Modeling one Dimensional Dispersion in Natural River Channels

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Abstract: The modeling of one Dimensional Dispersion in Natural River Channels was targeted at developing an equation which can be applied in predicting longitudinal dispersion in natural river channels. Data in literature comprising 11 rivers and 29 sets of data consisting of hydraulic and geometric parameters were used to undertake a multivariate regression analysis which led to the development of the model equation. The relationships between the variables were investigated through the application of the Karl Pearson coefficient of correlation, while the accuracy of the model equation was ascertained through comparative analysis, discrepancy ratio and mean absolute error. The developed equation was further evaluated and analyzed by comparing its results with the experimental measured field data. A high degree of consistency was observed in the analysis which confirmed the suitability of the developed equation for calculating the one- dimensional dispersion values.

Nomenclature

С	=	average concentration at a river reach
А	=	flow cross sectional area at a river reach
V	=	average velocity of flow at
\Box t	=	change in time
$\Box \mathbf{x}$	=	change in the longitudinal direction of flow
D	=	longitudinal dispersion coefficient
Κ	=	Von Karman constant
Н	=	depth of flow at the given reach
U	=	bed shear stress velocity
S	=	slope of energy line
F _{ir}	=	Froude number
W	=	width of channel at the given reach
Φ	=	constant derived through regressional analysis in Koussis and Rodriguez Marisol model,
		$(\theta = 0.6)$
R	=	correlation coefficient
DR	=	discrepancy ratio
D _m	=	predicted longitudinal dispersion
D _e	=	measured longitudinal dispersion
ME	=	mean Absolute Error
U _*	=	shear bed velocity
\mathbf{S}_{f}	=	shape factor

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1. INTRODUCTION

The mass-transport equation is a fundamental principle in the prediction of water quality and sediment transportation in natural rivers and channels. The velocity of flow in rivers and other natural channels can be determined by solving the one-dimensional equation of motion. The concentration of this research is to focus on one-dimensional transport of sediments in natural rivers and channels. In the case of unsteady non- uniform flow, the one dimensional advective dispersion equation (ADE)

becomes an important tool in the prediction of water quality indicator and the suspended sediment – concentration distributions in rivers and natural channels. The one dimensional advective mass balance cum dispersion equation can be stated thus:

$$\frac{\delta CA}{\delta t} + \frac{\delta CAV}{\delta x} = \frac{\delta}{\delta x} AD \cdot \frac{\delta C}{\delta x} + r$$
 Eqn(1)

where

C = average concentration at a river reach

A =flow cross sectional area at a river reach

V = average velocity of flow at a cross section within the reach

r = rate of reaction material

 δt = change in time

 δx = change in the longitudinal direction of flow, while

D = longitudinal dispersion coefficient

The advective dispersion equation shown in eqn. 1 is mostly used when the tracer material is reactive. For the case of a conservative material, the reactive term becomes zero i.e. r = 0 and the above equation 1 becomes

$$\frac{\delta CA}{\delta t} + \frac{\delta CAV}{\delta x} = \frac{\delta}{\delta x} AD \cdot \frac{\delta C}{\delta x} Eqn(2)$$

The above two equations 1 and 2 illustrate the two main transport processes of advection and dispersion. Immediately an effluent is discharged into the river or a natural channel, the advective term plays an indispensable role in the diffusion process due to turbulent flow velocities with the attendant consequence of high level mixing. Following this, there exists an imbalance between the two terms of advection and dispersion and therefore the Taylor's analysis dispersion cannot be applied in this action of flow. This imbalance leads to a skewness in the longitudinal distribution of the concentration, with the average concentration being more downstream of the original source and this makes the one-dimensional dispersion equation not strictly being applicable. It is therefore recommended that the reach where the concentration of contaminant is measured should be far enough downstream of the point of discharge so that the Taylor's shear flow dispersion theory can be applied appropriately. It should be understood that, in modeling contaminant or tracer concentration distributions, the empirical data such as the longitudinal dispersion coefficient and the reactivity condition of the solution obtained from the one dimensional advective dispersion is dependent on the accuracy of the empirical data.

Hydrodynamic parameters such as roughness height of rivers and natural channels differ and these factors affect the longitudinal dispersion coefficients of these rivers. This explains why the dispersion coefficient of rivers and natural channels differ from river to river and from one natural channel to another. The assumptions made in the analysis of dispersion differ from one theory to another and this has resulted to varied results of dispersion coefficients, D. This study has developed a new equation for dispersion coefficient, based upon about 29 data sets collected for 11 rivers in Anambra Imo River Basin of Nigeria.

