



## The Meta-technology Distance Function and the Gap Ratio of Directional Technology: Case of Tunisian Companies

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**Abstract:** The purpose of this paper is to study the influence of innovation production variables on the technological frontier and to develop an index to measure the productivity of the innovation production system. We first incorporated innovation production variables into the technological directional distance function proposed by Chambers et al. (1996, 1998). Then we decompose the Luenberger productivity indicator and we identify an indicator to measure the productivity of the innovation production system. Finally, we seek to validate the impact of sectoral variables on the nature of the relationship between the production of innovation and the efficient frontier of Tunisian exporting companies.

### 1. INTRODUCTION

Over the past decade, there has been an increase in empirical studies at the aggregate level that explain the efficiency of a company or a country referring to a single border, established for a number of companies belonging to different sectors. However, the judgment on the effectiveness in this case will be biased. The efficiency of a firm might necessarily be related to the specific movements of its industry, such as the size of the industry, the public expenditure on research and development of the industry. These sector-specific movements influence the behavior of the company. The response and sensitivity to these changes differs from company to company.

Therefore, all industry conditions should affect firm efficiency. Berger and De Young (1996) interpret several reasons that cause business inefficiency. One possible reason cited by these authors is the economic slowdown in the sector. Berger, et al (2000) also indicated that the performance of companies is sensitive to economic shocks of sectors. They also explained that business profitability improves during periods of industry growth.

It is a common practice to use borders to assess the level of efficiency of a company, a sector or a country as a guide to analyze its situation and to take part accordingly. his performance. These boundaries are identified using non-parametric or parametric methods that rely on various non-stochastic and stochastic assumptions.

Therefore, once, a frontier is established, the efficiency of each firm is calculated relative to the frontier using efficiency measures that were proposed by Farrell (1957). Consequently, the frontiers are estimated using cross-sectional data on the levels of inputs used and the results obtained by firms. Although the technical efficiency of firms, which is measured against a global frontier, is not normally the same for firms that operate with different technologies due to differences between business sectors. These problems arise when the comparison between companies from different sectors is involved. Different techniques are used to calculate the deviation of a particular company from the global frontier, established for the different sectors. Intra-sector divergence ends up misleading the decision-making process and thus over- or under-reacting. Battese and Rao (2002) and Battese, Rao and O'Donnell (2004) provide theoretical frameworks for solving this problem. While these studies shed light on a viable solution for providing industry comparisons, they did not examine the analytical framework needed to make such comparisons.

The main objective of this paper is to establish a framework for meta-boundaries based on the axioms associated with different sub-boundaries. The concept of meta-boundary used in this section is based on the concept of different sub-boundaries which can be considered as the envelopes of commonly conceived exporting firms belonging to each sector. The meta-boundary represents the envelope of the sub-envelope borders. To render a verdict of the effectiveness of a firm, we use the meta-technology directional distance function (Battese and Rao (2002) and Battese, Rao and O'Donnell (2004)).

The application of this technique aims to encompass the nine sectors studied in the first studied through this article. We use a parametric approach to compare the efficiency of exporting firms in different sectors that operate under different technologies.

## 2. ECONOMETRIC METHODOLOGY

Assuming a sector  $K$  based economy  $k = (1, 2, \dots, K)$  and the exporting firms in each sector operate under sector-specific technology. The definition of all possible input and output pair sets is usually expressed as follows:

$$T^k \equiv \{(x, y) : x \geq 0, y \geq 0; x \text{ can produce } y\} \quad (1)$$

Where, the input vector  $x \in \mathfrak{R}_+^N$ , while  $y \in \mathfrak{R}_+^M$  the output vector for each exporting firm.

