International Journal of Advanced Research in Physical Science (IJARPS)
Volume 8, Issue 2, 2021, PP 27-29
ISSN No. (Online) 2349-7882
www.arcjournals.org

Quantization of Klein-Gordon Scalar Field in Cosmological Inertial Frame

Sangwha-Yi*
Department of Math , Taejon University 300-716 , South Korea

Corresponding Author:** ***Sangwha-Yi, *Department of Math , Taejon University 300-716 , South Korea*

Abstract: *In the Cosmological Special Theory of Relativity, we quantized Klein-Gordon scalar field. We treat Lagrangian density and Hamiltonian in quantized Klein-Gordon scalar field in the Cosmological Special Theory of Relativity*

Keywords: *Cosmological inertial frame; Klein-Gordon scalar field; Hamiltonian; Quantization*

PACS Number: *03.30, 41.20*

1. Introduction

Our article's aim is that we make quantization of Klein-Gordon scalar field in Cosmological Special Theory of Relativity (CSTR).

At first, space-time relations are in cosmological special theory of relativity (CSTR).[1]

$$ct = \gamma(ct' + \frac{v_0}{c}\Omega^2(t_0)x') \ , x\Omega(t_0) = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\Omega(t_0)y = \Omega(t_0)y', \quad \Omega(t_0)z = \Omega(t_0)z' \ , \ \gamma = 1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)}, \ t_0 \text{ is cosmological time} \tag{1}$$

Proper time is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2]$$

$$= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)[dx'^2 + dy'^2 + dz'^2], \ t_0 \text{ is cosmological time} \tag{2}$$

Angular frequency-wave number relation is in CSTR.

$$\omega' = \gamma(\omega - v_0\Omega(t_0)k_1), \ k_1' = \gamma(k_1 - \frac{v_0}{c^2}\Omega(t_0)\omega)$$

$$k_2' = k_2, k_3' = k_3, \gamma = 1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)} \tag{3}$$

2. Quantization of Klein-Gordon Scalar Field in CSTR

Lagrangian density of Klein-Gordon scalar field in CSTR,

$$L = -\frac{1}{2}[-(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi + \frac{m_0{}^2c^2}{\hbar^2}\phi^2] \tag{4}$$

Hence, Euler-Lagrange equation is in CSTR,

$$\partial_\mu[\frac{\partial L}{\partial(\partial_\mu\phi)}] - \frac{\partial L}{\partial\phi} = [\Omega(t_0)\frac{1}{c^2}(\frac{\partial}{\partial t})^2 - \frac{1}{\Omega(t_0)}\nabla^2 + \frac{m_0{}^2c^2}{\hbar^2}]\phi = 0) \tag{5}$$

Hamiltonian of Klein-Gordon scalar field is in CSTR,

$$H = \frac{1}{2}[(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi + \frac{m_0{}^2c^2}{\hbar^2}\phi^2] \tag{6}$$

The Klein-Gordon scalar field is divided by positive frequency mode and negative frequency mode.

$$\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x) \tag{7}$$

The positive frequency mode is

$$\phi^{(+)}(x) = \int\frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}a(k)f_k(x) \tag{8}$$

The negative frequency mode is

$$\phi^{(-)}(x) = \int\frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}a^{(+)}(k)f_k(x) \tag{9}$$

In this time, $f_k(x)$ is

$$f_k(x) = \frac{1}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}\exp[i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})] \tag{10}$$

In this time,

$$\frac{\omega_k}{c} = (k^2 + \frac{m_0{}^2c^2}{\hbar^2})^{\frac{1}{2}} \tag{11}$$

Quantization of complex scalar field is in CSTR,

$$\phi(x) = \int\frac{d^3k}{(2\pi)^3 2\omega_k}[a(k)\exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}]$$

$$+\int\frac{d^3k}{(2\pi)^2 2\omega_k}[b^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \tag{12}$$

$$\phi^+(x) = \int\frac{d^3k}{(2\pi)^3 2\omega_k}[b(k)\exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}]$$

$$+\int \frac{d^3k}{(2\pi)^2 2\omega_k}[a^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \quad (13)$$

Hence, Hamiltonian H is in CSTR,

$$H = \int \frac{d^3k}{(2\pi)^3 2\omega_k}[a^+(k)a(x) + b^+(k)b(k)] \quad (14)$$

In this time,

$$[a(k), a^+(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}')$$

$$[b(k), b^+(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}') \quad (15)$$

3. Conclusion

We quantized Klein-Gordon scalar field in CSTR. We treat Lagranian density and Hamiltonian.

References

[1]S.Yi, "Cosmological Special Theory of Relativity", International Journal of Advanced research in Physical Science,**7,**11(2020),pp4-9

[2]S.Yi, "Klein-Gordon Equation and Wave Function in Cosmological Special Theory of Relativity", International Journal of Advanced research in Physical Science,**7,**12(2020),pp4-6

[3]C.Itzykson & J.B.Zuber, Quantum Field Theory, McGraw-Hill, (1980)

[4]S.S.Schweber, An Introduction to Relativistic Quantum Field Theory, Haper and Raw, (1962)

[5]G.Kallen, Quatum Electrodynamics, Springer-Verlag, (1972)

[6]T.D.Lee, Particle Physics and Introduction to Field Theory, Ch. 2 Harwood Academic Publishers, (1981)

[7]L.H.Ryder, Quantum Field Theory, Cambridge University Press, (1985)

[8]J.D.Bjorken and S.D.Drell, Relativistic Quantum Fields, McGraw-Hill Co., New York, (1980)

Citation: *Sangwha-Yi (2021). Quantization of Klein-Gordon Scalar Field in Cosmological Inertial Frame. International Journal of Advanced Research in Physical Science (IJARPS) 8(2), pp.27-29, 2021.*