1.1 Mathematical Models of Physical Systems

Most of the systems of concern in water quality management have peculiar and significant flow characteristics. The actual systems vary from lakes, rivers and estuaries to treatment process units but with few exceptions, all have continuous flow inputs and outputs and therefore the equations (3), (4) and (5) stated below are applicable. The material balance for materials flowing in a natural system can be stated in words statement thus;

Rate of accumulation of mass within		(Rate of flow mass into the system)	١.
boundary system) -	boundary	J

The equation (3) can be further simplified as

Accumulation = Inflow - Outflow + Generation

A number of approaches to the analyses of these systems are in vogue. Most of the approaches begin with an assumption about the hydraulic characteristics of the system. Two basic hydraulic models are used to simulate natural systems. They are the complete – mix model and the plug-flow model. These two ideal models are used to define an envelope within which other models fall. Often these models are combined in various ways to simulate large complex models with spatially varying characteristics. The objective of water quality modeling is to study the behavior of natural systems in response to external and internal inputs. In an initial step of the modeling process, it is important to predict the hydraulic performance of the system.

1.2 Review of Previous Works on Dispersion

In open channel flow, Elder (1959) presented the first published analysis of the longitudinal dispersion coefficient based on laboratory measurements and Taylor's method, by assuming a logarithmic vertical – velocity distribution to give the well-known equation;

$$D = \frac{0.04041}{K^3} + \frac{K}{6} HU_* \text{ or } D = 5.93HU_*$$
 Eqn(5)

Where

K = Von Kariman constant which is approximately equal to 0.41

 $U_* =$ bed shear stress velocity

H = Dept of flow. However, it has been found that Elder's equation does not accurately describe longitudinal dispersion in natural rivers, streams and channels, and generally, significant underestimates of the dispersion coefficients are observed. This is thought to be mainly due to the exclusion of the transverse variation of velocity profile across the stream in the deviation of Elder's equation. The vertical and transverse shear components are important in the determination of dispersion coefficient. This is confirmed by Guyer and West through the measurements of velocity and salinity distribution on a cross section in an estuary. Studies undertaken using many measured data sets for natural rivers have shown that the value of D/HU_{*} may vary from 8.6 to 7500. This range of values of D/HU_{*} is by far greater than that obtained from Elder equation which stood at 5.93.

Fischer applied Taylor's assumptions of the mass-conservation equation for turbulent flow by assuming that transverse or lateral variations are more important in comparison to the vertical variations in the velocity profile. In this light, Fischer presented a new equation for D. The equation was based on integrating the time-independent portion of the resultant conservation of mass equation over the depth and including the boundary condition of no mass flux across the bed and water surface. The resulting equation was stated thus;

$$D = -\frac{1}{A} \int_0^w \bigcup^1(y) H(y) \int_0^y \frac{1}{\sum y H(y)} \int_0^y \bigcup^1(y) H(y) dy dy dy$$
Eqn(6)

where

W = channel width

Y = Cartesian coordinate in the transverse flow direction

 $\sum y =$ lateral turbulent mixing coefficient in y direction

U' = spatial deviation of the velocity from cross sectional mean velocity as a function of distance in the direction. By experiment, Σy was found to be within the region of 0.23HU_{*} – 0.7HU_{*}.

Eqn(4)

Efforts made in investigations revealed that Eqn. (6) estimates the longitudinal dispersion coefficient more accurately than the other existing empirical equations in natural channels. Practically, the integrals of Eqn. (6) are replaced by similarly summations and thus, in using this equation, extensive dye dispersion-field data are required in the transverse and longitudinal directions of flow. It is advisable, that for practical engineering applications equations based on hydraulic and geometrical parameters should always be emloyed. McQuivey and Keefer presented such an equation which combined the linear one-dimensional flow dispersion equation and it is stated thus;

$$D = 0.058 \frac{Q}{SW}$$
 for $Fn < 0.5$

Where

S = slope of energy line

Fn = Froude number and

W = width of channel

Fischer developed a simple method to predict the longitudinal dispersion in a non-integral form as shown in eqn. (8) below;

$$D = 0.011U2W2$$

$$HU_{*}$$
Eqn(8)

Using the original theory and equation proposed by Fischer Kouscis and Rodriguez-Mirasol applied the Von Karman's defect law and derived an equation of the form:

$$D = \Phi U_* W_2$$
 Eqn(9)

They proposed that the value of $\Phi = 0.6$

2. METHODOLOGY

The study employed the survey design research method and sampled eleven rivers which included Oramirikwa, Urasi, Iyingodo, Mmam, Chaocha, Njaba, Oji, Otamiri, Aboine and Iyinwoba and all of them are located with Anambra Imo River Basin. Hydraulic, fluid and geometric data were collected on these rivers. It has been established that most real life applications demand the use of multiple regression model and a sample of this is represented in eqn. 9

$$Y = Bo + b1X1 + b2X2 \dots + bnXn \qquad Eqn(10)$$

These parameters were correlated with the measured dispersion coefficient D to establish the relationships existing between D and other parameters through multivariate regression analysis. Graphs of the measured experimental dispersion D were plotted against various independent variables and the lines of best fit were obtained in each case.

