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Comparison of Mathematical Approximation Methods for Mine Ventilation Network Analysis

Farhang Sereshki
Associate Professor
Faculty of Mining Engineering
Petroleum and Geophysics
Shahrood University of Technology
Shahrood, Iran
farhang@shahroodut.ac.ir

Amir Saffari
Ph.D Candidate
Faculty of Mining Engineering
Petroleum and Geophysics
Shahrood University of Technology
Shahrood, Iran
amirsaffari5710@yahoo.com

Ebrahim Elahi
Faculty of Engineering
Department of Mining Engineering
University of Sistan & Baluchestan, Zahedan, Iran
elahi@eng.usb.ac.ir

Abstract: *Various manual and computerized methods for analysis of mine ventilation network are presented. The selection of a suitable method depends on the target of network analysis. If the purpose of analysis is investigation of the effect of fans on mine network, using computerized methods based on mathematical approximation methods is better. The most notable feature has been the development of computerised mine ventilation network analysis. This has made prediction of fan requirements and airflow distributions in complex mine ventilation circuits feasible using desktop computers. A number of methods of mathematical approximations are presented such as the Hardy Cross method and its modified versions, Newton-Raphson technique, critical path, linear analysis, non-linear programming and optimization techniques. Using of Hardy Cross method in ventilation software has become common. In recent years, the use of Newton-Raphson technique in mine ventilation network analysis has been growing. Accordingly, in this paper, a model is developed for faster convergence to the final solution based on the Hardy Cross method that is named ''conflation model of Hardy Cross method''. Also, a more precise use of the Newton-Raphson technique in mine ventilation network analysis is also performed.*

Keywords: *Network Analysis, Mines Ventilation, Fans, optimization, Mathematical Approximations Methods.*

1. INTRODUCTION

The basic requirement for the mine ventilation system is to provide air for people to breath and in a condition that will not cause any immediate or future ill effects. Because of the processes of mining, if positive airflow through the workings was not provided the air would very quickly become stale, contaminated and unfit for human consumption. The ventilation system must therefore be sufficient to deal with the contaminants released during mining. If they are not adequately dealt with as they are identified, they may become at best a discomfort to mine workers, and at worst cause serious or even fatal illness.

Although the mine environment by tunnel, winze and shaft is connected to the surface, this connection alone is not adequate for providing good airflow in the networks. It is necessary to ensure the air is clean by using various instruments and artificial methods. Accordingly, for dilution of harmful gases in the mine ventilation networks requires knowledge of the natural and artificial ventilation circuits (Elahi, 2014; Madani, 2006).

Ventilation design methods in underground mines are based on principles such as mapping, identification of ventilation branches and nodes, calculation of ventilation resistance of the each branches, the airflow intensity of the each branch, modified flow intensity, pressure loss for each branch, natural ventilation, network regularization and finally selection of the regulator doors or underground fans, along with selection main fan.

Manual and computerized methods for the analysis of mine ventilation network are presented. The selection of the type of method depends on the target of network analysis. If the branch flow intensity and direction in a mine network have been fixed and determined in this state, using of manual methods for the analysis of the network is better.

The purpose of the mine ventilation network analysis in manual methods is only for the selection of main and auxiliary fans and the air regulators. But if the purpose of mine ventilation network analysis is the investigation of the effect of one or more fans on a mine network, use of mathematical approximation methods is better, because the manual method cannot calculate flow intensity in each of branches in complex networks.

The mathematical approximation methods begin with an assumption of flow intensity and direction in each of the branches of the mine network. The calculation error for each loop is determined using mathematical approximation equations, and then the assumed airflow intensity will be corrected. Airflow intensity corrections based on the provided mathematical equation continue to be conducted until calculation error is less than or equal to a required accuracy. Use of mathematical approximation methods for solving huge and complex networks by human is impossible; therefore utilisation of computers is necessary to solve them. A number of computer software's is available for analysis of mines ventilation networks. One of the most popular of them is Ventsim software that is based on Hardy Cross equation (Elahi, 2014; Madani, 2003). The first mathematical equations for estimating the flow intensity error in each loop, was presented by Hardy Cross in 1936. Although this equation was proposed for the analysis of water networks, this equation was later used and improved for mine ventilation network analysis by Wang (1982a) and El-Nagdy (2008). In addition to the mentioned methods, other techniques such as, Newton-Raphson technique (Madani & Maleki, 2007; Wang, 1989), critical path (Wang, 1982b), linear analysis (Bhamidipati & Procarione, 1985; Kamba et al., 1995), non-linear programming (Hu & Longson, 1990; Wang, 1984) and optimization techniques (Collins, 1978) are also presented.

2. A REVIEW OF APPROXIMATE MATHEMATICS METHODS

2.1. Hardy- Cross Method

Hardy Cross's famous equation is as follows and according to equation 1, can be used to estimate the flow intensity error in each loop (Cross, 1936; Elahi, 2014).

$$\begin{cases} Q = Q_0 + \Delta \\ \Delta P = R\,Q^n \end{cases} \rightarrow R\,Q^n = R\,(Q_0 + \Delta)^n \rightarrow$$

$$R\,Q^n = R\,(Q_0{}^n + n\,\Delta\,Q_0{}^{n-1} + \cdots) \cong R\,Q_0{}^n + n\,R\,\Delta\,Q_0{}^{n-1}$$

$$\rightarrow \sum R\,Q^n = \sum R\,Q_0{}^n + \Delta \sum n\,R\,Q_0{}^{n-1}$$

$$\rightarrow \begin{cases} \sum R\,Q^n = 0 \\ \sum R\,Q_0{}^n = -\Delta \sum n\,R\,Q_0{}^{n-1} \end{cases}$$

$$\rightarrow \Delta = -\frac{\sum R\,Q_0{}^n}{\sum n\,R\,Q_0{}^{n-1}} \xrightarrow{n=2} \Delta = -\frac{\sum R\,Q_0{}^2}{2\sum R\,Q_0}$$

$$\rightarrow \Delta = -\frac{\sum \pm \Delta P_i}{2\sum R_i\,Q_i} = -\frac{\sum \pm R_i\,Q_i{}^2}{2\sum R_i\,Q_i} \tag{1}$$

