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Yukawa Potential in Klein-Gordon Equation in Cosmological Inertial Frame

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Abstract: *We study Yukawa potential dependent about time in cosmological inertial frame. If we solve Klein-Gordon equation, we obtain Yukawa potential dependent about time in cosmological inertial frame.*

Keywords: *Yukawa potential; Klein-Gordon equation; Cosmological inertial frame*

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1. Introduction

Our article's aim is that we make Yukawa potential theory in cosmological inertial frame.

At first, Robertson-Walker metric is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{1}$$

According to ΛCDM model, our universe's k is zero. In this time, if t_0 is cosmological time[3],

$$k = 0, t = t_0 >> \Delta t, \ \Delta t \text{ is period of matter's motion} \tag{2}$$

Hence, the proper time is in cosmological time,

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dr^2 + r^2 d\Omega^2]$$

$$= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2]$$

$$= dt^2(1 - \frac{1}{c^2}\Omega^2(t_0)V^2), \quad V^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2} \tag{3}$$

In this time,

$$d\bar{t} = dt, d\bar{x} = \Omega(t_0)dx, d\bar{y} = \Omega(t_0)dy, d\bar{z} = \Omega(t_0)dz \tag{4}$$

Cosmological special theory of relativity's coordinate transformations are

$$c\bar{t} = ct = \gamma(c\bar{t}' + \frac{v_0}{c}\Omega(t_0)\bar{x}') = \gamma(ct + \frac{v_0}{c}\Omega(t_0)x'\Omega(t_0))$$

$$\bar{x} = x\Omega(t_0) = \gamma(\bar{x}' + v_0\Omega(t_0)\bar{t}') = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\bar{y} = \Omega(t_0)y = \bar{y}' = \Omega(t_0)y', \quad \bar{z} = \Omega(t_0)z = \bar{z}' = \Omega(t_0)z', \quad \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)} \tag{5}$$

2. Yukawa Potential in Klein-Gordon Equation in Cosmological Inertial Frame

If we focus Klein-Gordon equation about Yukawa potential ϕ dependent about time,

$$\frac{m_\pi^2 c^2}{\hbar^2}\phi + \partial_\mu \partial^\mu \phi = \frac{m_\pi^2 c^2}{\hbar^2}\phi + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi - \nabla^2\phi = 0 \tag{6}$$

In this time, Yukawa potential ϕ dependent about time is.

$$\phi = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar}) + A_0 \sin\omega t$$

Frequency $\omega = \frac{m_\pi c^2}{\hbar}$, m_π is meson's mass (7)

Eq(6)-Klein-Gordon equation is satisfied by Eq(7)-Yukawa potential dependent about time

In cosmological inertial frame, Klein-Gordon equation is[2]

$$-\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\phi'}{\partial t^2} + \frac{1}{\Omega(t_0)}\nabla^2\phi' = \frac{m_\pi^2 c^2}{\hbar^2}\phi' \tag{8}$$

In this point, in cosmological inertial frame, space-time transformations in the type A of wave function and the other type B of the expanded distance are

Type A: $r \to r\sqrt{\Omega(t_0)}, t \to \frac{t}{\sqrt{\Omega(t_0)}}$, Type B: $r \to r\Omega(t_0), t \to t$ (9)

Space-time transformation of Yukawa potential ϕ' is depend on Type A

Hence, Yukawa potential ϕ' dependent about time is

$$\phi' = -\frac{g^2}{r\sqrt{\Omega(t_0)}}\exp[-\frac{m_\pi r\sqrt{\Omega(t_0)}c}{\hbar}] + A_0 \sin(\frac{\omega t}{\sqrt{\Omega(t_0)}})$$

Frequency $\omega = \frac{m_\pi c^2}{\hbar}$, m_π is meson's mass (10)

Eq(8)-Klein-Gordon equation is satisfied by Eq(10)-the solution.

3. Conclusion

We solve Klein-Gordon equation in cosmological inertial frame. Hence, we found Yukawa potential dependent time in cosmological inertial frame.

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