The Karl Pearson coefficient of correlation was used in calculating the coefficient and Karl Pearson coefficient of correlation R is stated thus;

$$R = \sqrt{\frac{\sum_{i=1}^{n} (x_{1} - \bar{x})(y_{1} - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_{1} - \bar{x})^{2} \sqrt{\sum_{i=1}^{n} (y_{1} - \bar{y})^{2}}}} Eqn(11)$$

The Karl Pearson coefficient of correlation was to determine the degree of association between variables. The range of the correlation coefficient R is within the range of -1 to +1. When the correlation coefficient, R is negative, then the variables are inversely proportional and it is maximum in that respect when R = -1. If the coefficient of correlation R = 0, there is no association between the variables. When the coefficient is positive, then the variables are associated directly and it is maximum when R = +1.

Eqn(7)

The discrepancy ratio, DR was used to estimate the difference in the experimental field data and that of the model. The discrepancy ratio DR is given by White et al as

$$DR = \log 10 De = \underbrace{[Log10De - Log10Dm}_{Dm} Eqn(12)$$

where Dm = dispersion value calculated from the developed equation

De = dispersion value obtained from the field experimental data

If the predicted value is equal to the measured value, the DR becomes zero. For all values greater than zero, the model overestimates the value of dispersion and conversely when it is less than zero. Another method of determining the accuracy of developed equations was introduced by Moog and Forka and it was a little similar to the discrepancy ratio. It was developed to assess the predicted and measured stream re-aeration coefficient. Whenever the coefficient $\beta = De/Dm$ for the model is equal to one or close to that and the correlation coefficient is high, this is a pointer to a more accurate result. The mean absolute error ME was also used and it is defined as

$$ME = \frac{1}{N} \sum_{i=1}^{N} (DR_i)$$
 Eqn(13)

Data in literature used consisted of geometric hydraulic parameters and experimental dispersion monitored in eleven rivers as presented in Table 1:

Channel	H(m)	W(m)	U(m/s)	U _* (m/s)	$D(m^2/s)$
Mmam River	0.400	10.800	0.320	0.060	8.290
	0.420	19.800	0.430	0.069	14.152
	0.610	20.400	0.520	0.081	24.550
Otamiri River	0.610	83.000	0.160	0.046	6.009
	0.650	51.200	0.6620	0.044	79.600
	1.100	97.500	0.320	0.058	33.800
	0.400	40.500	0.230	0.040	6.500
Njaba River	0.840	25.900	0.340	0.067	38.620
	0.810	30.600	0.400	0.069	29.970
	0.410	10.800	0.290	0.044	10.940
Urasi River	0.860	46.900	0.280	0.067	12.930
	2.430	59.400	0.860	0.104	203.880
	2.190	53.300	0.790	0.107	145.450
Nwaorie River	0.360	12.500	0.310	0.043	6.870
	0.410	15.800	0.370	0.055	13.840
Oramirikwa River	0.700	40.200	0.230	0.064	8.800
	0.420	59.700	0.150	0.081	2.410
	1.230	43.000	0.630	0.084	59.300
Oji River	0.910	13.400	0.370	0.077	13.940
	1.300	19.500	0.450	0.093	33.520
Aboine River	0.990	20.100	0.590	0.098	34.500
	0.760	12.900	0.430	0.085	193.900
Iyingodo River	2.050	103.600	0.560	0.054	129.140
	4.850	127.400	0.640	0.081	288.456
Iyinwoba River	0.400	30.600	0.450	0.046	17.430
	0.520	50.900	0.460	0.046	20.900
Chaocha River	0.340	36.100	0.210	0.043	4.650
	0.490	36.600	0.320	0.051	13.940
	0.870	47.500	0.440	0.070	29.160