Where:

Q : The actual flow intensity (m^3/sec)

Q_0 : The primary or assumed flow intensity (m^3/sec)

Δ : The flow intensity error in loop (m^3/sec)

R : The mining work resistance in each branch (K Murgue)

ΔP : The pressure loss for each branch (mm water)

The Hardy- Cross equation solution steps are as follows:

2.1.1. First Stage

According to the junction law of equal flow into and out of every junction, assumed flow intensity with assumed direction for each of the branches of the mine ventilation network is established.

2.1.2. Second Stage

Identify closed loops in mine ventilation network according to equation (2) and select an assumed direction for them.

$$N_R = N_B - N_J + 1 \tag{2}$$

Where:

N_R : Number of useful loops (with a surface node)

N_B : Number of branches in mine ventilation network

N_J : Number of junctions in mine ventilation network

2.1.3. Third Stage

Calculation of air pressure loss for each of branches in loop according to equation (3):

$$\Delta P = R\,Q^2 \tag{3}$$

2.1.4. Fourth Stage

The calculation of flow intensity error for each loop according to equation (1): It should be noted for the numerator part of the equation, if the airflow direction in branch was aligned with the loop flow direction, the pressure loss is a positive sign, otherwise it is negative. For the denominator part of the equation it is always summed as a positive sign.

2.1.5. Fifth Stage

The calculation of the new flow for each branch of mine ventilation network: The error correction for each branch is determined by the algebraic summation of flow intensity error corrections of the loops contained the branch. It should be noted, if the loop direction was aligned with the branch flow direction, the error is applied as position; otherwise it is subtracted from the assumed flow. Also, if the value of new flow intensity in the branch is negative, the airflow direction in the branch is reversed.

2.1.6. Sixth Stage

Repeat the above operations from third to fifth stages until the calculation error is less than or equal to the required accuracy.

2.2. Wang Modified Method

Wang's correction is relating to fourth stage of the Hardy-Cross method. At this stage, the amount of flow intensity error of each loop is estimated according to equation (4). The purpose of this modification is that it eliminates the positive or negative sign of the pressure loss in equation (1), and also removes the decision to change the airflow direction if a flow is negative. Accordingly, if the flow direction changes, the amount of flow intensity changes to a positive sign, otherwise a negative sign should be appear in equation (4). In fact, the equations (1) and (4) for the flow intensity with positive value are quite similar.

$$\Delta = -\frac{\sum_{i=1}^{i=n} b_{ki}\,(R_i \left|Q_i\right| Q_i - P_{ni} - P_{Fi})}{2\sum_{1}^{i=n} b_{ki}^2\, R_i \left|Q_i\right|} \tag{4}$$

Where:

b_{ki} : Fundamental matrix element of the loop. If the airflow direction in branch was aligned with the loop flow direction, in this case, this element is equivalent to 1 in otherwise equivalent to -1 which appearance in equation (4).

P_{ni} : Increasing pressure by reason of natural ventilation in the branch.

P_{Fi} : Increasing pressure by reason of installed fan in the branch.

2.3. Newton– Raphson technique

One of methods to solving numerical computation is using the Newton- Raphson technique. This technique is based on the definition of the derivative and the correction of it. In this technique, the initial guess of error value for the solution of the equation is estimated and then iteratively corrected. The mathematical equation can be expressed as follows:

$$f(x) = 0 \rightarrow f'(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Where:

x_1 : Initial guess.

x_2 : Answer of the next step.

$f(x_1)$: The value of the function on the basis of initial guess.

$f(x_2)$: The value of the function on the basis of finally answer (is equivalent to 0).

$f'(x_1)$: The value of the derivative of function.

This technique for the first time was presented for the analysis of mine ventilation networks by Wang and Li in 1985. Also, this technique was used by Madani and Maleki in 2008 for $\Delta Q, H = \Delta P$ equations analysis in mine ventilation networks. Based on $\Delta Q, \Delta P = H$ equations analysis in mine ventilation networks by using Newton-Raphson technique is expressed as follows:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \rightarrow \begin{cases} \Delta Q_{n+1} = \Delta Q_n - \dfrac{f(H_n)}{\dfrac{\partial f_n}{\partial \Delta Q}} \\ \\ H_{n+1} = H_n - \dfrac{f(H_n)}{\dfrac{\partial f_n}{\partial H}} \end{cases}$$

$$\begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ . \\ . \\ . \\ \Delta Q_L \end{bmatrix}^{n+1} = \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ . \\ . \\ . \\ \Delta Q_L \end{bmatrix}^{n} - \begin{bmatrix} \frac{\partial f_1}{\partial \Delta Q_1} & \frac{\partial f_1}{\partial \Delta Q_2} \cdots\cdots & \frac{\partial f_1}{\partial \Delta Q_L} \\ \frac{\partial f_2}{\partial \Delta Q_1} & \frac{\partial f_2}{\partial \Delta Q_2} \cdots\cdots & \frac{\partial f_2}{\partial \Delta Q_L} \\ . & . & . \\ . & . & . \\ . & . & . \\ \frac{\partial f_1}{\partial \Delta Q_L} & \frac{\partial f_1}{\partial \Delta Q_L} \cdots\cdots & \frac{\partial f_1}{\partial \Delta Q_L} \end{bmatrix}^{-1} \times \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ . \\ . \\ . \\ f_L \end{bmatrix}^{n} = \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ . \\ . \\ . \\ \Delta Q_L \end{bmatrix}^{n} - \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ . \\ . \\ . \\ Z_L \end{bmatrix}^{n}$$

