International Journal of Advanced Research in Physical Science (IJARPS)
Volume 8, Issue 3, 2021, PP 19-22
ISSN No. (Online) 2349-7882
www.arcjournals.org

Expressing a Formula of Throwing Object Vertically Upward or Vertically Downward by Using Limit

Salman Mahmud*

Student of Government Azizul Haque College, Bogura, Bangladesh

Corresponding Author:** ***Salman Mahmud, *Student of Government Azizul Haque College, Bogura, Bangladesh*

Abstract: *We know, $v^2 = {v_0}^2 + 2gh$ is an important formula to calculate the velocity of a throwing object (vertically downward or upward) as well as the distance overcome by the object. In this paper we will apply a method to represent this formula by using limit.*

Keywords: *Gravitational acceleration, throwing object, limit.*

1. Introduction

We know, $v^2 = {v_0}^2 + 2gh$ is an important formula to calculate the velocity of a throwing object (vertically downward or upward) as well as the distance overcome by the object. For a small distance we consider gravitational acceleration g is constant and value of g is $9.8\,ms^{-2}$. But g will be different in two places if the distance between the places and the center of earth are different. We know, the gravitational acceleration at any point on earth surface is,

$$g = \frac{GM}{(R+r)^2}$$

Where $G =$ gravitational constant, $M =$ mass of the earth and $R + r =$ the distance between the point and center of earth. If we throw an object, the distance between the object and center of earth will be changing in every moment. So the gravitational acceleration will also be changed. That's why, here we will apply a method to express the formula by using limit to get the exact final velocity of a throwing object. In this paper it has been shown that if we throw a object vertically upward with an initial velocity v_0, the final velocity v after overcoming a height h will be

$$v^2 = \lim_{n \to 1^+} {v_0}^2 - 2\frac{GM}{x}\left\{1 - \left(\frac{1}{n}\right)^{\frac{ln\left(n+\frac{h}{x}\right)}{ln(n)} - 1}\right\}$$

Again, if we throw a object vertically downward with an initial velocity v_0, the final velocity v after overcoming a distance h will be

$$v^2 = \lim_{n \to 1^+} {v_0}^2 + 2\frac{GM}{xn^2}\left\{1 - \left(\frac{1}{n}\right)^{\frac{ln\left(n+\frac{h}{x}\right)}{ln(n)} - 1}\right\}$$

2. Expressing the Formula by Using Limit

To express the formula by using limit in case of throwing object vertically upward or vertically downward we have to understand and use the figure below.

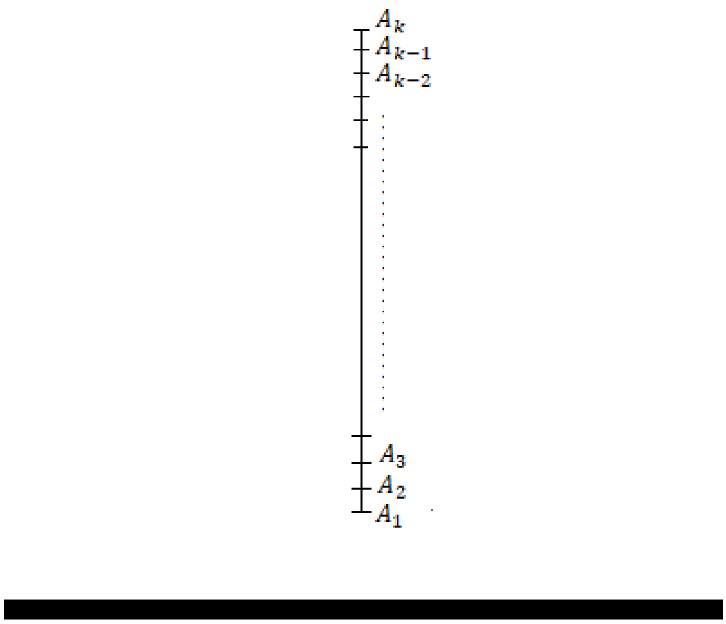

Figure1. *Here A_1A_k is a vertical line on earth's surface and the line is divided into k-1 parts.*

Suppose, the radius of the earth is x, the distance between the point A_1 and the center of earth is xn, the distance between the point A_2 and the center of earth is xn^2, the distance between the point A_3 and the center of earth is xn^3, the distance between the point A_4 and the center of earth is xn^4,….., the distance between the point A_k and the center of earth is xn^k. The distance between the point A_1 and A_k is h.