Hayami and Ruttan (1971) define the meta-production function as the envelope of commonly designed production functions. Referring to this definition, (Battese and Rao (2002) and Battese, Rao and O'Donnell (2004)) define the concept of meta-technology as an overly encompassing technology, which envelops the technology of each sector. The meta-technology function can be presented as follows:

$$T^* \equiv \{(x, y) : x \in \mathfrak{R}_+^N, y \in \mathfrak{R}_+^M; x \text{ minimum pour produire } y \text{ avec la technologie } T^k\} \quad (2)$$

Meta-technology can also be expressed as:

$$T^* \equiv \text{Convex Hull} \{T^1 \cup T^2 \cup \dots \cup T^K\} \quad (3)$$

T technologies can be fully characterized by the technology directional distance function originally introduced by Chambers et al (1996). This function allows exporting companies to obtain the optimal composition of input and output by simultaneously seeking the maximization of output or the minimization of inputs. It is usually expressed as follows:

$$\bar{D}_{T^k}(x, y; g_x, g_y) = \max \{\beta^k : (x - \beta^k g_x, y + \beta^k g_y) \in T^k\} \quad (4)$$

where  $\beta^k$  gives the distance between the observation  $(x, y)$  and a point on the technological frontier defined for the sector  $k$ ,  $g = (g_x, g_y)$  is a directional vector, with  $g_x \in \mathfrak{R}_+^N$  and  $g_y \in \mathfrak{R}_+^M$  establishes the direction in which technical efficiency is measured. It is generally accepted that  $(g_x, g_y) = (1, 1)$ . In the case where  $\bar{D}(x, y; g_x, g_y) = 0$  then the exporting firm is considered technically efficient. While if  $\bar{D}(x, y; g_x, g_y) > 0$ , the company is assumed to be technically inefficient the company is assumed to be technically inefficient  $T^*$ , We conceptualize the meta-technology directional distance function  $\bar{D}_{T^*}(x, y; g_x, g_y)$  assumed to be an envelope function of the technology distance directional functions of the various sectors and can be expressed as follows.

$$\bar{D}_{T^*}(x, y; g_x, g_y) = \max \{\beta^* : (x - \beta^* g_x, y + \beta^* g_y) \in T^*\} \quad (5)$$

For a determined  $k$  country, and following the definition of meta-technology, we have:

$$\bar{D}_{T^*}(x, y; g_x, g_y) \geq \bar{D}_{T^k}(x, y; g_x, g_y) \quad (6)$$

Färe et al. (2005) opt for a quadratic form to parameterize the technology directional distance function. This shape must satisfy the constraints imposed by the translation property and the symmetry constraint. This function is often expressed as follows:

$$\begin{aligned} \bar{D}(x, y; g_x, g_y, t, \theta) = & \alpha_0 + \sum_{n=1}^N \alpha_n x_n + \sum_{m=1}^M \beta_m y_m + 1/2 \sum_{n=1}^N \sum_{n'=1}^N \alpha_{nn'} x_n x_{n'} + 1/2 \sum_{m=1}^M \sum_{m'=1}^M \beta_{mm'} y_m y_{m'} \\ & + \sum_{n=1}^N \sum_{m=1}^M \gamma_{nm} y_m x_n + \delta_1 t + 1/2 \delta_2 t^2 + \sum_{n=1}^N \psi_n t x_n + \sum_{m=1}^M \eta_m t y_m \end{aligned} \quad (7)$$

To study the influence of the innovation production system on the technological frontier, we incorporate in expression (4) innovation production variables interacting with inputs, outputs and time trend. Let  $I = (I_1, I_2 \dots I_K)$  be the vector of innovation production variables for each firm. Thus, the new technological directional distance function is parameterized as follows:

$$\begin{aligned} \bar{D}(x, y, I; g_x, g_y, t, \theta) = & \alpha_0 + \sum_{n=1}^N \alpha_n x_n + \sum_{m=1}^M \beta_m y_m + 1/2 \sum_{n=1}^N \sum_{n'=1}^N \alpha_{nn'} x_n x_{n'} + 1/2 \sum_{m=1}^M \sum_{m'=1}^M \beta_{mm'} y_m y_{m'} \\ & + \sum_{n=1}^N \sum_{m=1}^M \gamma_{nm} y_m x_n + \sum_{k=1}^K \lambda_k I_k + \sum_{n=1}^N \sum_{k=1}^K \chi_{nk} x_n G_k + \sum_{m=1}^M \sum_{k=1}^K \varphi_{mk} y_m I_k + 1/2 \sum_{k=1}^K \sum_{k'=1}^K \tau_{kk'} I_k I_{k'} \\ & + \delta_1 t + 1/2 \delta_2 t^2 + \sum_{n=1}^N \psi_n t x_n + \sum_{m=1}^M \eta_m t y_m + \sum_{k=1}^K \phi_k t I_k \end{aligned} \quad (8)$$