$$\rightarrow \begin{bmatrix} \frac{\partial f_1}{\partial \Delta Q_1} & \frac{\partial f_1}{\partial \Delta Q_2} \cdots\cdots & \frac{\partial f_1}{\partial \Delta Q_L} \\ \frac{\partial f_2}{\partial \Delta Q_1} & \frac{\partial f_2}{\partial \Delta Q_2} \cdots\cdots & \frac{\partial f_2}{\partial \Delta Q_L} \\ . & . & . \\ . & . & . \\ . & . & . \\ \frac{\partial f_1}{\partial \Delta Q_L} & \frac{\partial f_1}{\partial \Delta Q_L} \cdots\cdots & \frac{\partial f_1}{\partial \Delta Q_L} \end{bmatrix} \times \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ . \\ . \\ . \\ Z_L \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ . \\ . \\ . \\ f_L \end{bmatrix} \rightarrow \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ . \\ . \\ . \\ \Delta Q_L \end{bmatrix}^{n+1} = \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ . \\ . \\ . \\ \Delta Q_L \end{bmatrix}^{n} - \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ . \\ . \\ . \\ Z_L \end{bmatrix}^{n}$$

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ . \\ . \\ . \\ H_L \end{bmatrix}^{n+1} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ . \\ . \\ . \\ H_L \end{bmatrix}^{n} - \begin{bmatrix} \frac{\partial f_1}{\partial H_1} & \frac{\partial f_1}{\partial H_2} \cdots\cdots & \frac{\partial f_1}{\partial H_L} \\ \frac{\partial f_2}{\partial H_1} & \frac{\partial f_2}{\partial H_2} \cdots\cdots & \frac{\partial f_2}{\partial H_L} \\ . & . & . \\ . & . & . \\ . & . & . \\ \frac{\partial f_1}{\partial H_L} & \frac{\partial f_1}{\partial H_L} \cdots\cdots & \frac{\partial f_1}{\partial H_L} \end{bmatrix}^{-1} \times \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ . \\ . \\ . \\ f_L \end{bmatrix}^{n} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ . \\ . \\ . \\ H_L \end{bmatrix}^{n} - \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ . \\ . \\ . \\ Z_L \end{bmatrix}^{n}$$

$$\rightarrow \begin{bmatrix} \frac{\partial f_1}{\partial H_1} & \frac{\partial f_1}{\partial H_2} \cdots\cdots & \frac{\partial f_1}{\partial H_L} \\ \frac{\partial f_2}{\partial H_1} & \frac{\partial f_2}{\partial H_2} \cdots\cdots & \frac{\partial f_2}{\partial H_L} \\ . & . & . \\ . & . & . \\ . & . & . \\ \frac{\partial f_1}{\partial H_L} & \frac{\partial f_1}{\partial H_L} \cdots\cdots & \frac{\partial f_1}{\partial H_L} \end{bmatrix} \times \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ . \\ . \\ . \\ Z_L \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ . \\ . \\ . \\ f_L \end{bmatrix} \rightarrow \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ . \\ . \\ . \\ H_L \end{bmatrix}^{n+1} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ . \\ . \\ . \\ H_L \end{bmatrix}^{n} - \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ . \\ . \\ . \\ Z_L \end{bmatrix}^{n}$$

The Newton-Raphson technique solution steps based on ΔQ equation are as follows:

2.3.1. First stage

According to branches' law, assumption flow intensity with assumption direction in each of the branches of the mine ventilation network is considered.

2.3.2. Second stage:

Identification of useful loops in mine ventilation network according to equation (2) and select an assumed direction for them.

2.3.3. Third stage:

Calculation of ΔQ equation based on air pressure loss for each branch in the network ($\Delta P = R\,Q^2$).

2.3.4. Fourth stage:

Repeat the above operations from third stage until calculation error is less than or equal to the required accuracy.

3. CONFLATION MODEL OF HARDY- CROSS METHOD

This correction is relating to fourth and fifth stages of the Hardy-Cross method. In the primary and modified methods, these two stages were calculated separately. Meaning that after completing the fourth stage calculations for all of the loop errors, the fifth stage of the operation to apply the loops errors to the branches begins. But if these two stages are applied to each loop together, the influence of each on each loop correction can be considered on others, and the investigation has led to more rapid access to the final answer. In other words, after the first loop error calculation is done, the relevant branches are corrected for flow intensity and only then are the second loop calculation error performed. Therefore, solving of fourth and fifth stages step by step together of Hardy-Cross method the investigation has led to more rapid access to the final answer. To prove this claim, consider the following two models.

3.1. First Model

An example mine's ventilation network, according to Figure 1 is considered by a fan with a pressure of 100 (mm water). The mining resistance and flow intensity of each branch in this network is shown in Table 1. Initially, this network was simulated with Ventsim software that so far is the most complete mine ventilation network analysis software known, and then the results are compared with manual method to solving the Hardy- Cross equations, the Wang modified method and the conflation model. The result of simulation with Ventsim software for flow intensity distribution of each branch in the mentioned network is shown in Figure 2.

