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Vibration of Yukawa Potential Dependent Time and Extended Klein-Gordon Equation in Rindler Space-Time

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Abstract: *Atom's nucleus force understand by Yukawa potential independent time. We study Yukawa potential dependent about time. We make Klein-Gordon equation is satisfied by Yukawa potential dependent about time. Yukawa potential satisfy Proca equation or Klein-Gordon equation. If we represent Yukawa potential dependent time in Rindler space-time, this Yukawa potential satisfy the extended Klein-Gordon equation in Rindler space-time. We understand Yukawa force in Rindler space-time.*

Keywords: *Nucleus vibration; Yukawa potential; Klein-Gordon equation Rindler Space-time; Extended Klein-Gordon equation Yukawa force*

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1. Introduction

Atom's nucleus force understand by Yukawa potential. We study Yukawa potential dependent about time. We make Klein-Gordon equation is satisfied by Yukawa potential dependent about time.

At first, Yukawa potential V describes nucleus's combine force in semi-classical method.[7]

$$V=-\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar})$$

g is real number, m_π is the meson's mass (1)

Klein-Gordon equation is satisfied by Yukawa potential V .

$$-\partial_i\partial^i V+\frac{m^2c^2}{\hbar^2}V=-\nabla^2 V+\frac{m_\pi^2c^2}{\hbar^2}V=0$$

$$V=-\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar}) \tag{2}$$

If we focus Klein-Gordon equation make 4-dimential partial differential equation about Yukawa potential ϕ dependent time,

$$\frac{m_\pi^2c^2}{\hbar^2}\phi+\partial_\mu\partial^\mu\phi=\frac{m_\pi^2c^2}{\hbar^2}\phi+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi-\nabla^2\phi=0 \tag{3}$$

In this time, Yukawa potential ϕ dependent time is.

$$\phi=-\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar})+A_0\sin\omega t,\quad \text{Frequency}\ \ \omega=\frac{m_\pi c^2}{\hbar} \tag{4}$$

Absolutely, if we calculate, Eq(3) is satisfied by Eq(4). Yukawa potential ϕ is vibrated about the amplitude A_0, but we know the nuclear strong force doesn't vibrate about time in inertial frame..

2. Yukawa Potential Dependent Time from Extended Klein-Gordon Equation in Rindler-Space-Time

Rindler coordinates are

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0}{c}\xi^0) \quad , \quad x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}$$
$$y = \xi^2, z = \xi^3 \tag{5}$$

If we write Yukawa potential ϕ in inertial frame,

$$\phi = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar}) + A_0 \sin\omega t, \quad \text{Frequency} \quad \omega = \frac{m_\pi c^2}{\hbar} \tag{6}$$

If we rewrite Yukawa potential ϕ_ξ in Rindler space-time,

$$\phi = \phi^1 + \phi^2 = \phi_\xi = \phi_\xi^{\ 1} + \phi_\xi^{\ 2} \tag{7}$$

$$\phi^1 = \phi_\xi^{\ 1} = -\frac{g^2}{\sqrt{x^2 + y^2 + z^2}}\exp(-\frac{m_\pi c}{\hbar}\sqrt{x^2 + y^2 + z^2})$$

$$= -\frac{g^2}{\sqrt{\{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}\}^2 + (\xi^2)^2 + (\xi^3)^2}}\exp[-\frac{m_\pi c}{\hbar}\sqrt{\{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}\}^2 + (\xi^2)^2 + (\xi^3)^2}\,] \tag{8}$$

And,

$$\phi^2 = \phi_\xi^{\ 2} = A_0 \sin\omega t = A_0 \sin[\omega\{(\frac{c}{a_0} + \frac{\xi^1}{c})\sinh(\frac{a_0\xi^0}{c})\}] \tag{9}$$

This Yukawa potential satisfy the extended Klein-Gordon equation. At first, energy and momentum are in Rindler space-time[1],

$$E_\xi = i\hbar\frac{1}{(1 + \frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^0}, \vec{p}_\xi = -i\hbar\vec{\nabla}_\xi \tag{10}$$

Energy-Momentum equation is in Rindler space-time[1],

$$E_\xi^{\ 2} = \vec{p}_\xi c \cdot \vec{p}_\xi c + m^2 c^4 \tag{11}$$

Hence, normal Klein-Gordon equation is in Rindler-spacetime,

$$\frac{m_\pi^{\ 2} c^2}{\hbar^2}\phi_\xi + \frac{1}{c^2}\frac{1}{(1 + \frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2}{(\partial\xi^0)^2}\phi_\xi - \nabla_\xi^{\ 2}\phi_\xi = 0 \tag{12}$$

In this time, we focus the gauge Λ equation in Rindler space-time[1],

$$\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2}{(\partial\xi^0)^2}\Lambda-\nabla_\xi{}^2\Lambda-\frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}=0 \tag{13}$$

Hence, Eq(12) change extended Klein-Gordon equation in Rindler space-time.

Extended Klein-Gordon Equation is in Rindler space-time,

$$\frac{m_\pi{}^2c^2}{\hbar^2}\phi_\xi{}^1+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2\phi_\xi{}^1}{(\partial\xi^0)^2}-\nabla_\xi{}^2\phi_\xi{}^1-\frac{\partial\phi_\xi{}^1}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}=0 \tag{14}$$

And

$$\frac{m_\pi{}^2c^2}{\hbar^2}\phi_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2\phi_\xi{}^2}{(\partial\xi^0)^2}-\nabla_\xi{}^2\phi_\xi{}^2-\frac{\partial\phi_\xi{}^2}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}=0 \tag{15}$$

Hence,

$$\frac{m_\pi{}^2c^2}{\hbar^2}\phi_\xi+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2\phi_\xi}{(\partial\xi^0)^2}-\nabla_\xi{}^2\phi_\xi-\frac{\partial\phi_\xi}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}=0 \tag{16}$$

Eq(8) ,Eq(9), Yukawa potentials $\phi_\xi{}^1,\phi_\xi{}^2$ satisfy Eq(14),Eq(15), extended Klein-Gordon equations in Rindler space-time. Therefore, Eq(7), Yukawa potential ϕ_ξ satisfy Eq(16), extended Klein-Gordon equation in Rindler space-time

Yukawa force $\vec{f}$ is

$$\vec{f}=-\vec{\nabla}\phi=-\frac{g^2}{r^3}[\exp(-\frac{m_\pi rc}{\hbar})](1+\frac{m_\pi rc}{\hbar})\vec{r} \tag{17}$$

In this time, Yukawa force $\vec{f}_\xi$ is Rindler space-time,

$$\vec{f}_\xi=-\vec{\nabla}_\xi\phi_\xi=-\vec{\nabla}_\xi\phi_\xi{}^1-\vec{\nabla}_\xi\phi_\xi{}^2=-\frac{g^2}{r^3}[\exp(-\frac{m_\pi rc}{\hbar})](1+\frac{m_\pi rc}{\hbar})(x\cosh(\frac{a_0\xi^0}{c}),\xi^2,\xi^3)$$

$$-\frac{\omega}{c}A[\cos(\omega t)](\sinh(\frac{a_0\xi^0}{c}),0,0) \tag{18}$$

Hence, according to Yukawa force $\vec{f}_\xi$ in Rindler space-time, the nuclear force strongly acts and vibrates in accelerated frame rather than inertial frame in x-axis.

3. Conclusion

We found Yukawa potential dependent time. Hence, the nuclear strong force vibrates about time in Rindler spacetime. We found Yukawa potential mechanism in Rindler Space-time. We understand nuclear force in Rindler space-time.

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