✓ The symmetry constraints are formulated as follows:

$$\alpha_{nm'} = \alpha_{n'n} \quad n \neq n'$$

$$\beta_{mm'} = \beta_{m'm} \quad m \neq m'$$

$$\tau_{kk'} = \tau_{k'k} \quad k \neq k' \quad (9)$$

✓ The other constraints imposed are:

✓ t :

$$\sum_{m=1}^M \beta_m g_y - \sum_{n=1}^N \alpha_n g_x = -1$$

$$\sum_{m=1}^M \gamma_{nm} g_y - \sum_{n'=1}^N \alpha_{nm'} g_{x'} = 0$$

$$\sum_{m'=1}^M \beta_{mm'} g_{y'} - \sum_{n=1}^N \gamma_{nm} g_x = 0$$

$$\sum_{m=1}^M \varphi_{km} g_{y'} - \sum_{n=1}^N \chi_{kn} g_x = 0$$

$$\sum_{m=1}^M \eta_m - \sum_{n=1}^N \psi_n = 0 \quad (10)$$

Or  $\theta = (\alpha, \beta, \gamma, \lambda, \chi, \varphi, \tau, \delta, \eta, \psi)$  is the vector of the parameters to be estimated

To estimate the parameters of equation (5), we use the stochastic method used by Kumbhakar and Lovell (2000) and Färe et al. (2005). This stochastic specification takes the following form:

$$\bar{D}(x, y, I; g_x, g_y, t, \theta) + \varepsilon^k = 0$$

In the first step, we need to estimate the parameters of the boundary  $\theta^k = (\alpha^k, \beta^k, \gamma^k, \delta^k, \eta^k, \psi^k)$  of each sector using a linear programming procedure proposed by Aigner and Chu (1968). In a second

step, we estimate the parameters of the meta-boundary  $\theta^* = (\alpha^*, \beta^*, \gamma^*, \delta^*, \eta^*, \psi^*)$  for the different sectors. In the third and final step, one can estimate the directional technology deviation rate for each sector.

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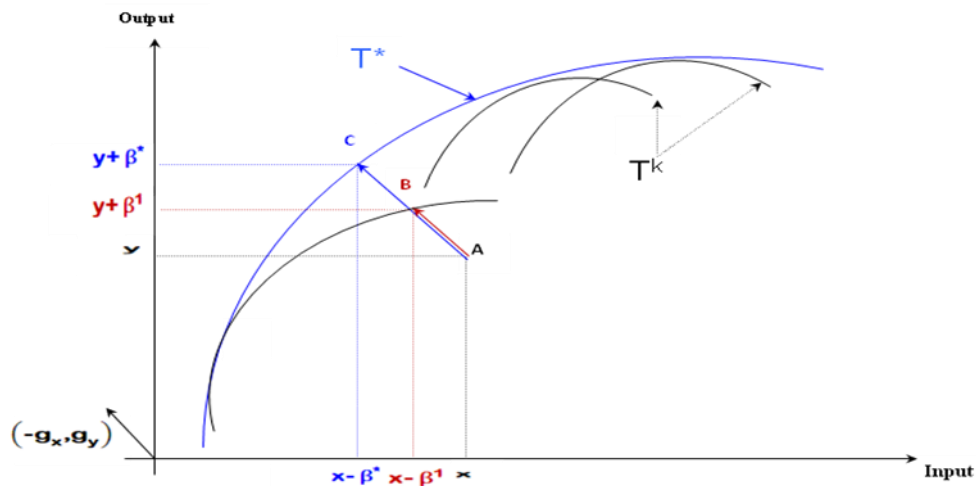


Figure1. The meta-technology directional distance function

### 3. RESULTS AND INTERPRETATIONS

The sectoral variables used are: credit rationing, size of the sector and public spending on research and development. In the different sector variables presented above, there is a great divergence between most sectors. Indeed, we expect that these discrepancies may influence the technology under which companies in each sector operate. In particular, we expect these variables to have a considerable effect in the directional technology gap ratio.