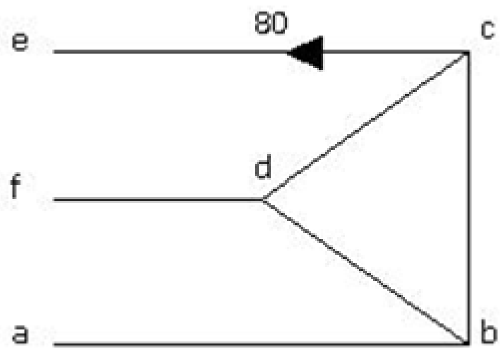

Figure1. *Mine ventilation hypothetical network*

Table1. *Mine ventilation hypothetical network properties*

Branch	Mining resistance (k Murgue)	Flow intensity (m^3/sec)
ab	0.15	14
bd	0.14	4
df	0.11	16
bc	0.12	18
dc	0.13	12
ce	0.10	30

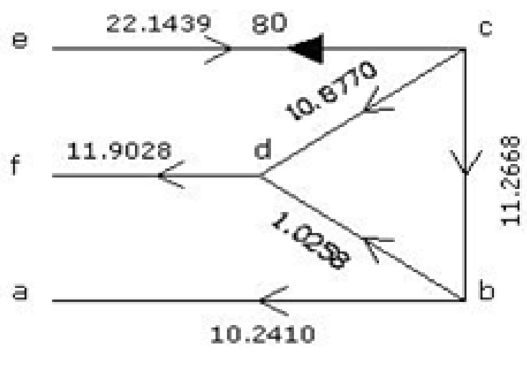

Figure2. *Distribution of flow intensity with Ventsim software*

For analysis of flow intensity of each branches by Hardy- Cross method, requires to assumption flow intensity. The assumed flows intensity with required loops selection is shown in Figure 3.

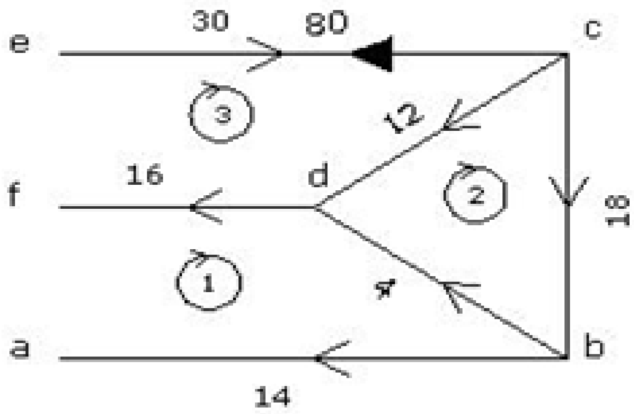

Figure3. *Hypothetical flows intensity with loop selection*

The first stage of calculation of Hardy- Cross method and Wang modified method (change the airflow direction where flow intensity has a negative value) is according to equation (1) and (2) which is as follows. Results of other stages are shown in Table 2.

$$\Delta_1 = -\frac{-0.11\times16^2 - 0.14\times4^2 + 0.15\times14^2}{2\,(0.11\times16 + 0.14\times4 + 0.15\times14)} = 0.1131$$

$$\Delta_2 = -\frac{0.14\times4^2 - 0.13\times12^2 + 0.12\times18^2}{2\,(0.14\times4 + 0.13\times12 + 0.12\times18)} = -2.6168$$

$$\Delta_3 = -\frac{0.1\times30^2 + 0.13\times12^2 + 0.11\times16^2 - 80}{2\,(0.1\times30 + 0.13\times12 + 0.11\times16)} = -4.5$$

$$Q_{ab} = Q_{ab} + \Delta_1 = 14 + 0.1131 = 14.1131$$

$$Q_{bd} = Q_{bd} - \Delta_1 + \Delta_2 = 4 - 0.1131 + (-2.6168) = 1.2701$$

$$Q_{df} = Q_{df} - \Delta_1 + \Delta_3 = 16 - 0.1131 + (-4.5) = 11.3869$$

$$Q_{bc} = Q_{bc} + \Delta_2 = 18 + (-2.6168) = 15.3832$$

$$Q_{dc} = Q_{dc} - \Delta_2 + \Delta_3 = 12 - (-2.6168) + (-4.5) = 10.1168$$

$$Q_{ce} = Q_{ce} + \Delta_3 = 30 + (-4.5) = 25.5$$

According to Table 2 and after 23 iterations, the method reached conclusion with the results obtained from Ventsim software being quite similar. Therefore, the Wang modified method in this model is not able to converge faster to reach the final solution.

Table2. *The results of Hardy Cross Method and its corrected stages*

Stage	Q_{ab}	Q_{bd}	Q_{bc}	Q_{ce}	Q_{df}	Q_{dc}
1	14.1131	1.2701	15.3832	25.5	11.3869	10.1168
2	11.9441	1.1454	13.0895	24.2697	12.3256	11.1802
3	11.2632	1.1206	12.3838	23.1369	11.8737	10.7531
4	10.7327	1.0679	11.8006	22.7312	11.9985	10.9306
5	10.5244	1.0522	11.5766	22.4281	11.9038	10.8516
10	10.2522	1.0268	11.2790	22.1567	11.9045	10.8776
15	10.2415	1.0258	11.2674	22.1444	11.9029	10.8770
20	10.2411	1.0258	11.2669	22.1439	11.9028	10.8770
23	10.2410	1.0258	11.2668	22.1439	11.9028	10.8770

The first stage of calculation of conflation model of Hardy-Cross method is as follows and results of other stages are shown in Table 3.