Here A_1A_k is a vertical line on earth's surface and the line is divided into *k-1* parts. We have to consider that the length of every part is closest to 0. In other words, $(xn^2 - xn) \to 0, (xn^3 - xn^2) \to 0, \dots, (xn^k - xn^{k-1}) \to 0$. That's why, here $n \to 1^+$. Here g will be different in every part of the line. But in case of a single part we will consider g is same from the starting point to ending point.

Now suppose, we have thrown an object with an initial velocity v_1 from the point A_1 to the point A_k . The gravitational acceleration at the point A_1 will be

$$g_1 = \frac{GM}{(xn)^2}$$

The gravitational acceleration will be same from A_1 to A_2 because $(xn^2 - xn) \to 0$. When the object will reach at the point A_2 the final velocity will be v_2. So we can write,

$$v_2{}^2 = v_1{}^2 - 2\frac{GM}{(xn)^2}(xn^2 - xn) = v_1{}^2 - 2\frac{GM}{xn}(n-1)$$

Again, The gravitational acceleration at the point A_2 will be

$$g_2 = \frac{GM}{(xn^2)^2}$$

The gravitational acceleration will be same from A_2 to A_3 because $(xn^3 - xn^2) \to 0$. When the object will reach at the point A_3 the final velocity will be v_3. So we can write,

$$v_3{}^2 = v_2{}^2 - 2\frac{GM}{(xn^2)^2}(xn^3 - xn^2) = v_1{}^2 - 2\frac{GM}{xn}(n-1) - 2\frac{GM}{(xn^2)^2}(xn^3 - xn^2)$$

$$= v_1{}^2 - 2\frac{GM(n-1)}{x}\left(\frac{1}{n} + \frac{1}{n^2}\right)$$

Again, when the object will reach at the point A_4 the final velocity will be v_4. By using the above process we can write,

$$v_4{}^2 = v_1{}^2 - 2\frac{GM(n-1)}{x}\left(\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3}\right)$$

We can see, there is a beautiful pattern. When the object will reach at the point A_k the final velocity will be v_k. According to the pattern we can write,

$$v_k{}^2 = v_1{}^2 - 2\frac{GM(n-1)}{x}\left(\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} + \cdots + \frac{1}{n^{k-1}}\right)$$

$$= v_1{}^2 - 2\frac{GM(n-1)}{x} \times \frac{\left\{1 - \left(\frac{1}{n}\right)^{k-1}\right\}}{(n-1)}$$

$$= v_1{}^2 - 2\frac{GM}{x}\left\{1 - \left(\frac{1}{n}\right)^{k-1}\right\}$$

Again,

$$xn^k - xn = h \quad \Rightarrow k = \frac{ln\left(n + \frac{h}{x}\right)}{ln(n)}$$

Now, we can express the formula by using limit in case of throwing object vertically upward. We can write, if we throw a object vertically upward with an initial velocity v_0, the final velocity v after overcoming a height h will be

$$v^2 = \lim_{n \to 1^+} v_0{}^2 - 2\frac{GM}{x}\left\{1 - \left(\frac{1}{n}\right)^{\frac{ln\left(n+\frac{h}{x}\right)}{ln(n)} - 1}\right\}$$