The objective of this paper is to highlight the impact of the divergence of sectoral data on the relationship between the production of innovation and the productivity of exporting companies belonging to various sectors. First of all, we calculated the level of efficiency of exporting companies based on a common frontier by pooling all the data of all exporting companies belonging to the various sectors, so we calculated this level on the different meta-borders specific to each sector. As a result, we obtain two efficiency estimates for each exporting firm, one relating to the meta-frontier and another to the common frontier of the exporting firms. The specifications of the output, input and industry variables were found to be statistically significant for both models (the meta-model and the common boundary model).

As already mentioned before, in the economic literature, common boundaries are generally estimated to control the different technologies inherent in different sectors. However, this approach does not allow us to adequately compare efficiency levels across sectors. On the other hand, the common border approach does not take into account the specific environmental and sectoral conditions of each sector. This approach allows for a good comparison of technical efficiency levels in a national scenario and for determining potential differences in efficiency across the economy. In a second step of our analysis, we address the issue of comparing the efficiency of exporting companies in different sectors. Using the linear programming method, we estimate a meta-frontier for each sector that includes the deterministic components of the individual frontier for exporting firms that operate in different environments and sectoral data and have access to different technologies. . On average, inefficiency scores vary widely between common function and industry-specific levels.

Table1. Estimation of the parameters of common borders and technological meta-borders

Var.	Par	S1	S2	S3	S4	S5	S6	S7	S8	S9	$\overrightarrow{D_{T^*}}$	Previously. Model
C	$\alpha_0$	-0,6715	-0,6957	0,0764	-0,8989	0,0755	0,5381	-0,1529	-0,8855	0,4425	0,6954 (0,0710)	0,0615 (0,0445)