$$\Delta_1 = -\frac{-0.11\times16^2 - 0.14\times4^2 + 0.15\times14^2}{2\,(0.11\times16 + 0.14\times4 + 0.15\times14)} = 0.1131$$

$$Q_{ab} = Q_{ab} + \Delta_1 = 14 + 0.1131 = 14.1131$$

$$Q_{bd} = Q_{bd} - \Delta_1 = 4 - 0.1131 = 3.8869$$

$$Q_{df} = Q_{df} - \Delta_1 = 16 - 0.1131 = 15.8869$$

$$\Delta_2 = -\frac{0.14\times3.8869^2 - 0.13\times12^2 + 0.12\times18^2}{2\,(0.14\times3.8869 + 0.13\times12 + 0.12\times18)} = -2.6119$$

$$Q_{bd} = Q_{bd} + \Delta_2 = 3.8869 + (-2.6119) = 1.2750$$

$$Q_{dc} = Q_{dc} - \Delta_2 = 12 - (-2.6119) = 14.6119$$

$$Q_{bc} = Q_{bc} + \Delta_2 = 18 + (-2.6119) = 15.3881$$

$$\Delta_3 = -\frac{0.13\times14.6119^2 + 0.11\times15.8869^2 + 0.1\times30^2 - 80}{2\,(0.13\times14.6119 + 0.11\times15.8869 + 0.1\times30)} = -4.9284$$

$$Q_{df} = Q_{df} + \Delta_3 = 15.8869 + (-4.9284) = 10.9585$$

$$Q_{dc} = Q_{dc} + \Delta_3 = 14.6119 + (-4.9284) = 9.6835$$

$$Q_{ce} = Q_{ce} + \Delta_3 = 30 + (-4.9284) = 25.0716$$

According to Table 3 and after 12 times iterations doing the method reached the conclusion with the results obtained from the Ventsim software being quite similar. Therefore, the conflation model of Hardy-Cross method is able to converge faster to reach the solution the final answer.

Table3. *The results of conflation model of Hardy- Cross Method stages*

Stage	Q_{ab}	Q_{bd}	Q_{bc}	Q_{ce}	Q_{df}	Q_{dc}
1	14.1131	1.2750	15.3881	25.0716	10.9585	9.6835
2	11.7652	1.1230	12.8882	23.1244	11.3592	10.2363
3	10.7573	1.0371	11.7944	22.4456	11.6883	10.6512
4	10.3994	1.0249	11.4243	22.2323	11.8330	10.8080
5	10.2874	1.0250	11.3124	22.1693	11.8819	10.8570
10	10.2411	1.0258	11.2669	22.1439	11.9028	10.8770
12	10.2410	1.0258	11.2668	22.1439	11.9028	10.8770

3.2. Second Model

An assumed mine's ventilation network, according to Figure 4 is considered with two fans with the 100 and 80 (mm water) pressures respectively. The mining resistance and flow intensity of each branch in this network is shown in Table 1. Initially, this network was simulated with Ventsim software, and then the results are compared with the manual method of solving the Hardy Cross equations, the Wang modified method and the conflation model. The result of simulation with Ventsim software for flow intensity distribution of each branch is shown in Figure 5.

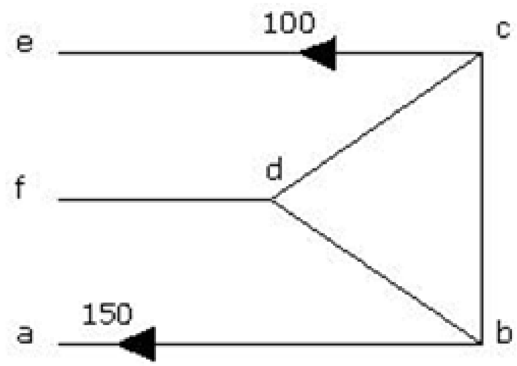

Figure4. *Mine ventilation hypothetical network*

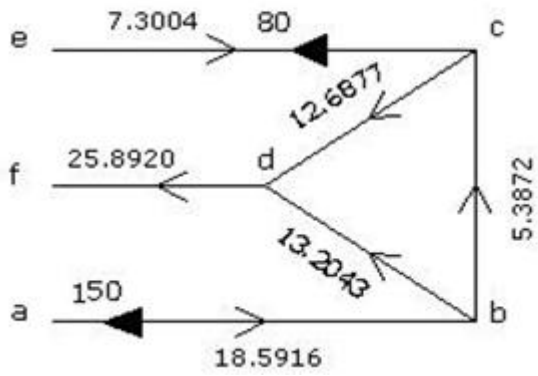

Figure5. *Distribution of flow intensity with Ventsim software*

For analysis of flow intensity of each branches by Hardy- Cross method, requires to assumption flow intensity. These assumption flows intensity with required loops selection is shown in Figure 6.

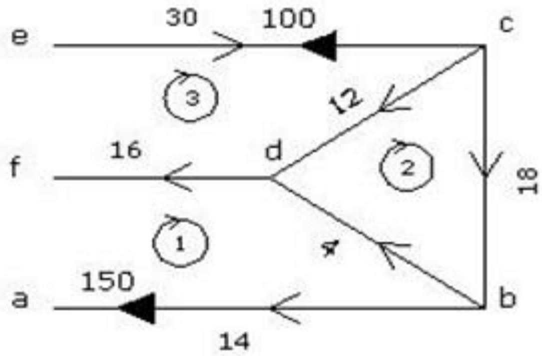

Figure6. *Hypothetical flows intensity with loop selection*

The first stage calculation of the Hardy- Cross method and the Wang modified method (change the airflow direction where flow intensity is a negative value) is according to Table 4. After 54 times iterations the results compared with the Ventsim software are quite similar. Therefore, the Wang modified method in this model is not able to converge faster to reach the final answer solution.