Again suppose, we have thrown an object with an initial velocity v_1 from the point A_k to the point A_1 The gravitational acceleration at the point A_k will be

$$g_1 = \frac{GM}{(xn^k)^2}$$

The gravitational acceleration will be same from A_k to A_{k-1} because $\left(xn^k - xn^{k-1}\right) \to 0$. When the object will reach at the point A_{k-1} the final velocity will be v_2. So we can write,

$$v_2{}^2 = v_1{}^2 + 2\frac{GM}{(xn^k)^2}\left(xn^k - xn^{k-1}\right) = v_1{}^2 + 2\frac{GM}{xn^{k+1}}(n-1)$$

Again, The gravitational acceleration at the point A_{k-1} will be

$$g_2 = \frac{GM}{(xn^{k-1})^2}$$

The gravitational acceleration will be same from A_{k-1} to A_{k-2} because $\left(xn^{k-1} - xn^{k-2}\right) \to 0$. When the object will reach at the point A_{k-2} the final velocity will be v_3. So we can write,

$$v_3{}^2 = v_2{}^2 + 2\frac{GM}{(xn^{k-1})^2}(xn^{k-1} - xn^{k-2}) = v_1{}^2 + 2\frac{GM}{xn^{k+1}}(n-1) + 2\frac{GM}{(xn^{k-1})^2}(xn^{k-1} - xn^{k-2})$$

$$= v_1{}^2 + 2\frac{GM(n-1)}{x}\left(\frac{1}{n^{k+1}} + \frac{1}{n^k}\right)$$

Again, when the object will reach at the point A_{k-3} the final velocity will be v_4. By using the above process we can write,

$$v_3{}^2 = v_1{}^2 + 2\frac{GM(n-1)}{x}\left(\frac{1}{n^{k+1}} + \frac{1}{n^k} + \frac{1}{n^{k-1}}\right)$$

We can see, there is a beautiful pattern. When the object will reach at the point A_1 the final velocity will be v_k. According to the pattern we can write,

$$v_k{}^2 = v_1{}^2 + 2\frac{GM(n-1)}{x}\left(\frac{1}{n^{k+1}} + \frac{1}{n^k} + \frac{1}{n^{k-1}} + \cdots + \frac{1}{n^3}\right)$$

$$= v_1{}^2 + 2\frac{GM}{xn^2}\left\{1 - \left(\frac{1}{n}\right)^{k-1}\right\}$$

Now we can write, if we throw a object vertically downward with an initial velocity v_0, the final velocity v after overcoming a distance h will be

$$v^2 = \lim_{n\to 1^+} v_0{}^2 + 2\frac{GM}{xn^2}\left\{1 - \left(\frac{1}{n}\right)^{\frac{ln\left(n+\frac{h}{x}\right)}{ln(n)}-1}\right\}$$

We can use also this method to express the formula by using the limit $n \to 1^-$.

3. CONCLUSION

These two formula is not so much important. We can easily solve these kind of problems by using calculus. But this paper will obviously help us to think about this formula from a new perspective. The interesting part is that we can only apply the above method in case of this formula. This method cannot be applied to express the other formulas by using limit. This makes the expression of the formula unique.

ACKNOWLEDGEMENT

I should thank my parents MD. Shah Alam and MST. Sabina Yesmin for providing me with the help materials. I am very lucky to have a special person in my life who always supports me. So a special thanks to her.

REFERENCES

[1] Wikipedia n. d.Gravitational acceleration, viewed 17 March 2021, https://en.wikipedia.org/wiki /Gravitational_acceleration

AUTHOR'S BIOGRAPHY

This is Salman Mahmud. I am an undergraduate student of Government Azizul Haque College, Bogura, Bangladesh (2021). My core research interest is in modern physics and pure mathematics. But I am interested in all the sectors of science. I have contributed in so many researches. In my leisure period I like to play video games. I believe mathematics is the ornaments of every subject. Email:

Citation: *Salman Mahmud (2021). Expressing a Formula of Throwing Object Vertically Upward or Vertically Downward by Using Limit. International Journal of Advanced Research in Physical Science (IJARPS) 8(3), pp.19-22, 2021.*