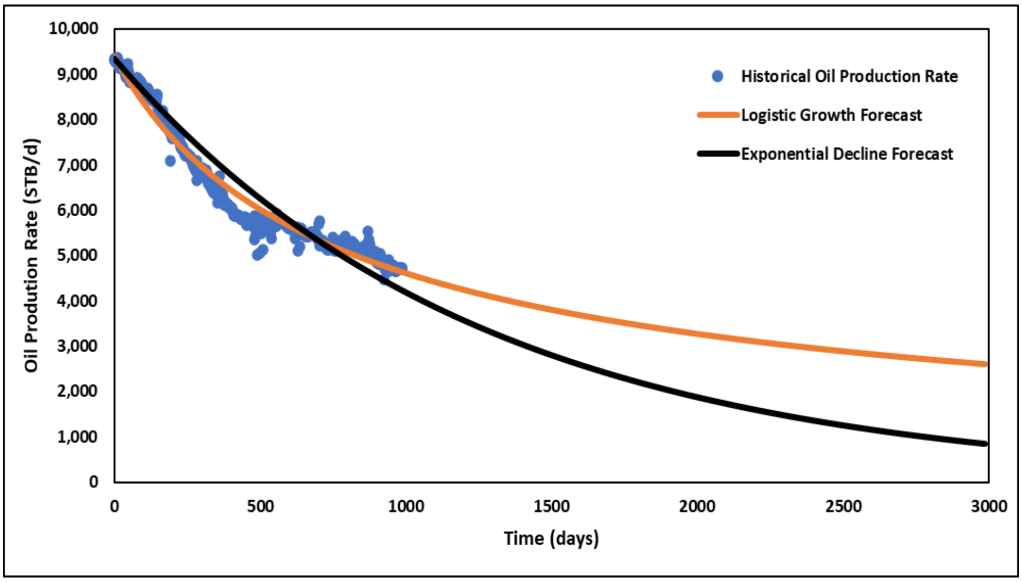

Figure10. *Arps vs. LGM Forecasts*

4. Conclusions

1. In the gas well, both forecast methods provided good fits to the production data, with the LGM providing a slightly better fit. Both models were good enough to predict future well performance.
2. In the oil well, the LGM provided a better match to the production data than the Arps model. It better modelled the latter portion of the well's data trend, which most likely will define the well's future performance.
3. The Arps model was unable to accurately predict the oil well's performance and may lead to underestimations of EUR.
4. It was seen that prior knowledge of the carrying capacity parameter in the logistic growth model greatly simplified the forecast process and led to few uncertainties.
5. The results obtained in this study are only valid for the conditions surrounding the candidate wells. In other wells that are subject to different production constraints, ailed by different problems or have different pressure support mechanisms, results may vary.
6. Overall, the logistic growth model appears to be superior to the Arps exponential decline curve model.
7. All fits will have to be rechecked over time with the addition of new production data. The conclusions drawn here are solely valid for the period that was analyzed.

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