

Schrodinger Equation in Cosmological Inertial Frame

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Abstract: Schrodinger equation is a wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Schrodinger equation from Klein-Gordon free particle's wave function in cosmological special theory of relativity.

Keywords: Klein-Gordon Free Particle's Wave Function; Cosmological Inertial Frame; Schrodinger equation;

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1. INTRODUCTION

At first, Klein-Gordon equation is for free particle field ϕ in cosmological inertial frame.[1]

$$\frac{m^2 c^2}{\hbar^2} \phi + \Omega(t_0) \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} - \frac{1}{\Omega(t_0)} \nabla^2 \phi = 0$$

m is free particle's mass, $\Omega(t_0)$ is the ratio of universe's expansion in cosmological time t_0 (1)

If we write wave function as solution of Klein-Gordon equation for free particle,[1]

$$\phi = A_0 \exp \left[i \left(\frac{\omega t}{\sqrt{\Omega(t_0)}} - \vec{k} \cdot \vec{x} \sqrt{\Omega(t_0)} \right) \right]$$

A_0 is amplitude, ω is angular frequency, $k = |\vec{k}|$ is wave number (2)

Energy and momentum is in cosmological inertial frame,[1]

$$E = \hbar \omega, \vec{p} = \hbar \vec{k} / \Omega(t_0) \quad (3)$$

Hence, energy-momentum relation is in cosmological inertial frame [1]

$$E^2 = \hbar^2 \omega^2 = \Omega^2(t_0) p^2 c^2 + m^2 c^4 = \hbar^2 k^2 c^2 + m^2 c^4 \quad (4)$$

Or angular frequency- wave number relation is

$$\frac{\omega^2}{c^2} = k^2 + \frac{m^2 c^2}{\hbar^2} \quad (5)$$

Hence, wave function is in cosmological inertial frame,

$$\phi = A_0 \exp \left[\left(-\frac{i}{\hbar} \right) \hbar \left(\frac{\omega t}{\sqrt{\Omega(t_0)}} - \hbar \vec{k} \cdot \vec{x} \sqrt{\Omega(t_0)} \right) \right]$$

$$= A_0 \exp \left[-\frac{i}{\hbar} \left(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p} \cdot \vec{x} \Omega(t_0) \sqrt{\Omega(t_0)} \right) \right] \quad (6)$$

2. SCHRODINGER EQUATION FROM KLEIN-GORDON FREE PARTICLE FIELD IN COSMOLOGICAL INERTIAL FRAME

Because, Schrodinger equation is made from Klein-Gordon free particle's wave function in cosmological special theory of relativity,

$$\phi = A_0 \exp \left[-\frac{i}{\hbar} \left(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p} \cdot \vec{x} \Omega(t_0) \sqrt{\Omega(t_0)} \right) \right] \quad (7)$$

If we calculate the derivation of Schrodinger equation,

$$\sum_i \left(\frac{\partial}{\partial x^i} \right)^2 \phi = -\sum_i \frac{(p^i)^2}{\hbar^2} \Omega^3(t_0) \phi = -\frac{p^2}{\hbar^2} \Omega^3(t_0) \phi \quad (8)$$

$$\frac{\partial \phi}{\partial t} = -\frac{i}{\hbar} E \frac{\phi}{\sqrt{\Omega(t_0)}} \quad (9)$$

Energy E is

$$E = \frac{p^2}{2m} \Omega^2(t_0) + V, \quad V \text{ is the potential energy} \quad (10)$$

Hence,

$$E\phi = \frac{p^2}{2m} \Omega^2(t_0) \phi + V\phi, \quad V \text{ is the potential energy} \quad (11)$$

Therefore, by Eq(8),Eq(9)

$$E\phi = i\hbar \frac{\partial \phi}{\partial t} \sqrt{\Omega(t_0)}, \quad \Omega^2(t_0) p^2 \phi = -\hbar^2 \nabla^2 \phi \frac{1}{\Omega(t_0)} \quad (12)$$

Therefore, Schrodinger equation in cosmological inertial frame,

$$E\phi = i\hbar \frac{\partial \phi}{\partial t} \sqrt{\Omega(t_0)} = -\frac{\hbar^2}{2m} \frac{1}{\Omega(t_0)} \nabla^2 \phi + V\phi \quad (13)$$

If the energy E is not concerned by time t,

$$\frac{\partial E}{\partial t} = 0 \quad (14)$$

$$\begin{aligned} \phi &= A_0 \exp \left[-\frac{i}{\hbar} \left(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p} \cdot \vec{x} \Omega(t_0) \sqrt{\Omega(t_0)} \right) \right] \\ &= \phi \exp \left[-\frac{i}{\hbar} \left(\frac{Et}{\sqrt{\Omega(t_0)}} \right) \right] \end{aligned} \quad (15)$$

Hence, stationary state of Schrodinger equation is in cosmological inertial frame,

$$\begin{aligned} E\phi \exp \left[-\frac{iE}{\hbar} \frac{t}{\sqrt{\Omega(t_0)}} \right] \\ = \left[-\frac{\hbar^2}{2m} \frac{1}{\Omega(t_0)} \nabla^2 \phi + V\phi \right] \exp \left[-\frac{iE}{\hbar} \frac{t}{\sqrt{\Omega(t_0)}} \right] \end{aligned} \quad (16)$$

Hence, stationary state of Schrodinger equation is

$$E\varphi = -\frac{\hbar^2}{2m} \frac{1}{\Omega(t_0)} \nabla^2 \varphi + V\varphi \quad (17)$$

Or,

$$\frac{1}{\Omega(t_0)} \nabla^2 \varphi + \frac{2m}{\hbar^2} (E - V)\varphi = 0 \quad (18)$$

3. CONCLUSION

We found Schrodinger equation from Klein-Gordon's free particle equation in cosmological special theory of relativity. The wave function uses as a probability amplitude.

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