Table4. *The results of Hardy Cross Method and its corrected stages*

Stage	Q_{ab}	Q_{bd}	Q_{bc}	Q_{ce}	Q_{df}	Q_{dc}
1	-2.8552	18.2384	15.3832	27.0823	29.9375	11.6991
2	3.1437	13.6982	10.5545	21.1182	24.2619	10.5637
3	8.8274	16.6249	7.7975	19.1800	28.0074	11.3825
4	9.8175	14.5423	4.7248	16.0994	25.9169	11.3746
7	14.6612	14.2574	-0.4037	11.9361	26.5973	12.3399
10	16.7412	13.6145	3.1267	9.4873	26.2285	12.6140
20	18.4633	13.2324	5.2310	7.4562	25.9195	12.6871
30	18.5829	13.2062	5.3767	7.3110	25.8939	12.6876
40	18.5910	13.2045	5.3865	7.3011	25.8921	12.6877
50	18.5915	13.2043	5.3872	7.3005	25.8920	12.6877
54	18.5916	13.2043	5.3872	7.3004	25.8920	12.6877

The first stage of calculation of the conflation model of Hardy-Cross method is as follows and results of other stages are shown in Table 5.

$$\Delta_1 = -\frac{-0.11\times16^2 + 0.15\times14^2 - 0.14\times4^2 + 150}{2\,(0.11\times16 + 0.15\times14 + 0.14\times4)} = -16.8552$$

$$Q_{ab} = Q_{ab} + \Delta_1 = 14 + (-16.8552) = -2.8552$$

$$Q_{bd} = Q_{bd} - \Delta_1 = 4 - (-16.8552) = 20.8552$$

$$Q_{df} = Q_{df} - \Delta_1 = 16 - (-16.8552) = 32.8552$$

$$\Delta_2 = -\frac{0.14\times 20.8552^2 - 0.13\times 12^2 + 0.12\times 18^2}{2\,(0.14\times 20.8552 + 0.13\times 12 + 0.12\times 18)} = -6.1035$$

$$Q_{bd} = Q_{bd} + \Delta_2 = 20.8552 + (-6.1035) = 14.7517$$

$$Q_{dc} = Q_{dc} - \Delta_2 = 12 - (-6.1035) = 18.1035$$

$$Q_{bc} = Q_{bc} + \Delta_2 = 18 + (-6.1035) = 11.8965$$

$$\Delta_3 = -\frac{0.13\times 18.1035^2 + 0.11\times 32.8552^2 + 0.1\times 30^2 - 100}{2\,(0.13\times 18.1035 + 0.11\times 32.8552 + 0.1\times 30)} = -8.4386$$

$$Q_{df} = Q_{df} + \Delta_3 = 32.8552 + (-8.4386) = 24.4166$$

$$Q_{dc} = Q_{dc} + \Delta_3 = 18.1035 + (-8.4386) = 9.6649$$

$$Q_{ce} = Q_{ce} + \Delta_3 = 30 + (-8.4386) = 21.5614$$

As shown in Table 5, after 29 times iterations the conclusion reached is similar to Ventsim software. Therefore, the conflation model of Hardy-Cross method is able to converge faster to reach the solution.

Table5. *The results of conflation model of Hardy Cross Method stages*

Stage	Q_{ab}	Q_{bd}	Q_{bc}	Q_{ce}	Q_{df}	Q_{dc}
1	-2.8552	14.7517	11.8965	21.5614	24.4166	9.6649
2	7.9459	14.3534	6.4075	16.6701	24.6159	10.2625
3	11.7563	14.1362	2.3799	13.4210	25.1773	11.0412
4	14.1798	13.6850	-0.4947	11.2403	25.4201	11.7351
5	15.8277	13.3255	2.5022	9.7304	25.5581	12.2327
10	18.3870	13.2122	5.1749	7.4790	25.8661	12.6539
15	18.5772	13.2049	5.3723	7.3130	25.8901	12.6853
20	18.5906	13.2044	5.3862	7.3013	25.8919	12.6875
25	18.5915	13.2043	5.3872	7.3005	25.8920	12.6876
29	18.5916	13.2043	5.3872	7.3004	25.8920	12.6877

4. Comparison between Conflation Model of Hardy-Cross Method and Newton-Raphson Technique

So far, the comparison of the Newton- Raphson technique in the analysis of the mines ventilation network is not fully accurate. Therefore, for a more precise evaluation of this technique, a comparison was done between this model and conflation model of Hardy- Cross method, resulting in two different evaluation models.

4.1. First Model

An example mine ventilation network is considered according to Figure 1 and Table 1. According to Table 3, the conflation model of Hardy-Cross method is able to solve after 12 times iterations and the flow intensity is calculated as shown in Figure 2. The solving stages of calculation of Newton-Raphson technique in mines ventilation networks analysis according to Figure 3 and Table 6 are shown in 4 stages. Comparison of these results with the results obtained from the Ventsim software which as shown in Figure 2 is quite similar.