Table1. Experimental Measurements of Longitudinal Dispersion in Natural Channels

Source: Anambra Imo River Development Authority

3. DEVELOPMENT OF MODEL EQUATION

3.1 Mathematical Relationship of Parameters

Dispersion coefficients have mathematical relationships with fluid properties, hydraulic characteristics and geometric parameters. In order to establish the relationships to calculate the longitudinal dispersion based on hydraulic and geometric parameters, 29 data sets in 11 rivers located **International Journal of Constructive Research in Civil Engineering (IJCRCE)** Page 5

in the Anambra-Imo River Basin of Nigeria were collected. Longitudinal dispersion coefficient can be directly related to the depth of water H, channel width W, and velocity of flow V. Following the review of key literature on dispersion, it was postulated that fluid properties, hydraulic and geometric characteristics were functions of dispersion as stated in Eqn. 12,

$$D = f(U, H, W, U_*. Sf, \mu)$$

Eqn(14)

where H = depth of water, U = velocity of flow, U_* = shear bed velocity, μ = kinematic viscosity and Sf =shape factor. The major parameters employed in the equation development were H, U and U_{*}.

The shape factor was neglected in developing the equation due to its relationship with shear bed velocity and since the flow in river channels are normally turbulent and rough, with Reynolds number effects being negligible, the kinematic viscosity in eqn. 12 can also be neglected.

3.2 Derived Parameter

The geometric hydraulic characteristics were used to determine the combined parameters and the dispersion. The primary data collected (refer to Table 1) were combined to obtain the derived values. These values were employed in the regression analysis. The values are shown in Table 2:

Table2.Calculated Parameters and Dispersion

Channel	$HU(m^2/s)$	$WU(m^2/s)$	U/U _*	$WU(U/U_*)$	$HU(U/U_*)$	Dispersion
				(m^2/s)	(m^2/s)	(m^{2}/s)
Mmam	0.128	3.456	5.333	18.431	0.683	8.290
River	0.181	8.084	6.232	50.379	1.128	14.152
	0.317	10.556	6.420	67.770	2.035	24.550
Otamiri	0.098	13.280	3.478	46.188	0.341	6.009
River	0.403	19.344	14.091	272.576	5.679	79.600
	0.352	30.880	5.517	170.365	1.942	33.800
	0.092	9.315	5.750	53.561	0.529	6.500
Njaba River	0.286	8.126	5.075	41.239	10451	38.620
_	0.324	12.240	5.970	73.073	1.934	29.970
	0.119	3.132	6.591	20.643	0.784	10.940
Urasi River	0.241	13.132	4.179	54.879	1.007	12.930
	2.090	51.084	8.269	422.414	17.282	203.880
	1.730	42.107	7.383	310.876	12.773	145.450
Nwaorie	30112	3.255	7.209	23.465	0.807	6.870
River	0.152	5.846	6.727	39.326	1.023	13.840
Oramiriukwa	0.161	9.246	3.594	33.230	0.579	8.800
River	0.063	8.955	1.852	16.585	1.167	2.410
	0.775	25.200	7.500	189.000	5.813	59.300
Oji River	0.337	4.958	4.805	23.823	1.619	13.940
	0.585	8.775	4.839	42.462	2.831	33.520
Aboine	0.584	11.859	6.020	71.391	3.516	34.500
River	0.327	5.547	5.059	28.062	1.654	193.900
Iyingodo	1.142	58.016	10.370	601.626	11.843	129.140
River	3.104	81.536	7.901	644.216	24.525	288.456
Iyinwaoba	0.180	13.770	9.783	134.712	1.761	17.430
River	0.239	23.414	10.000	234.140	2.390	20.900
Chaocha	0.071	7.581	4.884	37.026	0.347	4.650
River	0.157	11.712	6.275	73.493	0.985	13.940
	0.383	20.900	6.286	131.377	2.408	23.160

3.3 Regression Analysis of Relationships

At the commencement of regressional analysis, the dependent variable D was related to all the independent variables H, W, U, and U_{*}. The relationships are illustrated in fig 1a - d and the coefficient of correlation R of the data in each graph was calculated to determine whether the dependent variable D and the independent variables of H, W, U and U_{*} are associated. The analysis portrayed that the correlation coefficient R for fig. 1a-d are 0.35, 0.38, 0.39 and 0.33 respectively. The results showed that there were mathematical relationships between the independent variables and the dependent variable but the relationship is relatively low judging from the values of the coefficient of correlation. It was observed in further regression analysis of the data, that when the dependent