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$x_1$	$\alpha_1$	-0,1442	4.42E-19	-0,1851	-0,1898	-0,0931	-0,0854	-0,0452	-0,1770	0,0000	-0,1238 (0.0093)	0,0206 (0.0048)
$x_2$	$\alpha_2$	-0,1596	7.51E-18	-0,3010	-0,1985	-0,4006	-0,3467	-0,3866	-0,0620	-0,4501	-0,3463 (0.0088)	-0,0784 (0.0046)
$x_3$	$\alpha_3$	-0,1758	-0,4828	-0,0022	-0,1034	-0,0226	-0,1237	-0,0720	-0,2393	-0,0938	-0,0930 (0.0053)	0,5258 (0.0031)
$y_1$	$\beta_1$	0,1384	-0,2701	0,0840	-0,0619	-0,0243	-0,0762	-0,0022	-0,2162	0,0270	0,0891 (0.0063)	-0,0821 (0.0035)
$y_2$	$\beta_2$	-0,2060	0,1138	0,1803	-0,1424	0,0309	0,1066	0,0128	0,1176	-0,0306	-0,0590 (0.0061)	-0,3494 (0.0033)
$y_3$	$\beta_3$	0,5881	0,6736	0,2476	0,7127	0,4771	0,4139	0,4855	0,6203	0,4598	0,4068 (0.0124)	-0,1005 (0.0092)
$x_1^2$	$\alpha_{11}$	0,0032	0,0075	-0,0032	-0,0029	-0,0115	-0,0073	0,0350	0,0137	0,0010	0,0188 (0.0015)	-0,0021 (0.0006)
$x_2^2$	$\alpha_{22}$	-0,0040	0,0061	-0,0049	0,0058	-0,0303	-0,0048	0,0272	0,0197	-0,0177	0,0062 (0.0013)	-0,0013 (0.0005)
$x_3^2$	$\alpha_{33}$	-0,0042	-0,0335	-0,0058	-0,0091	-0,0002	0,0016	0,0170	-0,0379	0,0033	-0,0138 (0.0004)	-0,0952 (0.0002)
$y_1^2$	$\beta_{11}$	-0,0137	-0,0146	-0,0212	-0,0249	-0,0167	-0,0084	0,0075	-0,0210	-0,0016	0,0079 (0.0007)	0,0100 (0.0003)
$y_2^2$	$\beta_{22}$	-0,0231	-0,0099	-0,0169	-0,0110	-0,0114	-0,0107	-0,0354	-0,0317	-0,0126	-0,0351 (0.0006)	-0,0137 (0.0003)
$y_3^2$	$\beta_{33}$	0,0378	0,0292	0,0565	0,0448	0,0781	0,0347	-0,0849	0,0429	0,0235	-0,0011 (0.0054)	-0,0018 (0.0009)
$x_1x_2$	$\alpha_{12}$	-0,0036	0,1150	0,0312	-0,0087	0,0423	0,0065	-0,0350	0,0216	0,0123	0,0014 (0.0012)	0,0088 (0.0005)
$x_1x_3$	$\alpha_{13}$	0,0042	-0,0616	-0,0169	0,0149	-0,0031	0,0046	-0,0382	-0,0128	0,0056	-0,0112 (0.0007)	0,0046 (0.0003)
$x_1y_1$	$\gamma_{11}$	-0,0050	0,0142	-0,0074	-0,0103	-0,0273	-0,0084	0,0563	-0,0367	-0,0090	-0,0084 (0.0008)	-0,0010 (0.0004)
$x_1y_2$	$\gamma_{12}$	-0,0197	-0,0948	-0,0713	-0,0411	-0,0701	-0,0706	-0,0954	-0,0204	-0,0819	-0,0528 (0.0009)	-0,0018 (0.0004)
$x_1y_3$	$\gamma_{13}$	0,0352	-0,1220	0,0383	0,0455	0,0492	0,0668	0,1225	0,0008	0,0806	0,0671 (0.0012)	-0,0059 (0.0006)
$x_2x_3$	$\alpha_{23}$	0,0046	-0,0332	-0,0006	-0,0001	0,0027	-0,0002	-0,0065	-0,0048	-0,0038	0,0003 (0.0007)	0,0858 (0.0003)
$x_2y_1$	$\gamma_{21}$	-0,0130	0,0311	0,0024	0,0138	-0,0007	-0,0019	0,0173	-0,0504	0,0003	-0,0064 (0.0009)	-0,0297 (0.0004)
$x_2y_2$	$\gamma_{22}$	-0,0418	0,0328	-0,0048	-0,0264	-0,0086	-0,0197	-0,0338	0,0316	-0,0020	-0,0141 (0.0008)	-0,0543 (0.0004)
$x_2y_3$	$\gamma_{23}$	0,0435	0,1343	0,0242	0,0095	0,0075	0,0185	0,0454	0,0844	0,0035	0,0445 (0.0016)	-0,0089 (0.0008)
$x_3y_1$	$\gamma_{31}$	0,0041	-0,0090	0,0060	0,0000	-0,0053	-0,0095	-0,0064	0,0174	-0,0002	0,0013 (0.0005)	-0,0036 (0.0003)
$x_3y_2$	$\gamma_{32}$	0,0153	-0,0077	0,0030	-0,0022	0,0019	0,0033	0,0215	-0,0395	-0,0008	0,0019 (0.0005)	0,0769 (0.0003)
$x_3y_3$	$\gamma_{33}$	0,0022	0,0075	0,0077	0,0249	0,0506	0,0344	-0,0798	0,1163	0,0096	-0,0021 (0.0034)	0,0289 (0.0022)
$y_1y_2$	$\beta_{12}$	0,0105	0,0279	0,0371	-0,0045	-0,0001	-0,0182	-0,0159	0,0214	-0,0004	0,0202 (0.0006)	0,0159 (0.0003)
$y_1y_3$	$\beta_{13}$	0,0682	0,0293	0,0195	0,0895	0,0817	0,1113	0,1571	0,0102	0,0966	0,0675 (0.0014)	-0,0039 (0.0005)
$y_2y_3$	$\beta_{23}$	-0,1002	-0,0476	-0,0736	-0,1078	-0,1289	-0,1210	-0,0771	-0,1263	-0,1039	-0,0870 (0.0012)	-0,0058 (0.0005)
$t$	$\delta_1$	-0,0232	-0,0003	0,0160	-0,0051	0,0243	0,0129	-0,0132	0,0095	0,0276	0,0727 (0.0420)	0,0013 (0.0203)
$t^2$	$\delta_2$	-0,0003	0,0027	-0,0008	0,0010	-0,0002	0,0004	0,0048	-0,0005	-0,0013	0,0006 (0.0849)	-0,0006 (0.0338)
$tx_1$	$\psi_1$	0,0009	0,0049	-0,0009	0,0025	0,0007	0,0012	0,0010	-0,0016	-0,0015	0,0031 (0.0052)	-0,0032 (0.0021)
$tx_2$	$\psi_2$	-0,0013	0,0057	-0,0010	0,0010	-0,0010	-0,0015	-0,0043	0,0011	-0,0016	-0,0009 (0.0051)	0,0022 (0.0021)
$tx_3$	$\psi_3$	0,0010	-0,0096	0,0015	-0,0031	0,0001	-0,0002	0,0034	0,0006	0,0026	-0,0038 (0.0030)	0,0009 (0.0014)
$ty_1$	$\eta_1$	-0,0015	0,0043	0,0010	-0,0013	0,0018	-0,0016	0,0002	-0,0056	-0,0016	0,0016 (0.0036)	0,0014 (0.0017)
$ty_2$	$\eta_2$	-0,0008	0,0044	-0,0014	0,0031	0,0023	0,0006	0,0007	0,0084	0,0011	0,0033 (0.0034)	0,0005 (0.0016)
$ty_3$	$\eta_3$	0,0028	-0,0078	0,0000	-0,0014	-0,0044	0,0005	-0,0008	-0,0027	0,0000	-0,0065 (0.0067)	-0,0015 (0.0019)