$$\begin{cases} f_1 = 0.15(Q_{ab} + \Delta Q_1)^2 - 0.11(Q_{df} - \Delta Q_1 + \Delta Q_3)^2 - 0.14(Q_{bd} - \Delta Q_1 + \Delta Q_2)^2 \\ f_2 = 0.14(Q_{bd} - \Delta Q_1 + \Delta Q_2)^2 - 0.13(Q_{dc} - \Delta Q_2 + \Delta Q_3)^2 + 0.12(Q_{bc} + \Delta Q_2)^2 \\ f_3 = 0.11(Q_{df} - \Delta Q_1 + \Delta Q_3)^2 + 0.1(Q_{ce} + \Delta Q_3)^2 + 0.13(Q_{dc} - \Delta Q_2 + \Delta Q_3)^2 - 80 \end{cases}$$

$$\begin{cases} \dfrac{\partial f_1}{\partial \Delta Q_1} = 0.3(Q_{ab} + \Delta Q_1) + 0.22(Q_{df} - \Delta Q_1 + \Delta Q_3) + 0.28(Q_{bd} - \Delta Q_1 + \Delta Q_2) \\ \dfrac{\partial f_1}{\partial \Delta Q_2} = -0.28(Q_{bd} - \Delta Q_1 + \Delta Q_2) \\ \dfrac{\partial f_1}{\partial \Delta Q_3} = -0.22(Q_{df} - \Delta Q_1 + \Delta Q_3) \end{cases}$$

$$\begin{cases} \dfrac{\partial f_2}{\partial \Delta Q_1} = -0.28(Q_{bd} - \Delta Q_1 + \Delta Q_2) \\ \dfrac{\partial f_2}{\partial \Delta Q_2} = 0.28(Q_{bd} - \Delta Q_1 + \Delta Q_2) + 0.26(Q_{dc} - \Delta Q_2 + \Delta Q_3) + 0.24(Q_{bc} + \Delta Q_2) \\ \dfrac{\partial f_2}{\partial \Delta Q_3} = -0.26(Q_{dc} - \Delta Q_2 + \Delta Q_3) \end{cases}$$

$$\begin{cases} \dfrac{\partial f_3}{\partial \Delta Q_1} = -0.22(Q_{df} - \Delta Q_1 + \Delta Q_3) \\ \dfrac{\partial f_3}{\partial \Delta Q_2} = -0.26(Q_{dc} - \Delta Q_2 + \Delta Q_3) \\ \dfrac{\partial f_3}{\partial \Delta Q_3} = 0.22(Q_{df} - \Delta Q_1 + \Delta Q_3) + 0.2(Q_{ce} + \Delta Q_3) + 0.26(Q_{dc} - \Delta Q_2 + \Delta Q_3) \end{cases}$$

$$\begin{bmatrix} 8.84 & -1.12 & -3.52 \\ -1.12 & 8.56 & -3.12 \\ -3.52 & -3.12 & 12.64 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 22.4 \\ 56.88 \end{bmatrix} \rightarrow \begin{cases} Z_1 = 3.28397 \\ Z_2 = 5.51631 \\ Z_3 = 6.776144 \end{cases} \rightarrow \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}^1 = \begin{bmatrix} -3.28397 \\ -5.51631 \\ -6.77614 \end{bmatrix}$$

$$\begin{bmatrix} 6.461475 & -0.49494 & -2.75172 \\ -0.49494 & 6.283473 & -2.79244 \\ -2.75172 & -2.79244 & 10.18894 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} -0.42148 \\ 4.142896 \\ 6.139427 \end{bmatrix} \rightarrow \begin{cases} Z_1 = 0.4701 \\ Z_2 = 1.16211 \\ Z_3 = 1.048013 \end{cases} \rightarrow \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}^2 = \begin{bmatrix} -3.75407 \\ -6.67842 \\ -7.82416 \end{bmatrix}$$

$$\begin{bmatrix} 5.999542 & -0.30118 & -2.62458 \\ -0.30118 & 5.84047 & -2.82211 \\ -2.62458 & -2.82211 & 9.881858 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} -0.07063 \\ 0.22741 \\ 0.148264 \end{bmatrix} \rightarrow \begin{cases} Z_1 = 0.004927 \\ Z_2 = 0.054609 \\ Z_3 = 0.031908 \end{cases} \rightarrow \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}^3 = \begin{bmatrix} -3.759 \\ -6.73303 \\ -7.85606 \end{bmatrix}$$

$$\begin{bmatrix} 5.978217 & -0.28727 & -2.61865 \\ -0.28727 & 5.819355 & -2.82801 \\ -2.61865 & -2.82801 & 9.875443 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} -0.00042 \\ 0.000636 \\ 0.000249 \end{bmatrix} \rightarrow \begin{cases} Z_1 = -4.1\text{E-}05 \\ Z_2 = 0.000133 \\ Z_3 = 5.22\text{E-}05 \end{cases} \rightarrow \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}^4 = \begin{bmatrix} -3.75896 \\ -6.73316 \\ -7.85612 \end{bmatrix}$$

Table6. *The results of Newton- Raphson Technique stages*

Stage	Q_{ab}	Q_{bd}	Q_{bc}	Q_{ce}	Q_{df}	Q_{dc}
1	10.7160	1.7677	12.4837	23.2239	12.5078	10.7402
2	10.2459	1.0756	11.3216	22.1758	11.9299	10.8543
3	10.2410	1.0260	11.2670	22.1439	11.9029	10.8770
4	10.2410	1.0258	11.2668	22.1439	11.9028	10.8770

4.2. Second Model

An example mine ventilation network is considered according to Figure 4 and Table 1. According to Table 5, the conflation model of Hardy-Cross method is able to solve after 29 times iterations, the flow intensity is calculated as shown in Figure 5.

