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New Coordinate Vacuum Solution in Cosmological General Theory of Relativity

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Abstract: *In the general relativity theory, we discover new vacuum solution by Einstein's gravity field equation. We investigate the new coordinate in cosmological general theory of relativity (CGTR).*

Keywords: *Cosmological General Theory of Relativity; Gravity Field Equation; New Coordinate Vacuum Solution*

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1. Introduction

We solve new vacuum solution by gravity field equation in cosmological general theory of relativity. New spherical coordinate is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dr^2 + V(t,r)\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

$$V(t,r) = C_1(act+br)^2, \quad C_1 = \frac{1}{b^2-a^2}$$

a, b, C_1 is constant, c is light's velocity. (1)

In this time, Einstein's gravity equation is

$$R_{tt} = \frac{\ddot{V}}{V} - \frac{\dot{V}^2}{2V^2}$$

$$= \frac{2a^2}{(act+br)^2} - \frac{1}{2}\frac{4a^2}{(act+br)^2} = 0 \quad (2)$$

$$R_{rr} = \frac{V''}{V} - \frac{1}{2}\frac{V'^2}{V^2}$$

$$= \frac{2b^2}{(act+br)^2} - \frac{1}{2}\frac{4b^2}{(act+br)^2} = 0$$

(3)

$$R_{\theta\theta} = -\frac{\ddot{V}}{2} + \frac{V''}{2} - 1$$

$$= -C_1 a^2 + C_1 b^2 - 1 = 0 \quad (4)$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta} = 0 \tag{5}$$

$$R_{tr} = \frac{\dot{V}'}{V} - \frac{\dot{V}V'}{2V^2}$$

$$= \frac{2C_1 ab}{(act+br)^2} - \frac{1}{2}\frac{4C_1 ab}{(act+br)^2} = 0 \tag{6}$$

In this time**,**

$$V' = 2C_1 b(act+br), \dot{V} = 2C_1 a(act+br), V'' = 2C_1 b^2, \ddot{V} = 2C_1 a^2$$

$$A' = \frac{\partial A}{\partial r}, \dot{A} = \frac{1}{c}\frac{\partial A}{\partial t}$$

2. New Vacuum Solution in Cosmological General Theory of Relativity

Hence, new vacuum solution is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dr^2 + \frac{1}{b^2-a^2}(act+br)^2\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

a, b, C_1 are constant, c is light's velocity. (7)

In this time, if r' is

$$r' = \frac{1}{\sqrt{b^2-a^2}}(act+br)$$

As

$$dr' = \frac{1}{\sqrt{b^2-a^2}}(acdt + bdr)$$

Or

$$dr = \frac{\sqrt{b^2-a^2}}{b}dr' - \frac{a}{b}cdt \tag{8}$$

If new solution Eq(7) is inserted by transformation Eq(8),

$$dr^2 = \frac{b^2-a^2}{b^2}dr'^2 - 2\frac{a}{b^2}\sqrt{b^2-a^2}dr'cdt + \frac{a^2}{b^2}c^2dt^2 \tag{9}$$

In this time, if α_0 is

$$\alpha_0 = \frac{a}{b} \tag{10}$$

Hence, proper time $d\tau$ of new solution is

$$d\tau^2 = (1-\alpha_0{}^2)dt^2 + 2\alpha_0\sqrt{1-\alpha_0{}^2}dr'\frac{dt}{c} - \frac{1}{c^2}[(1-\alpha_0{}^2)dr'^2 + r'^2\{d\theta^2 + \sin^2\theta d\phi^2\}] \tag{11}$$

In this time, if dt' is

$$dt' = \sqrt{1-\alpha_0{}^2}dt \tag{12}$$

Therefore, new solution is

$$d\tau^2 = dt'^2 + 2\alpha_0 dr' \frac{dt'}{c} - \frac{1}{c^2}[(1-\alpha_0{}^2)dr'^2 + r'^2\{d\theta^2 + \sin^2\theta d\phi^2\}] \tag{13}$$

If we rewrite dt, dr instead of dt', dr', the proper time $d\tau$ of new solution is

$$d\tau^2 = dt^2 + 2\alpha_0 dr \frac{dt}{c} - \frac{1}{c^2}[(1-\alpha_0{}^2)dr^2 + r^2\{d\theta^2 + \sin^2\theta d\phi^2\}] \tag{14}$$

Therefore, new spherical solution in general relativity theory is

$$d\tau^2 = dt^2 + 2\alpha_0 dr \frac{dt}{c} - \frac{1}{c^2}[(1-\alpha_0{}^2)dr^2 + r^2\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

$\alpha_0 \neq 1$, α_0 is constant (15)

In this time, the coordinate transformation in cosmological general theory of relativity [1-3] is

$r \to r\Omega(t_0), t \to t$,

t_0 is cosmological time. $\Omega(t_0)$ is the ratio of universe's expansion in cosmological time t_0 . (16)

Hence, this vacuum solution is by the coordinate transformation in cosmological general theory of relativity,

$$d\tau^2 = dt^2 + 2\alpha_0\Omega(t_0)dr\frac{dt}{c} - \frac{\Omega^2(t_0)}{c^2}[(1-\alpha_0{}^2)dr^2 + r^2\{d\theta^2 + \sin^2\theta d\varphi^2\}]$$

$\alpha_0 \neq 1$, α_0 is constant (17)

3. CONCLUSION

In the general relativity theory, we discover new vacuum solution by Einstein's gravity field equation. We investigate the new coordinate in cosmological general theory of relativity.

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