Table 1 presents the results of the estimation of the parameters of the technological frontier of each sector. The last two columns of this table show the estimation of the meta-boundary and the common

boundary using parametric linear programming. The standard deviations attached to the meta-frontier and common frontier series are obtained by the bootstrap method. We draw randomly with replacement 50 new samples of the same size as the original sample. For each sample of the generated data, the new parameters of the meta-boundaries are estimated by linear programming. Therefore, there are 50 parameter estimates for each coefficient. The estimated standard deviation of a meta-boundary parameter is calculated by the standard deviation of the estimates of the 50 new parameters. However, there are substantial differences between the meta-boundary coefficients and the corresponding common-boundary coefficients. Furthermore, we observe that the majority of the bootstrap standard deviations of the meta-boundary parameters are relatively small compared to the corresponding coefficients of the common border.

Comparing the inefficiency scores, using the directional distance function, we find significant variation between the common boundary and meta-boundary efficiency scores (see Table 1). For example, the inefficiency score of exporting firms belonging to sector 1 decreased from 27.51% in the common frontier model to 10.61% in the meta-frontier. Overall, the scores obtained from the common model seem to underestimate the level of efficiency of the exporting firms in the sample. These results show that the study of the efficiency of innovation production and its impact on the productivity of exporting firms can lead to erroneous results, if they are based on a common frontier for all firms.