The solving stages of calculation of Newton-Raphson technique in mines ventilation networks analysis for this model is similar to first model, but the difference is the +150 value added to f_1 function. Therefore, in accordance with Figure 6, the first to fourth stages of the calculations are shown as follows in Table 7. After the fourth stage the results of this technique will diverge and its results and results obtained from Ventsim software which has been presented in Figure 5 is not the same.

$$\begin{bmatrix} 8.84 & -1.12 & -3.52 \\ -1.12 & 8.56 & -3.12 \\ -3.52 & -3.12 & 12.64 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 149 \\ 22.4 \\ 36.88 \end{bmatrix} \rightarrow \begin{cases} Z_1 = 22.75768 \\ Z_2 = 9.854478 \\ Z_3 = 11.68774 \end{cases} \rightarrow \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}^1 = \begin{bmatrix} -22.7577 \\ -9.85448 \\ -11.6877 \end{bmatrix}$$

$$\begin{bmatrix} 8.060979 & -4.7329 & -5.95539 \\ -4.7329 & 9.331173 & -2.64335 \\ -5.95539 & -2.64335 & 12.26119 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 40.89803 \\ 34.52533 \\ 27.57702 \end{bmatrix} \rightarrow \begin{cases} Z_1 = 79.20504 \\ Z_2 = 59.01308 \\ Z_3 = 53.44228 \end{cases} \rightarrow \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}^2 = \begin{bmatrix} -101.963 \\ -68.8676 \\ -65.13 \end{bmatrix}$$

$$\begin{bmatrix} -4.37898 & -10.3866 & -11.6232 \\ -10.3866 & 2.270191 & -4.09176 \\ -11.6232 & -4.09176 & 8.688953 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 810.9264 \\ 470.951 \\ 362.6513 \end{bmatrix} \rightarrow \begin{cases} Z_1 = -41.1124 \\ Z_2 = -30.0618 \\ Z_3 = -27.4154 \end{cases} \rightarrow \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}^3 = \begin{bmatrix} -60.8503 \\ -38.8058 \\ -37.7146 \end{bmatrix}$$

$$\begin{bmatrix} 1.847253 & -7.29249 & -8.60987 \\ -7.29249 & 5.702813 & -3.40371 \\ -8.60987 & -3.40371 & 10.47066 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 215.8013 \\ 124.6311 \\ 96.70772 \end{bmatrix} \rightarrow \begin{cases} Z_1 = -23.2938 \\ Z_2 = -17.1868 \\ Z_3 = -15.505 \end{cases} \rightarrow \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}^4 = \begin{bmatrix} -37.5565 \\ -21.6189 \\ -22.2095 \end{bmatrix}$$

Table7. *The results of Newton- Raphson Technique stages*

Stage	Q_{ab}	Q_{bd}	Q_{bc}	Q_{ce}	Q_{df}	Q_{dc}
1	-8.7577	16.9032	8.1455	18.3123	27.0699	10.1667
2	-87.9627	37.0952	-50.8676	-35.1300	52.8327	15.7375
3	-46.8503	26.0446	-20.8058	-7.7146	39.1358	13.0912
4	-23.5565	19.9376	-3.6189	7.7905	31.3470	11.4094
5	-2.4295	14.3553	11.9257	21.8638	24.2933	9.9381
10	-72.2536	32.7890	-39.4646	-24.6447	47.6089	14.8199
15	-61.7188	30.0079	-31.7109	-17.6276	44.0911	14.0833
20	4.1949	12.6069	16.8017	26.2761	22.0812	9.4743
25	-0.0391	13.7246	13.6855	23.4559	23.4950	9.7704
30	-64.8736	30.8407	-34.0329	-19.7290	45.1446	14.3039
35	-10.1318	16.3891	6.2573	16.7334	26.8652	10.4761
40	-25.0300	20.3221	-4.7078	6.8100	31.8400	11.5178
45	-8.8842	16.0597	7.1755	17.5644	26.4486	10.3889
50	109.2174	-15.1188	94.0987	96.2295	-12.9880	2.1308
55	-19.6218	18.8944	-0.7274	10.4123	30.0341	11.1397

5. CONCLUSIONS

Various methods of manual and computerized analysis of mine ventilation network are presented. The selection of type of method depends on target of network analysis. If the purpose of mine ventilation network analysis is investigation of the effect of one or more fans on mine network, use of computerized methods based on mathematical approximation methods is better.

The Hardy Cross method for mathematical approximation methods for mines ventilation network analysis has become more common. The convergence of this method depends on values, initial assumed flows and direction and loops selection. In this method, if the value of new flow in the branch is negative, it must reverse the direction of the branch.

Wang's correction is relating to fourth stage of the Hardy-Cross method. In this correction model, if the value of new flow intensity in branch is negative, in the direction of the branch stays the same, and the flow intensity becomes negative.

Conflation model of Hardy- Cross method is relating to fourth and fifth stages of the Hardy-Cross method. Accordingly, if these two stages are performed sequentially step by step together for each branch, and the influence of each on the other to be considered, in this case, the investigation has led to more rapid access to the final answer. In other words, after the first loop error calculation must be done to correct the relevant branches of flow intensity and then calculate the error of the second loop. According to the examples presented, conflation model is able to decrease the repeat of calculation of the Hardy- Cross method, approximately 50 percent.

Newton- Raphson technique is one of the mathematical approximation methods which, is using in mines ventilation network analysis in recent years, however, so far the exact validation is not performed. Based on the results of this study, it can be concluded that in some of the models, this technique is not able to calculate the real value of flow intensity in mines ventilation networks analysis.

If in a network, the assumed initial flows direction is done in accordance with the real flows direction, in this case, with full confidence it can be claimed that the most rapid way to reaching the final answer in the analysis of mines ventilation networks is using of Newton- Raphson technique.

If in a network, some of assumed flows directions are not be in accordance with the real flows direction, using of Newton- Raphson technique in the analysis of mines ventilation networks for reaching the final answer in some of models may result in divergence and incorrect answers.

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