

Transformation of 4-dimensional Rindler spacetime

Sangwha-Yi*

Department of Math, Taejon University 300-716, South Korea

*Corresponding Author: Sangwha-Yi, Department of Math, Taejon University 300-716, South Korea

Abstract: In special relativity theory, we discover 4-dimensional transformation of general Rindler spacetime from 4-dimensional Lorentz transformation in inertial frames.

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1. INTRODUCTION

In special relativity theory, we discover 4-dimensional transformation of general Rindler space-time from 4-dimensional Lorentz transformation in inertial frames.

At first, 2-dimensional-Lorentz transformation is in inertial frame,

$$ct = \gamma \left(ct' + \frac{v_0}{c} \cdot x' \right),$$

$$x = \gamma (x' + \vec{v}_0 t'), y = y', z = z' \quad \gamma = 1 / \sqrt{1 - \frac{v_0^2}{c^2}} \quad (1)$$

2-dimensional transformation is in Rindler spacetime,

$$ct = \frac{c^2}{a_0} + \xi^1 \sinh \left(\frac{a_0 \xi^0}{c} \right), x = \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh \left(\frac{a_0 \xi^0}{c} \right) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3 \quad (2)$$

2. 4-DIMENSIONAL TRANSFORMATION IN RINDLER SPACETIME

4-dimensional-Lorentz transformation is in inertial frame,

$$ct = \gamma \left(ct' + \frac{\vec{v}_0}{c} \cdot \vec{x}' \right)$$

$$\vec{x} = \vec{x}' + \gamma \vec{v}_0 t' \left(1 - \gamma \right) \frac{\vec{v}_0 \cdot \vec{x}'}{v_0^2} - \vec{v}_0, \quad \gamma = 1 / \sqrt{1 - \frac{v_0^2}{c^2}} \quad (3)$$

4-dimensional-differential Lorentz transformation is in inertial frame,

$$cdt = \gamma \left(cdt' + \frac{\vec{v}_0}{c} \cdot d\vec{x}' \right)$$

$$d\vec{x} = d\vec{x}' + \gamma \vec{v}_0 dt' - (1 - \gamma) \frac{\vec{v}_0 \cdot d\vec{x}'}{v_0^2} \vec{v}_0, \quad \gamma = 1 / \sqrt{1 - \frac{v_0^2}{c^2}} \quad (4)$$

Hence, the proper time is

$$\begin{aligned} d\tau^2 &= dt^2 - \frac{1}{c^2} d\vec{x} \cdot d\vec{x} = dt^2 - \frac{1}{c^2} (dx^2 + dy^2 + dz^2) \\ &= dt'^2 - \frac{1}{c^2} (dx'^2 + dy'^2 + dz'^2) \end{aligned} \quad (5)$$

If we suggest 4-dimensional transformation in Rindler spacetime,

$$\begin{aligned} ct &= \sinh\left(\frac{\vec{a}_0 \xi^0}{c}\right) \frac{c^2}{a_0} \left(1 + \frac{\vec{a}_0}{c^2} \cdot \vec{\xi}\right) \\ \vec{x} &= \vec{\xi} + \frac{c^2}{a_0^2} \cosh\left(\frac{\vec{a}_0 \xi^0}{c}\right) \vec{a}_0 - \left(1 - \cosh\left(\frac{\vec{a}_0 \xi^0}{c}\right)\right) \frac{\vec{a}_0 \cdot \vec{\xi}}{a_0^2} \vec{a}_0 - \frac{c^2}{a_0^2} \vec{a}_0 \end{aligned} \quad (6)$$

Therefore, 4-dimensional-differential transformation is in Rindler spacetime

$$\begin{aligned} cdt &= \cosh\left(\frac{\vec{a}_0 \xi^0}{c}\right) c d\xi^0 \left(1 + \frac{\vec{a}_0}{c^2} \cdot \vec{\xi}\right) + \sinh\left(\frac{\vec{a}_0 \xi^0}{c}\right) \frac{\vec{a}_0}{a_0} d\vec{\xi} \\ d\vec{x} &= d\vec{\xi} + \sinh\left(\frac{\vec{a}_0 \xi^0}{c}\right) \frac{c}{a_0} \vec{a}_0 d\xi^0 + \sinh\left(\frac{\vec{a}_0 \xi^0}{c}\right) \frac{d\xi^0}{c} \frac{\vec{a}_0 \cdot \vec{\xi}}{a_0} \vec{a}_0 \\ &\quad - \left(1 - \cosh\left(\frac{\vec{a}_0 \xi^0}{c}\right)\right) \frac{\vec{a}_0 \cdot d\vec{\xi}}{a_0^2} \vec{a}_0 \end{aligned} \quad (7)$$

Hence, the proper time is[8]

$$\begin{aligned} d\tau^2 &= dt^2 - \frac{1}{c^2} d\vec{x} \cdot d\vec{x} = dt^2 - \frac{1}{c^2} (dx^2 + dy^2 + dz^2) \\ &= \left(1 + \frac{\vec{a}_0 \cdot \vec{\xi}}{c^2}\right)^2 (d\xi^0)^2 - \frac{1}{c^2} ((d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2) \end{aligned} \quad (8)$$

3. CONCLUSION

We know general Rindler coordinate transformation from 4-dimensional Lorentz transformation.

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