**Table2.** Estimated efficiency by sector

	S1	S2	S3	S4	S5	S6	S7	S8	S9
<b>2015</b>									
<b>Model 1</b>	0,2597	0,1734	0,3661	0,2706	0,1635	0,3236	0,2421	0,1385	0,2550
<b>Model 2</b>									
$\bar{D}_{T^k}$	0,0020	0,0006	0,0095	0,0217	0,0027	0,0281	0,0249	0,0029	0,0020
$\bar{D}_{T^*}$	0,1099	0,0797	0,0755	0,1254	0,0880	0,1012	0,1065	0,1080	0,0560
<b>2016</b>									
<b>Model 1</b>	0,3507	0,1697	0,2692	0,3011	0,1629	0,287	0,3397	0,1428	0,1881
<b>Model 2</b>									
$\bar{D}_{T^k}$	0,0013	0,0004	0,0063	0,0215	0,0035	0,0215	0,0214	0,0023	0,0027
$\bar{D}_{T^*}$	0,1283	0,0887	0,0687	0,1255	0,0690	0,1016	0,0967	0,1018	0,0733
<b>2017</b>									
<b>Model 1</b>	0,3449	0,1357	0,2645	0,2813	0,1592	0,2295	0,3429	0,2526	0,2907
<b>Model 2</b>									
$\bar{D}_{T^k}$	0,0025	0,0011	0,0064	0,0220	0,0016	0,0286	0,0195	0,0068	0,0014
$\bar{D}_{T^*}$	0,1268	0,0753	0,0728	0,1200	0,0756	0,1118	0,0974	0,1150	0,0795
<b>2018</b>									
<b>Model 1</b>	0,2539	0,1510	0,2703	0,4095	0,3239	0,3094	0,3808	0,2847	0,2613
<b>Model 2</b>									
$\bar{D}_{T^k}$	0,0011	0,0005	0,0112	0,0223	0,0033	0,0274	0,0268	0,0042	0,0067
$\bar{D}_{T^*}$	0,1272	0,0857	0,0890	0,1489	0,0717	0,1194	0,1004	0,0918	0,0706
<b>2019</b>									
<b>Model 1</b>	0,2242	0,1094	0,2814	0,3326	0,1702	0,2674	0,4327	0,1749	0,3926
<b>Model 2</b>									
$\bar{D}_{T^k}$	0,0024	0,0011	0,0100	0,0183	0,0032	0,0252	0,0232	0,0077	0,0016
$\bar{D}_{T^*}$	0,1294	0,0909	0,0867	0,1105	0,0757	0,1314	0,0960	0,0892	0,0701
<b>2020</b>									
<b>Model 1</b>	0,3073	0,1468	0,2468	0,2723	0,1734	0,2337	0,3918	0,1758	0,3832
<b>Model 2</b>									
$\bar{D}_{T^k}$	0,0009	0,0003	0,0108	0,0210	0,0021	0,0220	0,0258	0,0043	0,0014
$\bar{D}_{T^*}$	0,1341	0,0936	0,0896	0,1011	0,0809	0,1284	0,1001	0,0859	0,0841

<b>15-20</b>									
<b>Model 1</b>	0,2901	0,1477	0,2830	0,3112	0,1922	0,2751	0,3550	0,1949	0,2952
<b>Model 2</b>									
$\bar{D}_{T^k}$	0,0017	0,0007	0,0090	0,0211	0,0027	0,0255	0,0236	0,0047	0,0026
$\bar{D}_T^*$	0,1260	0,0856	0,0804	0,1322	0,0755	0,1061	0,1080	0,0986	0,0723

In the common frontier model, the chemical industries sector is the most efficient sector compared to the other sectors in the sample. On the other hand, in the case of a meta-frontier model, the agro-food industries sector is the most efficient sector with respect to the other sectors.

From Table 2, we see a considerable discrepancy between the mean values of directional technology error rates across countries. From this table, we observe during our survey period that the lowest value of this ratio 0.0082 attributed to the mechanical and metallurgical industries sector. The largest value of the directional technology gap index is 0.2403 attributed to the agro-food industries sector.

These results lead to the conclusion that the specific technological frontier of the mechanical and metallurgical industries sector is furthest from the meta-frontier and as a consequence of the technology under which the exporting companies of this sector operate. This technology is less developed referring to meta-frontier technology with respect to other sectors. On the other hand, the specific technological frontier of the agro-food industries sector is closer to the meta-frontier technology. Indeed, the technology under which exporting companies in this sector operate is more developed.