variable D was associated with the combinations of HU, WU and U/U* as independent variables, the correlation factor increased to 0.45, 0.43 and 0.58 respectively. This was illustrated in fig. 2(a) - 2(c). When the independent variables were associated in the form of WU(U/U*) and HU(U/U*), the coefficients of correlation increased to 0.61 and 0.87 respectively. This is illustrated in fig. 3a and 3b respectively. The combinations in fig.3a and 3b introduced dimension harmony and homogeneity with the dependent variable D. A multiple regression analysis carried out by associating the longitudinal dispersion coefficient D with the independent variable of (HU) $\begin{pmatrix} U \\ U_* \end{pmatrix}$ and as shown fig. 3b yielded the highest correlation coefficient to the tune of 0.87 which showed a strong proportional relationship. The parameters in fig. 3b were analyzed and it was observed that the graph passed through the origin and therefore its intercept was zero and the resulting slope was 11.007. This resulted to the model equation as shown below;

$$D = 11.007 HU \begin{pmatrix} U \\ U_* \end{pmatrix} - Eqn(15)$$

With this correlation coefficient of 0.87, the above eqn. 13 stands as the developed model equation. The graphs were shown below:



Graphs



3.4 Evaluation of the Equation

The developed equation was evaluated by comparing the results of the measured dispersion and that of the developed equation through the discrepancy ratio, DR and the mean absolute error, ME. The results were illustrated in Table 3

River channel	Measured dispersion	Calculated	Discrepancy Ratio	Coefficient B =
	from experimental	dispersion from	DR=Log ₁₀ D _e -	D _m /D _e
	data $D_e (m^2/s)$	developed model D _m	$Log_{10}D_m$	
		$= 11.007 HU \begin{pmatrix} U \\ U_* \end{pmatrix}$		
		(m^2/s) –		
Mmam river	8.290	7.514	0.043	0.906
	14.152	12.388	0.058	0.875
	24.550	22.414	0.040	0.913
Otamiri River	6.009	3.737	0.206	0.622
	79.600	62.505	0.105	0.785
	33.800	21.376	0.199	0.632
	6.500	5.819	0.048	0.895
Njaba River	38.620	15.952	0.384	0.413
	29.970	21.291	0.148	0.710
	10.940	8.626	0.103	0.788
Urasi River	12.930	11.077	0.067	0.857
	203.880	190.213	0.030	0.933
	145.450	140.599	0.015	0.967
Nwaorie River	6.870	8.856	-0.110	1.289
	13.840	11.233	0.091	0.812
Oramiriukwa River	8.800	6.369	0.140	0.724
	2.410	1.284	0.273	0.533
	59.300	63.970	-0.033	1.079
Oji River	13.940	17.808	-0.106	1.277
	33.52	31.157	0.032	0.930
Aboine River	34.500	38.706	-0.050	1.122
	193.900	191.866	0.005	0.990
Iyingodo River	129.140	130.401	-0.004	1.010
	288.456	269.951	0.029	0.936
Iyinwoba River	17.430	19.382	-0.046	1.112
	20.900	26.329	-0.100	1.260
Chaocha River	4.650	3.838	0.083	0.825
	13.940	10.829	0.110	0.777
	29.160	26.485	0.042	0.908

Table3.Comparison between measured dispersion and developed model dispersion

From the Table 3, it can be seen that the differences between the measured experimental data and the values determined from the developed equation are highly minimal and the differences are neither solely on the underestimating nor on the overestimating side. The discrepancy ratio, DR was used to estimate the difference in the experiment field data and that of the values determined from the developed equation and it is shown in Table 3. A study of the table shows a slight but a balanced difference between the two results. This reveals that the equation has neither underestimation nor overestimation feature. The discrepancy ratio DR was used to determine the absolute mean error ME of the developed equation. The estimated value of the mean absolute error Absolute mean error

$$\mathrm{ME} = \frac{1}{N} \sum_{i=1}^{N} (DR_i)$$

Gave a result of 0.062 and this error tends towards zero and this suggests the high accuracy potentiality of the developed equation. For all the sets of data in Table 3, the coefficient B = Dm/De which represents the ratio of the value of the dispersion determined from the proposed equation to that of the experimental data is approximately equal to one. To obtain good predictions, the resulting value of the ME and DR should be as close as possible to zero while the coefficient B will be approximately equal to unity. These conditions are satisfied in the analysis done so far and this further strengthens the potency of the developed equation.

4. CONCLUSION

Following the findings of the research it can be concluded that the study has been able to establish a new relationship between longitudinal dispersion coefficient and other hydraulic and geometrical parameters. The process of evolving this model equation was sound as it was predicated on sequential and multiple regression analysis with consideration to dimensional analysis. The research was also able to use the concept of correlation coefficient to establish the relationship between the variables. It is therefore more advisable to use this model to predict the water quality or sediment – concentration distribution in a river or any natural channel.

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