**Table3.** Directional Technology Spread Ratio by Sector

	S1	S2	S3	S4	S5	S6	S7	S8	S9
<b>2015</b>									
$DTE^k$	0,0020	0,0006	0,0095	0,0217	0,0027	0,0281	0,0249	0,0029	0,0020
$DTE^*$	0,1099	0,0797	0,0755	0,1254	0,0880	0,1012	0,1065	0,1080	0,0560
$DTGR^k$	0,0182	0,0075	0,1258	0,1731	0,0307	0,2777	0,2338	0,0269	0,0357
<b>2016</b>									
$DTE^k$	0,0013	0,0004	0,0063	0,0215	0,0035	0,0215	0,0214	0,0023	0,0027
$DTE^*$	0,1283	0,0887	0,0687	0,1255	0,0690	0,1016	0,0967	0,1018	0,0733
$DTGR^k$	0,0101	0,0045	0,0917	0,1713	0,0507	0,2116	0,2213	0,0226	0,0368
<b>2017</b>									
$DTE^k$	0,0025	0,0011	0,0064	0,0220	0,0016	0,0286	0,0195	0,0068	0,0014
$DTE^*$	0,1268	0,0753	0,0728	0,1200	0,0756	0,1118	0,0974	0,1150	0,0795
$DTGR^k$	0,0197	0,0146	0,0879	0,1834	0,0212	0,2558	0,2002	0,0591	0,0176
<b>2018</b>									
$DTE^k$	0,0011	0,0005	0,0112	0,0223	0,0033	0,0274	0,0268	0,0042	0,0067
$DTE^*$	0,1272	0,0857	0,0890	0,1489	0,0717	0,1194	0,1004	0,0918	0,0706
$DTGR^k$	0,0086	0,0058	0,1258	0,1498	0,0460	0,2295	0,2670	0,0458	0,0949
<b>2019</b>									
$DTE^k$	0,0024	0,0011	0,0100	0,0183	0,0032	0,0252	0,0232	0,0077	0,0016
$DTE^*$	0,1294	0,0909	0,0867	0,1105	0,0757	0,1314	0,0960	0,0892	0,0701
$DTGR^k$	0,0185	0,0121	0,1153	0,1656	0,0423	0,1918	0,2417	0,0863	0,0228
<b>2020</b>									
$DTE^k$	0,0009	0,0003	0,0108	0,0210	0,0021	0,0220	0,0258	0,0043	0,0014
$DTE^*$	0,1341	0,0936	0,0896	0,1011	0,0809	0,1284	0,1001	0,0859	0,0841

$DTGR^k$	0,0067	0,0032	0,1205	0,2077	0,0260	0,1714	0,2577	0,0501	0,0166
<b>15-20</b>									
$DTE^k$	0,0017	0,0007	0,0090	0,0211	0,0027	0,0255	0,0236	0,0047	0,0026
$DTE^*$	0,1260	0,0856	0,0804	0,1322	0,0755	0,1061	0,1080	0,0986	0,0723
$DTGR^k$	0,0135	0,0082	0,1119	0,1596	0,0358	0,2403	0,2185	0,0477	0,0360

We also demonstrate empirically the influence of certain sector indicators in the value of this report.

As presented above in the previous section, we model the directional technology gap ratio as a linear function of industry variables to demonstrate the significant effect of industry divergences between industries on the value of the technology gap index. directional technology.

**Table4.** Sector Effect on Directional Technology Gap Ratio

variables	Coefficients	t-Stat	Prob
<b>C</b>	123330.5	2,0853	0,0435
<b>Z1</b>	-3,0341	-2,3505	0,0238
<b>Z2</b>	2,8420	1,3857	0,1735
<b>Z3</b>	4,6149	3,4468	0,0013
<b>R<sup>2</sup></b>	0,7864		
<b>Prob.</b>	0.000000		

Following the results presented in Table 4, we show the existence of a significant effect of credit rationing associated with a negative sign. The size of the sector and the public expenditure on research and development show a positive sign, respectively at the level of 1% and 5%. Furthermore, the R-squared has a value of 0.7864 which indicates that the industry variables we use in our regression can explain 78.64% of the directional technology gap index. Indeed, the technological frontier under which the exporting companies of each sector operate is influenced by the monetary and budgetary policies and the environmental characteristics of each sector.

#### 4. CONCLUSION

Despite the conformity of our results, we note that the divergences in the development between the sectors of activity, are a reality that we cannot hide.

Thus, we consider that each sector has its economic specificities. These factors influence the development of the industry and the production of innovation of each sector. In fact, the technology under which exporting companies in each sector operate is not the same. For this reason, we sought to highlight the variation in the efficiency of innovation production taking into account environmental specifications and sectoral variables in which Tunisian exporting companies operate.

To do this, it is necessary to take into consideration the technological frontier specific to each sector. From the different technological frontiers, we build a technological frontier that envelops all the meta frontiers.

Next, we assessed the directional technology gap ratio and estimated the key industry factors that can influence this ratio. As a result, first, we find a significant discrepancy between the results of using the meta-frontier technology and the common-frontier technology to estimate the efficiency of exporting firms in each industry. Second, the directional technology gap ratio allows us to determine the most developed sector in the production of innovation. This sector is the one that presents a technological frontier closer to the meta-frontier. Finally, the regression of the directional technology gap index on sectoral indicators shows that the latter have a significant influence on the production of innovation and subsequently on the efficiency of Tunisian exporting companies.

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